## Properties of Matter

## Introductory Properties of Matter Worksheets and Solutions

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# Workshop Tutorials for Introductory Physics <br> PI1: Pressure 

## A. Review of Basic Ideas:

## Use the following words to fill in the blanks:

$9.8 \mathrm{~m} . \mathrm{s}^{-2}$, greater, force, 321 kN , perpendicular, 13.5 kPa , gauge

## Under pressure

If the pressure of the air inside a car tyre is equal to atmospheric pressure, the tyre is flat. The pressure has to be $\qquad$ than atmospheric to keep the tyre firm, and the significant quantity is the pressure difference between the inside and outside. When we say that the pressure in a car tyre is 220 kPa , we mean that it is greater than atmospheric pressure $(101 \mathrm{kPa})$ by this amount. This is called a
$\qquad$ pressure. The total pressure, called the absolute pressure, is 321 kPa . A pressure of 321 kPa acting on a surface of $1.0 \mathrm{~m}^{2}$ will produce a force of $\qquad$ .

The compressed air, inside a car tyre, exerts an outward $\qquad$ on the inner surface of the car tyre. The direction of the outward force is always $\qquad$ to the inner surface of the car tyre. Thus at the top of the tyre the force is upwards and at the bottom it is downwards. This keeps all the surfaces of the car tyre firm.

The pressure difference, $\Delta P$, between two points in a fluid is $\Delta P=\rho g \Delta h$ where $g$ is the acceleration due to gravity, $\rho$ is the density of the fluid and $\Delta h$ is the height difference between the two points. In human beings, there is a difference in pressure between the blood at the feet and the heart. In the reclining position, the head, heart and feet are at the same elevation and the pressures are the same. For a standing adult whose heart is 1.30 m above his feet the pressure difference is:

$$
\begin{aligned}
\Delta P & =\rho_{\text {blood }} g \Delta h \\
& =1060 \mathrm{~kg} \cdot \mathrm{~m}^{-3} \times \ldots \times 1.30 \mathrm{~m}
\end{aligned}
$$

$=1.35 \times 10^{4} \mathrm{~Pa}=$ $\qquad$
So the blood has to be "pumped uphill" from the feet to the heart. This is achieved by one-way valves and the squeezing of veins during walking. Note that this is a first approximation, the actual processes are much more complicated.

## B. Activity Questions:

## 1. Suction cups and Magdeburg plates

How can you make the suction cup stick to a surface?
Explain what happens when it sticks and when it fails to stick.
When are the Magdeburg plates hard to pull apart?
When are the Magdeburg plates easy to pull apart? Explain why.

## 2. Hydrostatic paradox

Water is poured to the same level in each of the vessels shown below, all having the same base area. If the pressure is the same at the bottom of each vessel, the force experienced by the base of each vessel is the same. Why do the vessels have different weights when put on a scale? This apparently contradictory result is commonly known as the hydrostatic paradox. Use the activity to solve this issue.


## 3. Squirting

Use the activity to show that 'a fluid exerts an outward force on the walls of its container'.
Observe the way water 'squirts' out of the holes. What can you say about the direction of the water just as it leaves the holes?
Push a drinking straw into the water and then put your finger over the top. Lift the straw out of the water. What happens? Why? Observe what happens when you undo the lid of the "watering bottle". Explain your observations.

## 4. Hollow tube and disc

Hollow tube and disk: Why does the disk fall away in air but stay attached to the tube when there is air in the tube and water outside the tube?

## C. Qualitative Questions:

1. You are about to set out on a scuba diving trip, and are having a medical check. The doctor measures your blood pressure to be a healthy $120 / 80 \mathrm{mmHg}$. The 120 mmHg is the maximum pressure at the peak of each pulse, called the systole, and the 80 mmHg is the lowest pressure between pulses, called the diastole.
a. Given that normal atmospheric pressure is around 760 mmHg , why does blood spurt from a deep cut? You check the weather report, and it's going to be a fine weekend, with a high pressure front of 102 kPa bringing warm weather. You pack up and head off. You check your tyre pressure when you fill up with petrol, and inflate them to $25 \mathrm{psi}(17.2 \mathrm{kPa})$.
b. Which of the pressures given above are absolute and which are gauge pressures?

