Workshop Tutorials for Physics

Solutions to MR13: **Relativity**

A. Qualitative Questions:

1. Rebecca and the hat.

a. Light reach the two sensors on Rebecca's hat at the same time in her reference frame, as the light travels equal distances to get to each side of the hat. Hence she will take the hat off.

b. Julia and the other students are moving along with the same reference frame as Rebecca. They see the light reaching the two sides of the hat at the same time, they hear the buzzer buzz, and observe Rebecca taking off the hat.

c. If Rebecca has taken the hat off, and the buzzer has gone off, then this is what Brent will also observe. However in Brent's reference frame the light from the two transmitters *does not* reach the sensors at the same time. Different observers do not disagree on whether or not events happen, but they do disagree on whether they are simultaneous or not.

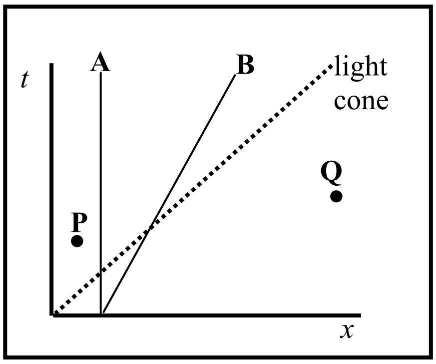
2. It is not possible for there to be a reference frame in which the cat falls off the couch before Brent drops the plate. For an observer to see the cat fall first they (and their reference frame) must be travelling faster than the speed of light, which is not possible. This would also violate causality, which is not possible when the speed of light is finite and the absolute speed limit.

B. Activity Questions:

1. Relativity and electromagnetism

It makes no difference whether it is the coil or the magnet which is moving. A reference frame attached to one is not preferred to a reference frame attached to the other. It was this observation, in electromagnetism, that led Einstein to his theory of relativity.

2. Space-time diagram

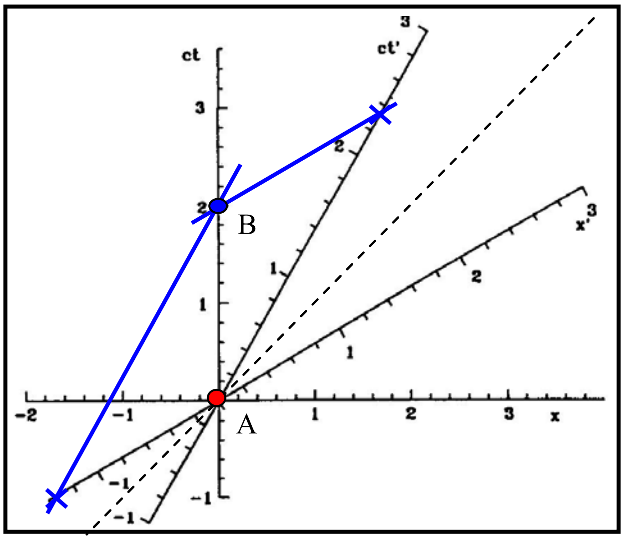


Usually when we draw displacement vs time graphs we plot the time on the horizontal and the position on the vertical axis.

Line **A** represents an object at rest (relative to the reference frame). Its position is not changing in time. Line **B** represents an object whose position is changing in time, hence this is a moving object.

It is not possible for any object to move from point P in space-time to point Q, as to do so it would have to travel faster than the speed of light. No path steeper than the light cone is permitted, and it is impossible to move from the left side of the light cone to the right.

3. Minkowski diagrams



The axes of frame S are the x and ct axes, the axes of frame S' are the x' and ct' axes.

Event A (see diagram) is the spacecraft passing Fox Mulder, at $x = 0$, $t = 0$ in S, and $x' = 0$, $t' = 0$ in S'.

Event B is Fox turning his phone on. This happens at $x = 0$, $t = 6$ ns $= 2$ ct in the S frame. In the S' frame this happens at $x' = -1.4$, $t' = 7.5$ ns. These coordinates are found by drawing lines through the point (point B in this case) parallel to the S' axes and finding the intersections with the x' and ct' axes. Note that the time axis is ct not t, so to find the time we need to divide by c.

