

Workshop Tutorials for Introductory Physics

Solutions to PI2: Buoyancy and Density

A. Review of Basic Ideas:

Fluids, floating bodies and density

Fluids play an important role in our everyday life. We drink them, breathe them, swim in them; they circulate through our bodies, they control our weather, aeroplanes fly through them, ships float in them. The list goes on and on. A fluid is any substance that can **flow**; we use the term for both **liquids** and **gases**. We usually think of a gas as easily compressed and a liquid as nearly incompressible.

The density of any material is defined as its **mass** divided by **volume**. Measuring density is an important analytical technique. For example, we can determine the charge condition of a storage battery by measuring the density of its electrolyte, a sulfuric acid solution. As the battery discharges the density decreases from about $1.30 \times 10^3 \text{ kg.m}^{-3}$ for a fully charged battery to $1.15 \times 10^3 \text{ kg.m}^{-3}$ for a discharged battery.

This measurement is performed routinely in service stations with the aid of a hydrometer, which measures density by observation of the level at which a calibrated body floats in a sample of the solution. The solution exerts an upward force, on the hydrometer, called the **buoyant** force. The calibrated float sinks into the fluid until the weight of the fluid it displaces is **equal** to its own weight which is also equal to the buoyant force. This is Archimedes' principle for floating bodies. The hydrometer floats **higher** in denser liquids than in less dense liquids. It is heavier at its bottom end so that the upright position is stable, and a scale in the top stem permits direct density readings.

B. Activity Questions:

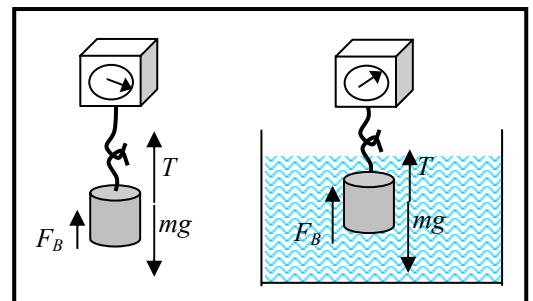
1. Archimedes and the king's crown

The volume of water displaced is equal to the volume, V , of the crown. You can use the scales to measure the mass, m , of the crown. The density of the crown is then $\rho = m/V$. Comparing this density to the density of gold, $19.3 \times 10^3 \text{ kg.m}^{-3}$, the crown is *not* made of gold.

2. Buoyant force

The cylinder will weigh less in water than air because water is more dense than air and hence the buoyant force is greater.

In both cases $F_B + T = mg$, and the scale measures the tension, T . F_B is greater in water, hence T is less.



3. Hydrometers

a. A hydrometer floats higher in denser liquids. The buoyant force, which is equal to the weight of water displaced, must be equal to the weight of the hydrometer if it is to float. A denser liquid needs less water displaced to give the same buoyant force, hence the denser the liquid, the higher the hydrometer floats.

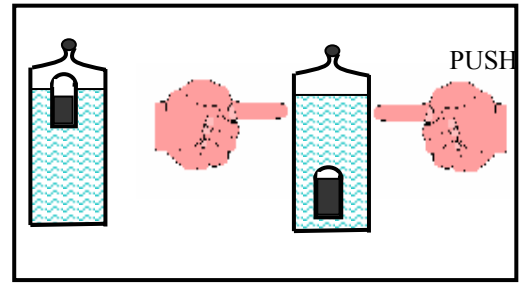
b. The buoyant force is equal to the weight of displaced liquid. The weight of displaced liquid is equal to the weight of the hydrometer, hence the buoyant force will be the same in each case as long as the hydrometers float and do not sink to the bottom.

c. Say you are using a hydrometer that has been designed so that the lowest density it can measure is that of water. This hydrometer sinks in kerosene because the hydrometer can at most displace its own volume of fluid. If this volume has a mass less than that of the hydrometer, ie the fluid is less dense than the hydrometer and it cannot provide a buoyant force great enough to balance the weight of the hydrometer.

d. To measure the density of kerosene using this hydrometer you would need to lower its density. perhaps by removing weights, or adding a block of foam or bubble of some sort.