You arrive at the diving class and are issued with instructions and equipment.
c. Why does the diving instructor tell you not to hold your breath when surfacing?
d. Why are you issued with lead belts and inflatable packets?
2. The diagram shows a reservoir wall.
a. Why is the wall thicker at the bottom than at the top?
b. Two reservoirs of the same depth are to be joined to form a single much larger reservoir. Is it necessary to reinforce the dam wall?


## D. Quantitative Question:

a.If a giraffe is 5.0 m tall, with his heart at approximately half that height, what pressure does the heart need to produce to keep the brain supplied with oxygen?
b. How does this pressure change when he drinks?
c. Why do giraffes spread their front legs to drink? What would happen if they didn't?


# Workshop Tutorials for Introductory Physics <br> Solutions to PI1: Pressure 

## A. Review of Basic Ideas:

## Under pressure

If the pressure of the air inside a car tyre is equal to atmospheric pressure, the tyre is flat. The pressure has to be greater than atmospheric to keep the tyre firm, and the significant quantity is the pressure difference between the inside and outside. When we say that the pressure in a car tyre is 220 kPa , we mean that it is greater than atmospheric pressure $(101 \mathrm{kPa})$ by this amount This is called a gauge pressure. The total pressure, called the absolute pressure, is 321 kPa . A pressure of 321 kPa acting on a surface of $1.0 \mathrm{~m}^{2}$ will produce a force of $321 \mathbf{k N}$.

The compressed air, inside a car tyre, exerts an outward force on the inner surface of the car tyre. The direction of the outward force is always perpendicular to the inner surface of the car tyre. Thus at the top of the tyre the force is upwards and at the bottom it is downwards. This keeps all the surfaces of the car tyre firm.

The pressure difference, $\Delta P$, between two points in a fluid is $\Delta P=\rho g \Delta h$ where $g$ is the acceleration due to gravity, $\rho$ is the density of the fluid and $\Delta h$ is the height difference between the two points. In human beings, there is a difference in pressure between the blood at the feet and the heart. In the reclining position, the head, heart and feet are at the same elevation and the pressures are the same. For a standing adult whose heart is 1.30 m above his feet the pressure difference is:

$$
\begin{aligned}
& \Delta P=\rho_{\text {blood }} g \Delta h \\
& =1060 \mathrm{~kg} \cdot \mathrm{~m}^{-3} \times \mathbf{9 . 8} \mathbf{~ m .} \cdot \mathbf{s}^{-2} \times 1.30 \mathrm{~m} \\
& =1.35 \times 10^{4} \mathrm{~Pa}=\mathbf{1 3 . 5} \mathbf{~ k P a}
\end{aligned}
$$

So the blood has to be "pumped uphill" from the feet to the heart. This is achieved by one way valves and the squeezing of veins during walking. Note that this is a first approximation, the actual processes are much more complicated.

## B. Activity Questions:

## 1. Suction cups and Magdeburg plates

The suction cup must have the air squeezed out of it and make a complete seal with the surface to stick to it. If the seal isn't complete, air can enter the cup, removing the pressure difference and allowing the cup to fall off.
The Magdeburg plates are hard to pull apart when there is a vacuum between them, but easy to pull apart when there is air. A fluid exerts a force perpendicular to a surface it comes in contact with: $F=P A$. If there is a difference in pressure across a surface this results in a net force which is directed from the region of greater to lower pressure. In the case of the Magdeburg plates, when air is removed from the region between the plates the pressure between the plates is less than the atmospheric pressure outside the plates. This difference in pressure results in a net force inwards, holding the plates together.