C. Quantitative Questions:

1. The proper length of the centipede is 10 cm.

a. Proper length is the length measured in the frame at which the object being measured is at rest. In this case it is the frame of reference of the centipede.

b. If the butcher measures the length of the centipede to be 8 cm, then we can use the length transformation: $l = l_p \frac{1}{\gamma} = l_p \sqrt{1 - v^2/c^2}$ which we can rearrange to get

$v = c\sqrt{1 - l^2/l_p{}^2} = c = \sqrt{1 - 0.08^2/0.10^2} = 0.6c$.

The butcher believes that as the centipede is 8cm long and the cleaver separation is 9 cm, the centipede should be safe. The centipede is not convinced, and is quite worried by the situation, as the cleavers approach at high velocity v.

c. To find the separation of the cleavers from the point of view of the centipede we again use the length transform: $l = l_p \frac{1}{\gamma} = l_p\sqrt{1 - v^2/c^2} = 0.09\sqrt{1 - (0.6c)^2/c^2} = 0.09 \times 0.8 = 7.2$ cm.

This is less than the length of the centipede!

(Note also from this that $1/\gamma = 0.8$, so $\gamma = 1.25$. This will be useful later.)

Let the reference frame of the butcher be frame S, and that of the centipede be frame S*.

Let's consider two events: cleaver A hitting the block and cleaver B hitting the block. These both happen at time t =0 according to the butcher.

d. In the butchers frame these events happen at positions $x_A = 0$ and x_B =9cm.

e. Using the Lorentz transformations event 1 (cleaver A hits the block) happens in the frame of the centipede, S*, at time $t_A{}^* = [t_A - x_A(v/c^2)]\gamma = 0\times \gamma = 0$ s. So cleaver A hits at time 0.

The position is $x_A{}^* = [x_A - vt_A]\gamma = 0\times \gamma = 0$ m. So cleaver A hits at time 0 and position 0 in both frames.

f. Event 2 (cleaver B hits the block) the frame S* at

$t_B{}^* = [t_B - x_B(v/c^2)]\gamma = [0 - 0.09 \text{ m} \times 0.6c/c^2]1.25 = -2.3 \times 10^{-10}$ s.

and $x_B{}^* = [x_B - vt_B]\gamma = [0.09 \text{ m} - 0.6c\times 0]1.25 = 11$ cm.

So in the centipede's frame cleaver B comes down first!

g. The centipede's tail is at 0 when cleaver A comes down in S*, which is at $x = 0$, $t = 0$ in both frames.

h. The centipede's head when cleaver B comes down in S* is at $x^* = 10$ cm, a good 1 cm behind cleaver B in her reference frame.

i. Is it the end for the super speedy centipede? No! The speedy centipede survives!!

2. A spaceship with length is $L_o = 350$ m has a speed of 0.82 c with respect to an observer on Earth, which gives a Lorentz factor of $\gamma = 1.75$.

a. The apparent length of the space ship according to an observer on Earth is $L = L_o/\gamma$, and the time it takes to pass a fixed point at speed v is $L_o/v\gamma$. The time taken for an object moving in the opposite direction at the same speed (the micrometeorite) to pass the space ship is half this time:

$t = \frac{1}{2} L_o/v\gamma = 4.1 \times 10^{-7}$ s = 0.41 μs.

b. For an observer on the space ship the speed of the micrometeorite will be $v_{\text{meteorite}} = \dfrac{2v}{1 + v^2/c^2} = 0.98c$.

c. The time taken for the micrometeorite to pass the space ship according to an observer on the space ship is therefore $t = L_o/v = 1.2$ μs.

d. $KE_{\text{meteorite}} = (\gamma\text{-}1)\, mc^2$ =12 GJ so $m = KE / ((\gamma\text{-}1)\, c^2) = 1.8 \times 10^7$ kg = 0.18 mg.

e. The Lorentz factor of the micrometeorite in the frame of the space ship is 5.1.

The rest energy is mc^2 = 12 GJ/(1.75-1), so the kinetic energy of the micrometeorite according to an observer on the space ship is 12 GJ × (5.1 – 1)/(1.75 – 1) = 66 GJ.

Energy is also relative.