4. Cartesian Diver

When you push the bottle the pressure you apply is transmitted evenly and without loss to all parts of the fluid. Water is almost incompressible, but air is very compressible, hence the air bubble in the diver is compressed, changing his average density. You should be able to see the bubble get smaller. The more you squeeze, the denser he becomes, and the faster he sinks. When you let go, he decompresses and rises again.



C. Qualitative Questions:

1. How to use Buoyant force to estimate body density and fat content.

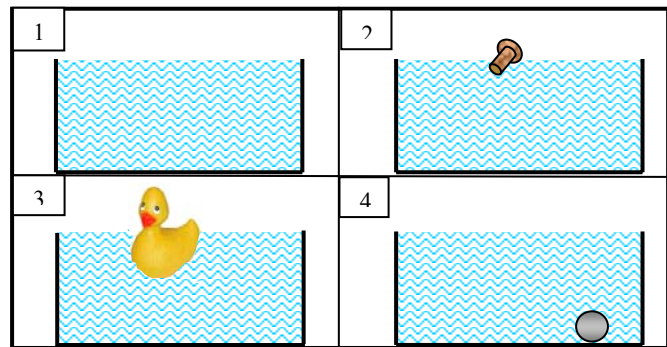
a. The body is weighed in air and then in water. The difference is equal to the buoyant force, which is equal to the weight of water displaced. As long as the body is properly submerged, the volume of displaced water will be equal to the volume of the body. From the weight of displaced water = (body weight in air – body weight in water) and the density of the water, you can find the volume of the displaced water which is the same as the volume of the body (the volume of water displaced = mass of water / density of water) = volume of body. You then know the volume and the mass of the body, from which you can calculate the density using $\rho = m / V$.

b. Air has a very low density, and hence the more air in your lungs, the lower your average density.

c. In general women have a higher fat content than men, approximately 20% : 12%, and fat is less dense than muscle, water or bone, hence women are less dense and float better.

d. Very salty water is more dense than pure water, hence the amount of water displaced is less, and you float higher.

2. Containers 2 and 3 both contain objects which are floating and hence displace a weight of water equal to the objects weight, so the total weight of containers 2 and 3 is the same as container 1 which has only water in it. Container 4 has a steel marble, which has sunk, and displaced a volume of water equal to the marble's volume.



However the marble is more dense than water, and hence weighs more than the volume of water it has displaced. So container 4 weighs more than the other three containers.

D. Quantitative Question:

a. For a floating body, such as an iceberg in the sea, the weight of displaced water is equal to the weight of the iceberg, $W_{\text{water displaced}} = W_{\text{iceberg}}$

$m_{\text{water displaced}} \times g = m_{\text{iceberg}} \times g$, and now using $m = \rho V$:

$\rho_{\text{water}} \times V_{\text{water displaced}} \times g = \rho_{\text{iceberg}} \times V_{\text{iceberg}} \times g$

The volume of water displaced must be equal to the volume of the iceberg which is submerged.

$V_{\text{submerged}} / V_{\text{iceberg}} = \rho_{\text{iceberg}} / \rho_{\text{water}}$

$V_{\text{submerged}} / V_{\text{iceberg}} = 917 \text{ kg.m}^{-3} / 1024 \text{ kg.m}^{-3} = 0.896 = 90\%$

So 90% of the iceberg is submerged.

b. This proportion does not depend on the shape or size of the iceberg, only the density of the ice and the water.

c. Most of the iceberg is underwater, and it may be much longer beneath than above making it difficult to see.

d. If water density increased on freezing, like most other liquids, icebergs would sink and not be a hazard. However this would lead to other problems, like aquatic plants and creatures being frozen.