## 2. Hydrostatic paradox

The containers have different masses (because they contain different amounts of water), so they must have different weights.
Another argument goes as follows: the pressure is the same at the bottom of each container (because they are filled to the same height). But they all have the same base area, so the force experienced by the base of each container is the same. Therefore, they should all give the same reading on the scale. This second argument is wrong because we have only considered the force of the water on the base of the containers. When calculating the force of the water on the container, we must include the forces on the sides, which may have a component in the vertical direction.

## 3. Squirting

The water will come out perpendicular to the container wall, as this is the direction of the net force.
In each of these activities the liquid is held in by the low pressure in the tube or bottle. When this pressure is increased to atmospheric pressure, by opening the lid or removing the finger, the water will come out.

## 4. Hollow tube and disc

The disc stays attached when there is a pressure difference exerting a force which holds it in place. When the pressure difference decreases such that the force falls below $m g$ of the disc, the disc falls.

## C. Qualitative Questions:

1. Absolute and gauge pressures.
a. Blood pressure is a measure of pressure above atmospheric, it is a relative or gauge pressure.
b. Atmospheric pressure is the only absolute pressure given here, both blood pressure and tyre pressure are gauge pressures, i.e. pressure above atmospheric.
c. You are told not to hold your breath when surfacing because as you get higher, the external pressure from the water decreases. The air in your lungs exerts a pressure outwards on your lungs, while the water outside you exerts an inward pressure. As you rise and the water pressure decreases, the air in your lungs is able to expand. If there is too much air in them pushing outwards, and not enough pressure outside them, they could rupture!
d. You are issued with lead belts and inflatable packets to adjust your buoyancy; lead to make you more dense, allowing you to sink, inflatable packets to make you less dense, allowing you to float.

2. Reservoir walls and water depth.
a. Pressure increases as depth as $P=\rho g h$. Pressure $=$ (force/area) so the wall needs to withstand greater force at the bottom, hence it is built to be thicker at the bottom.
b. Changing the surface area does not change the pressure because it does not change the depth, hence there is no need to further reinforce the wall.

## D. Quantitative Question:

Giraffe's blood pressure.
a. The heart needs to pump blood up by 2.5 m , again using $P=\rho g h$,
$P=\rho g h=1060 \mathrm{~kg} \cdot \mathrm{~m}^{-3} \times 9.8 \mathrm{~m} \cdot \mathrm{~s}^{-2} \times 2.5 \mathrm{~m}=26 \mathrm{kPa}$. This is the minimum pressure the heart must supply to get blood to the brain, in practice it would need to be a bit higher to get it to circulate once there.
b. When the giraffe drinks he will have double this pressure at his head if the heart is still supplying this pressure.
c. If he didn't bend down and thus lower his heart with respect to his head, he'd get a terrible headache (at least) from the high pressure at his head, and possibly burst capillaries. Fortunately the giraffe compensates for the pressure changes by having very tight skin on his legs and strong blood vessels. The heart also adjusts its pressure to suit the giraffe's posture.

# Workshop Tutorials for Introductory Physics <br> PI2: Buoyancy and Density 

## A. Review of Basic Ideas:

## Use the following words to fill in the blanks:

higher, volume, flow, gases, mass, equal, buoyant, liquids.

## Fluids, floating bodies and density

Fluids play an important role in our everyday life. We drink them, breathe them, swim in them; they circulate through our bodies, they control our weather, aeroplanes fly through them, ships float in them. The list goes on and on. A fluid is any substance that can $\qquad$ ; we use the term for both $\qquad$ and $\qquad$ . We usually think of a gas as easily compressed and a liquid as nearly incompressible.

The density of any material is defined as its $\qquad$ divided by $\qquad$ . Measuring density is an important analytical technique. For example, we can determine the charge condition of a storage battery by measuring the density of its electrolyte, a sulfuric acid solution. As the battery discharges the density decreases from about $1.30 \times 10^{3} \mathrm{~kg} . \mathrm{m}^{-3}$ for a fully charged battery to $1.15 \times 10^{3} \mathrm{~kg} . \mathrm{m}^{-3}$ for a discharged battery.

