Workshop Tutorials for Introductory Physics
Solutions to WI1: **Simple Harmonic Motion**

A. Review of Basic Ideas:

Periodic Motion

What is periodic motion? The expansion of your lungs as you breath, the pendulum of a grandfather clock, the back-and-forth motion of pistons in a car engine - all these are examples of motion that repeats itself over and over; **periodic** motion or oscillation. If the periodic motion is sinusoidal then it is called **simple harmonic motion.**

A body that undergoes periodic motion always has a stable **equilibrium** position. When it is moved away from this position a **force** pulls it back toward equilibrium. But by the time it gets there, it has picked up **kinetic energy**, so it overshoots, stopping somewhere on the other side, and is again pulled back towards equilibrium. This force is **proportional** to the distance the body has been displaced from its equilibrium position and is written $F=-kx$. The minus sign tells you that the force is in the opposite direction to the displacement, and is directed towards the equilibrium position. The k is the **spring** constant. Two simple examples are spring-mass systems and pendulums.

Oscillatory motion can be described by a **period**, which tells you how long it takes per oscillation, and a **frequency**, which is how many oscillations per second. The size of the oscillations is described by the **amplitude**.

When a mass on a spring oscillates it has **kinetic** energy and elastic **potential** energy and gravitational potential energy. The total energy is the sum of these and is **conserved**. However, we know that if we start a spring oscillating, it will eventually stop, because of **friction**. This is known as **damped** simple harmonic motion. To keep the spring oscillating we need to provide a **driving** force.

Discussion questions

A child on a swing will oscillate back and forth, and if you plotted the child's position as a function of time you would find that it could be described by the equation $x = A\cos\omega t$, where A is the largest displacement from equilibrium and ω is the angular frequency of the motion. Without pushing, if the child just sits on the swing, they will come to a halt after a while. This is because they lose energy due to friction in the swings attachment to the support, and due to air resistance – hence the motion is simple harmonic, and damped. To keep the child swinging a driving force needs to be applied – they need to be pushed, or swing their legs to make themselves move.

B. Activity Questions:

1. Oscillations of a spring-mass system

Two identical objects are attached to identical springs, hence they both have mass m attached to a spring with spring constant k, so the periods are the same and are equal to $T_1 = T_2 = 2\pi\sqrt{\frac{m}{k}}$.

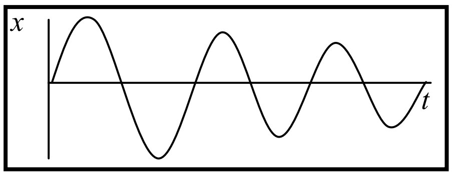
If one of the springs has a bigger mass attached to it, the period will be longer; if $m_2 > m_1$ then $T_2 > T_1$

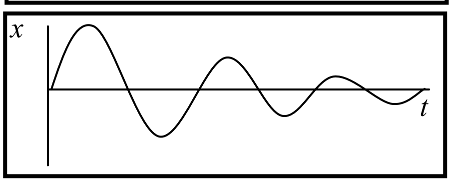
Changing the spring constant also changes the period, if $k_2 > k_1$ then $T_2 < T_1$

For simple harmonic motion the period is independent of amplitude, i.e. extension does *not* affect the period of oscillation.

2. Damped oscillations

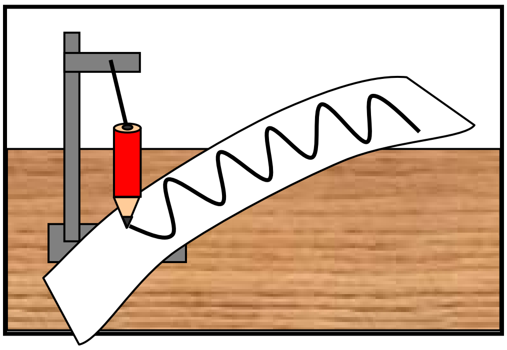
When the object is allowed to oscillate in air it takes a long time to stop, and the amplitude decreases very slowly. See top plot opposite. In water, the motion is strongly damped, and the oscillations decay and stop very quickly, as shown in the lower plot opposite.





Your knee joint is damped, as are all your joints. As you get older the damping usually increases as the joints are less lubricated.

