

Workshop Tutorials for Introductory Physics

Solutions to TI6: Entropy and the Second Law of Thermodynamics

A. Review of ideas in basic physics:

Entropy and the Second Law of Thermodynamics

If a china mug is dropped on the floor and breaks into many pieces, we accept this as a normal process. If the mug were to put itself back together and jump back into our hand we would consider this a most abnormal process. It is the **Second** Law of Thermodynamics which provides the principle that governs the ordering of events. It can be said that it determines the direction of **time**.

The Second Law of Thermodynamics can be stated in terms of the **entropy** of a system. The concept of entropy is used to describe the degree of order in a system. The number of ways a system can be organized can be used as a measure of its disorder or **randomness**. To illustrate this concept imagine a room full of molecules and then imagine the room divided up into many different equal volume cells. The probability of finding all the molecules in one cell is very **small**. The entropy of that situation has a low value. The probability of finding molecules spread throughout the imaginary cells is much greater and hence the entropy of this situation is higher. In fact if you pushed all the air molecules into a corner of a room and then released them you would expect some time later to find the molecules spread throughout the room – i.e. you would expect the entropy to **increase**.

The Second Law of Thermodynamics can be stated as follows: *In any naturally occurring system the entropy of an isolated system cannot decrease.* The word **isolated** means that there is no energy flow in or out of the system.

A broken mug is less ordered than an intact one. Hence it would be abnormal for the broken pieces of the mug to reassemble themselves into a more ordered state.

B. Activity Questions:

1. Macroscopic states and microscopic states

With two discs there are four possible microstates. B is blue side up, G is green side up. The possible states are BB, BG, GB, GG.

The macroscopic state of half of the discs facing up to be blue and the other half to be green has a probability of $\frac{1}{2}$, as two of the four possible microstates give this macrostate.

With four discs there are $2 \times 2 \times 2 \times 2 = 16$ possible microstates. These are BBBB, BBBG, BBGB, BBGG, BGBB, BGBG, BGGB, BGGG, GBBB, GBBG, GBGB, GBGG, GGBB, GGBG, GGGB, GGGG. The probability of half the discs green and half blue is now $\frac{6}{16} = \frac{3}{8}$. The probability has decreased.

In general, the more possible microstates there are, the less probable a given macrostate becomes. As the number of components increases, so does the possible number of microstates, and so does the entropy of the system.

2. Multiplicity

You have 6 identical “molecules” and a box with two parts.

The possible states, written (X,Y) where X is the number in one side and Y is the number in the other are: (6,0) (5,1) (4,2) (3,3) (2,4) (1,5) (0,6). The multiplicity, W , of a state is the number of different ways in which that state can be achieved. It is equal to $W = \frac{N!}{n_1!n_2!}$. So in this case the multiplicities are: 1, 6, 15, 20, 15, 6, 1. There are $1 + 6 + 15 + 20 + 15 + 6 + 1 = 64$ possible states in total.

The equilibrium condition is the most probable state – in this case the state with 3 molecules in each half of the box. This is also the most disordered state.

C. Qualitative Questions.

1. Plants do not violate the second law of thermodynamics. The plant uses energy from the sun to break down nutrients from the soil, and carbon dioxide from the air. These nutrients are then used to build complex molecules. Even though locally (inside themselves) they increase order by making complex molecules, the overall entropy of the plant and its surroundings increases, as they use the light and produce oxygen.

2. People who wear glasses sometimes walk into a humid place and have their glasses fog up.

a. The glasses fog up because water condenses out from the surrounding air onto the colder surface of the glasses. The water molecules lose energy as they change phase from vapour to liquid. You may also have seen this happen on the side of a cold can or bottle when you take it out of the fridge.

b. As the liquid is a more ordered state than vapour then the condensed water has lost entropy. The change is negative. The change in entropy depends on temperature – while the change in both temperature and entropy is negative for the water, it is positive for the glasses, which absorb heat from the water. The total change in entropy of the universe when the water condenses is still positive or zero.

D. Quantitative question

On a warm evening, $27^{\circ}\text{C} = 300\text{ K}$, Barry, who weighs 28 kg , runs into a tree. Barry bounces off the tree, lands on the ground and comes to a complete stop. Barry had a speed of $8\text{ m}\cdot\text{s}^{-1}$ just before he hit the tree. The change in entropy of the universe due to this collision is given by $\Delta S = W_{\text{lost}}/T$ where T is the temperature and W_{lost} is the work lost due an irreversible process – Barry's collision is an irreversible process – his mechanical energy (kinetic energy) is completely lost as thermal energy, and cannot be changed back into mechanical energy. The work lost due to the collision is the kinetic energy previously had by Barry – $KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 28\text{ kg} \times (8\text{ m}\cdot\text{s}^{-1})^2 = 896\text{ J}$.

The change in entropy of the universe is $\Delta S = W_{\text{lost}}/T = 896\text{ J} / 300\text{ K} = 3.0\text{ J}\cdot\text{K}^{-1}$.