This measurement is performed routinely in service stations with the aid of a hydrometer, which measures density by observation of the level at which a calibrated body floats in a sample of the solution. The solution exerts an upward force, on the hydrometer, called the $\qquad$ force. The calibrated float sinks into the fluid until the weight of the fluid it displaces is $\qquad$ to its own weight which is also equal to the buoyant force. This is Archimedes' principle for floating bodies. The hydrometer floats $\qquad$ in denser liquids than in less dense liquids. It is heavier at its bottom end so that the upright position is stable, and a scale in the top stem permits direct density readings.

## B. Activity Questions:

## 1. Archimedes and the king's crown

Repeat the experiment done by Archimedes. Use the overflow of water to measure the volume of the crown to find its density. A table of densities of different materials is provided.
Do you have a gold crown in your hands? What do you hold in your hands?

## 2. Buoyant force

An object is suspended from a spring balance. Will the reading on the spring balance be different when the object is in air compared to when the object is immersed in water? Draw a diagram showing the forces acting on the object to help explain your answer.

## 3. Hydrometers

There are hydrometers and several liquid samples on the activity benches. Walk over and take a few measurements. Can you identify the samples from the table of densities given below?
a. Why does a hydrometer float higher in denser liquids?
b. Identical hydrometers are placed in three different liquids. They float at different levels. Is the buoyant force on the hydrometers the same or different? Why?
c. Say you are using a hydrometer that has been designed so that the lowest density it can measure is that of water. This hydrometer sinks in kerosene. Why?
d. How would you alter this hydrometer to measure the density of kerosene?

## 4. Cartesian diver

What happens to the diver as you push on the bottle? Why?
What controls the motion of the diver?

## C. Qualitative Questions:

1. When you join a gym you may have a skin fold test done to tell you how much of your body is fat. A more accurate, but less pleasant, means of measuring body composition is via submersion in water. The person is weighed in air and then weighed again when completely submerged in water. (Don't try this at home!)
a. Explain how this process can be used to measure average density.
b. Why is it important to breathe out as much as possible when doing such a test?
c. In general, women float better than men. Why do you think this is the case?
d. Why is it easier to float in very salty water, for example the Dead Sea, than in fresh water.
2. The figure below shows four identical open-top containers. One container has just water. A cork floats in another container and a toy duck floats in the third. The fourth container has a steel marble in it. All four containers are filled to the brim with water. The containers are now placed on separate weighing scales without spillage. How do the readings on the weighing scales compare? Explain your answer.


## D. Quantitative Question:

In February 1995, an iceberg so big the entire Sydney region from the coast to the Blue Mountains could fit on its surface broke free of Antarctica. The iceberg was approximately rectangular with a length of 78 km , a width of 37 km and 200 m thick.
a. What fraction of this iceberg was underwater?
b. Do you actually need the shape and size of the iceberg to determine this fraction?
c. The "unsinkable" Titanic was sunk by an iceberg. Why do icebergs present such a problem for shipping?
d. Would icebergs be a problem if water density increased on freezing, like most other liquids?

## TABLE OF DENSITIES

| Ice | $917 \mathrm{~kg} \cdot \mathrm{~m}^{-3}\left(\mathrm{at} 1 \mathrm{~atm}\right.$ and $\left.0^{\circ} \mathrm{C}\right)$ |
| :--- | :--- |
| Sea water | $1024 \mathrm{~kg} \cdot \mathrm{~m}^{-3}\left(\right.$ at 1 atm and $\left.20^{\circ} \mathrm{C}\right)$ |
| Water | $998 \mathrm{~kg} \cdot \mathrm{~m}^{-3}\left(\right.$ at 1 atm and $\left.20^{\circ} \mathrm{C}\right)$ |

# Workshop Tutorials for Introductory Physics 

## Solutions to PI2: Buoyancy and Density

## A. Review of Basic Ideas:

## Fluids, floating bodies and density

Fluids play an important role in our everyday life. We drink them, breathe them, swim in them; they circulate through our bodies, they control our weather, aeroplanes fly through them, ships float in them. The list goes on and on. A fluid is any substance that can flow; we use the term for both liquids and gases. We usually think of a gas as easily compressed and a liquid as nearly incompressible.