3. Charting pendulum motion



The line drawn is sinusoidal, and can be described by the equation $x = A\cos\omega t$, where A is the initial, and maximum, displacement, ω is the angular frequency of the motion and is equal to $2\pi f$ where f is the frequency of oscillation, t is the time, and x is the displacement at that time t.

C. Qualitative Questions:

1. Imagine you have a spring and you cut it into two pieces, one a third the length of the original spring and one two thirds the length of the original.

a. The more coils a spring has the easier it is to stretch it. The shorter spring, with only half as many coils, will not stretch as much as the longer spring when the same force is applied to it by hanging a mass on it.

b. The spring constant, k, is given by the extension, Δx, for a given applied force F: $F = -k\Delta x$ or $k = F/\Delta x$. For the same force the longer spring has a greater extension, therefore it has a smaller spring constant.

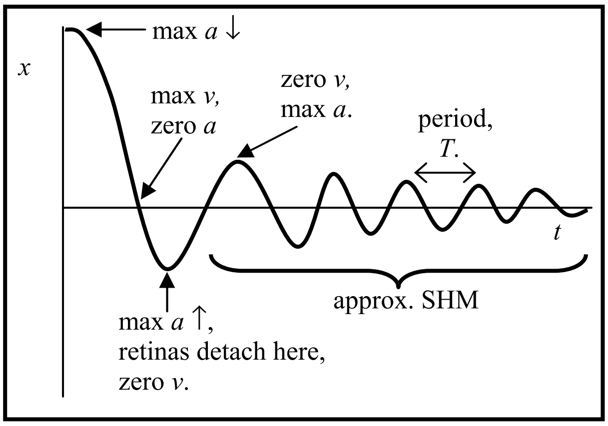
2. Bungy jumping is an increasingly popular sport.

a. See opposite.

b. See opposite, the region which is approximately simple harmonic motion is after the initial jump when you oscillate up and down before being untied.

c. See opposite. The speed is greatest as you pass through the equilibrium position, the acceleration is greatest as the maximum and minimum displacements.

d. You momentarily stop at the top and bottom of each oscillation. At these points your acceleration is maximum, and hence the force you experience will be maximum at these points.



e. When you jump you go head first, and the first minimum is where you are most likely to have your retinas detach, as this is where the acceleration and hence the force will be greatest.

D. Quantitative Question:

A 100 g bungy fish is bobbing up and down with amplitude 6 cm = 0.06 m and frequency 1 Hz. At time $t = 0$, when you first observe the bungy fish, it has a displacement of +0.06 m from its equilibrium position.

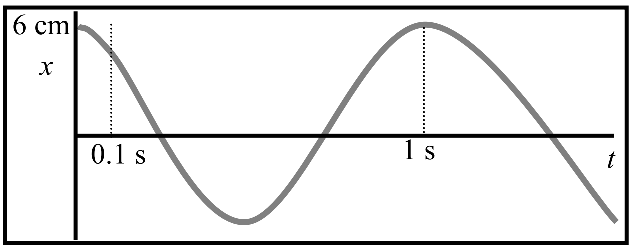
a. The angular frequency of the bungy fish is $\omega = 2\pi f = 2 \times \pi \times 1$ Hz $= 2\pi$ rad.s^{-1}.

b. The formula which describes simple harmonic is $x = A\cos(\omega t + \phi)$, where ϕ is the phase constant, A is the maximum displacement, $\omega = 2\pi f$ where f is the frequency, t is the time, and x is the displacement at time t. For the bungy fish: $x = 0.06$ m cos (2π rad.s$^{-1} \times t$)

When we first observe the bungy fish, at $t = 0$, it has its maximum displacement, $A = 6$ cm. Hence the phase angle is $\phi = 0$.

c. At $t = 1$ s, $x = 0.06$ m cos (2π rad.s$^{-1} \times 1$ s) $= 0.06$ m, back to the maximum.

This makes sense because the frequency is 1 Hz, so the period is 1 s. The bungy fish will have performed one complete oscillation and returned to its starting point at $t = 1$ s.



d. At time $t = 0.1$ s, $x = 0.06$ m cos (2π rad.s$^{-1} \times 0.1$ s) $= 0.049$ m $= 5$ cm.

e. We can find the force constant, k, from the angular frequency using $\omega = \sqrt{\frac{k}{m}}$, which we rearrange to give $k = \omega^2 m = (2\pi \text{ rad.s}^{-1})^2 \times 0.1 \text{ kg} = 3.9 \text{ kg.s}^{-2}$.