The density of any material is defined as its mass divided by volume. Measuring density is an important analytical technique For example, we can determine the charge condition of a storage battery by measuring the density of its electrolyte, a sulfuric acid solution. As the battery discharges the density decreases from about $1.30 \times 10^{3} \mathrm{~kg} \cdot \mathrm{~m}^{-3}$ for a fully charged battery to $1.15 \times 10^{3} \mathrm{~kg} . \mathrm{m}^{-3}$ for a discharged battery.

This measurement is performed routinely in service stations with the aid of a hydrometer, which measures density by observation of the level at which a calibrated body floats in a sample of the solution. The solution exerts an upward force, on the hydrometer, called the buoyant force. The calibrated float sinks into the fluid until the weight of the fluid it displaces is equal to its own weight which is also equal to the buoyant force. This is Archimedes' principle for floating bodies. The hydrometer floats higher in denser liquids than in less dense liquids. It is heavier at its bottom end so that the upright position is stable, and a scale in the top stem permits direct density readings.

## B. Activity Questions:

## 1. Archimedes and the king's crown

The volume of water displaced is equal to the volume, $V$, of the crown. You can use the scales to measure the mass, $m$, of the crown. The density of the crown is then $\rho=m / V$. Comparing this density to the density of gold, $19.3 \times 10^{3} \mathrm{~kg} . \mathrm{m}^{-3}$, the crown is not made of gold.

## 2. Buoyant force

The cylinder will weigh less in water than air because water is more dense than air and hence the buoyant force is greater.
In both cases $F_{B}+T=m g$, and the scale measures the tension, $T . F_{B}$ is greater in water, hence $T$ is less.

## 3. Hydrometers


a. A hydrometer floats higher in denser liquids. The buoyant force, which is equal to the weight of water displaced, must be equal to the weight of the hydrometer if it is to float. A denser liquid needs less water displaced to give the same buoyant force, hence the denser the liquid, the higher the hydrometer floats.
b. The buoyant force is equal to the weight of displaced liquid. The weight of displaced liquid is equal to the weight of the hydrometer, hence the buoyant force will be the same in each case as long as the hydrometers float and do not sink to the bottom.
c. Say you are using a hydrometer that has been designed so that the lowest density it can measure is that of water. This hydrometer sinks in kerosene because the hydrometer can at most displace its own volume of fluid. If this volume has a mass less than that of the hydrometer, ie the fluid is less dense than the hydrometer and it cannot provide a buoyant force great enough to balance the weight of the hydrometer.
d. To measure the density of kerosene using this hydrometer you would need to lower its density. perhaps by removing weights, or adding a block of foam or bubble of some sort.

## 4. Cartesian Diver

When you push the bottle the pressure you apply is transmitted evenly and without loss to all parts of the fluid. Water is almost incompressible, but air is very compressible, hence the air bubble in the diver is compressed, changing his average density. You should be able to see the bubble get smaller. The more you squeeze, the denser he becomes, and the faster he sinks. When you let go, he decompresses and rises again.


## C. Qualitative Questions:

1. How to use Buoyant force to estimate body density and fat content.
a. The body is weighed in air and then in water. The difference is equal to the buoyant force, which is equal to the weight of water displaced. As long as the body is properly submerged, the volume of displaced water will be equal to the volume of the body. From the weight of displaced water = (body weight in air - body weight in water) and the density of the water, you can find the volume of the displaced water which is the same as the volume of the body (the volume of water displaced = mass of water /density of water) $=$ volume of body. You then know the volume and the mass of the body, from which you can calculate the density using $\rho=m / V$.
b. Air has a very low density, and hence the more air in your lungs, the lower your average density.
c. In general women have a higher fat content than men, approximately $20 \%: 12 \%$, and fat is less dense than muscle, water or bone, hence women are less dense and float better.
d. Very salty water is more dense than pure water, hence the amount of water displaced is less, and you float higher.
2. Containers 2 and 3 both contain objects which are floating and hence displace a weight of water equal to the objects weight, so the total weight of containers 2 and 3 is the same as container 1 which has only water in it. Container 4 has a steel marble, which has sunk, and displaced a volume of water equal to the marble's volume.
However the marble is more dense than water, and hence
 weighs more than the volume of water it has displaced.
So container 4 weighs more than the other three containers.

## D. Quantitative Question:

a. For a floating body, such as an iceberg in the sea, the weight of displaced water is equal to the weight of the iceberg, $W_{\text {water displaced }}=W_{\text {iceberg }}$
$m_{\text {water displaced }} \times g=m_{\text {iceberg }} \times g$, and now using $m=\rho V$ :
$\rho_{\text {water }} \times V_{\text {water displaced }} \times g=\rho_{\text {iceberg }} \times V_{\text {iceberg }} \times g$
The volume of water displaced must be equal to the volume of the iceberg which is submerged.
$V_{\text {submerged }} / V_{\text {iceberg }}=\rho_{\text {iceberg }} / \rho_{\text {water }}$
$V_{\text {submerged }} / V_{\text {iceberg }}=917 \mathrm{~kg} \cdot \mathrm{~m}^{-3} / 1024 \mathrm{~kg} \cdot \mathrm{~m}^{-3}=0.896=90 \%$.
So $90 \%$ of the iceberg is submerged.
b. This proportion does not depend on the shape or size of the iceberg, only the density of the ice and the water.

c. Most of the iceberg is underwater, and it may be much longer beneath than above making it difficult to see.
d. If water density increased on freezing, like most other liquids, icebergs would sink and not be a hazard. However this would lead to other problems, like aquatic plants and creatures being frozen.

# Workshop Tutorials for Introductory Physics <br> PI3: Fluid Flow 

## A. Review of Basic Ideas:

## Use the following words to fill in the blanks:

Flow, increases, heart, lower, incompressible, water, capillaries, high, air, volume, decreases, tangent friction, viscosity, swim, birds

## Fluid flow

By definition, a fluid is a substance which can $\qquad$ As over 70\% of the earth's surface is covered with , and the $\qquad$ above is also a fluid, it is important to understand how fluids flow.
Fluids will flow from a region of $\qquad$ pressure to one of $\qquad$ pressure. This is how we breathe. When we inhale, we increase the volume of our chest. This $\qquad$ the pressure in our lungs, and air flows in. To exhale we allow the chest to collapse back to its smaller volume, which $\qquad$ the pressure inside the lungs. This causes the air to flow out again. When a fluid flows steadily along a pipe, the $\qquad$ rate of flow must be constant along the pipe. This means that if you measure the volume passing through one section of a pipe, as long as there are no holes in the pipe and the fluid is $\qquad$ , the same volume will flow through any other section of the pipe in a given time. This is true even if the pipe gets wider or narrower. In a narrow section the fluid will flow faster, and it will flow more slowly in a wide section.
This is very important in the lungs. The total blood flow going up the pulmonary arteries from the
$\qquad$ to the lungs must be the same as the flow through the capillaries, around the lungs, and back down the pulmonary veins to the heart to be pumped out to the body. In between, millions of tiny each take a tiny flow of blood, and all together give a large flow.
A useful way of visualizing the flow of fluids is with streamlines. A streamline is the path of a particle of the fluid. The velocity of the particle is always $\qquad$ to the streamlines. We can trace streamlines in air by adding a little smoke to the air, or in fluids by adding a few drops of dye. Flow rate also depends on $\qquad$ . Viscosity is a measure of the $\qquad$ between molecules in the fluid, the more friction, the slower it will flow. If there were no frictional forces between the molecules, fish couldn't $\qquad$ , and $\qquad$ couldn't fly.

## Discussion question:

How would rowing be different if water had zero viscosity? Could birds fly if air had zero viscosity?

## B. Activity Questions:

## 1. Hot honey

Honey is a good example of a viscous fluid.
Explain what happens to the density and viscosity when the honey is heated.
Does one change more than the other?

## 2. Chimney effect

Use the air jet to make the polystyrene balls rise up the tube.
Why do they rise? Can you think of an example where this effect would be useful?

## 3. Blowing and lifting

How is it possible to lift the foam block off the table by blowing down a hollow tube onto it?

## 4. Two sheets of paper

What happens if you blow between two sheets of paper held approximately parallel and about 2 cm apart? Can a similar phenomenon occur as two large trucks pass each other on a highway?

## C. Qualitative Questions:

1. Density and viscosity.
a. What is the difference between viscosity and density?
b. Give an example of a fluid which has high density and low viscosity.
c. Give an example of a fluid which has low density and high viscosity.
d. Spiders move by pumping fluid into and out of their legs. Why are spiders slower in winter?
2. Water flows through the pipe shown below from left to right.

a. Rank the volume rate of flow at the four points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$.
b. Rank the velocity of the fluid at the points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$. Explain your answer.
c. Rank the pressure in the fluid at points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$. Explain your answer.

## D. Quantitative Question:

Smoking causes inflammation of the bronchioles, the small air passages in the lungs, which tends to decrease the flow of air into the lungs and hence the oxygen supply to the blood.
Air is flowing down a normal section of a bronchiole with a diameter of 1 mm at a velocity of $0.5 \mathrm{~m} \cdot \mathrm{~s}^{-1}$. Part of the bronchiole is narrowed due to inflammation, and has a diameter of only 0.7 mm .

a. What is the velocity of the air in this section of the bronchiole?
b. What are the consequences of this for gas exchange in the lung?

# Workshop Tutorials for Introductory Physics Solutions to PI3: Fluid Flow 

## A. Review of Basic Ideas:

## Fluid flow

By definition, a fluid is a substance which can flow. As over $70 \%$ of the earth's surface is covered with water, and the air above is also a fluid, it is important to understand how fluids flow.
Fluids will flow from a region of high pressure to one of lower pressure. This is how we breathe.
When we inhale, we increase the volume of our chest. This decreases the pressure in our lungs, and air flows in. To exhale we allow the chest to collapse back to its smaller volume, which increases the pressure inside the lungs. This causes the air to flow out again.

When a fluid flows steadily along a pipe, the volume rate of flow must be constant along the pipe. This means that if you measure the volume passing through one section of a pipe, as long as there are no holes in the pipe and the fluid is incompressible, the same volume will flow through any other section of the pipe in a given time. This is true even if the pipe gets wider or narrower. In a narrow section the fluid will flow faster, and it will flow more slowly in a wide section.

This is very important in the lungs. The total blood flow going up the pulmonary arteries from the heart to the lungs must be the same as the flow through the capillaries, around the lungs, and back down the pulmonary veins to the heart to be pumped out to the body. In between, millions of tiny capillaries each take a tiny flow of blood, and all together give a large flow.

A useful way of visualizing the flow of fluids is with streamlines. A streamline is the path of a particle of the fluid. The velocity of the particle is always tangent to the streamlines. We can trace streamlines in air by adding a little smoke to the air, or in fluids by adding a few drops of dye.

Flow rate also depends on viscosity. Viscosity is a measure of the friction between molecules in the fluid, the more friction, the slower it will flow. If there were no frictional forces between the molecules, fish couldn't swim, and birds couldn't fly.

## Discussion question:

If water had zero viscosity it would not be possible to row, the water would move but not provide any resistance to the oar, and hence you couldn't push a boat along. Birds also rely on viscosity to be able to fly, by pushing the air with their wings. Birds would still be able to glide, but not fly if there was no viscosity.

## B. Activity Questions:

## 1. Hot honey

The mass and volume of the honey change very little when it is heated. However the viscosity changes a lot, the honey goes from flowing very slowly, to very quickly as its viscosity decreases.

## 2. Chimney effect

The air rushing past the top of the chimney has lower pressure than the air in the chimney. There is a net upward force on the air and foam balls in the chimney, and the air and the balls rise. This effect is also used by burrowing animals to ventilate their burrows


## 3. Blowing and lifting

As air rushes through the narrow gap it speeds up and the pressure drops. There is atmospheric pressure under the polystyrene so the pressure difference results in a force up, equal and opposite to the weight of the polystyrene.

## 4. Two sheets of paper

There is reduced pressure between the sheets of paper and they move inwards. Other examples are passing vehicles and the lower end of a shower curtain curling towards the water.

## C. Qualitative Questions:

1. Viscosity and density are different.
a. Viscosity is a measure of how easily a fluid flows, the less easily it flows, the greater the viscosity. Viscosity depends on the frictional forces of the fluids. Density is mass per unit volume, the greater the mass of a given volume, the greater the density.
b. Mercury liquid has high density and low viscosity, it is dense but flows easily.
c. Thickened cream has low density and high viscosity.
d. Spiders move by pumping fluid into and out of their legs, so when it gets colder the fluid becomes more viscous and it is harder for them to pump it in and out and they move slowly.
2. Water flows through the pipe shown below from left to right.

a. Water is incompressible, and as there is no water either entering or leaving between points A and D the volume flow rate must be same at all points, just as current must be the same at all points along an arm of an electrical circuit. This is called the principle of continuity $-A \times v=$ constant.
b. As the water flows from $\mathbf{A}$ to $\mathbf{B}$ the area increases, hence to maintain continuity $v$ must decrease, therefore $v_{\mathbf{A}}>v_{\mathbf{B}}$. As the water then flows downhill to point $\mathbf{C}$ it will gain energy, however the area has not changed so we know, because of continuity, that the velocity has not changed, $v_{\mathbf{B}}=v_{\mathbf{C}}$. When the water flows from $\mathbf{C}$ to $\mathbf{D}$ the area increases again, so the velocity will decrease again, $v_{\mathbf{C}}>v_{\mathbf{D}}$. The ranking is therefore $v_{\mathbf{A}}>v_{\mathbf{B}}=v_{\mathbf{C}}>v_{\mathbf{D}}$.
c. When the water flows from $\mathbf{A}$ to $\mathbf{B}$ there is no change in gravitational potential energy, however the velocity has decreased which means that the kinetic energy of the water has decreased. By conservation of energy we know that if kinetic energy decreases, some other form of energy must increase. If we look at Bernoulli's equation, $\rho g h{ }^{+}{ }_{-} \rho v^{2}+P=$ constant, (which is a statement of conservation of energy density), we can see that the pressure must have increased, $P_{\mathbf{B}}>P_{\mathbf{A}}$. When the water flows downhill from $\mathbf{B}$ to $\mathbf{C}$ it loses gravitational potential energy, but the velocity and hence kinetic energy does not change. The pressure must again increase in going from $\mathbf{B}$ to $\mathbf{C}, P_{\mathbf{C}}>P_{\mathbf{B}}$. Finally, as the water flows from $\mathbf{C}$ to $\mathbf{D}$ the velocity decreases again and the pressure must once more increase, $P_{\mathbf{D}}>P_{\mathbf{C}}$. So the final pressure ranking will be $P_{\mathrm{A}}<P_{\mathbf{B}}<P_{\mathbf{C}}<P_{\mathbf{D}}$

## D. Quantitative Question:

Smoking causes inflammation of the bronchioles. Air is flowing down a normal section of a bronchiole with a diameter of 1 mm at a velocity of $0.5 \mathrm{~m} . \mathrm{s}^{-1}$.
a. Part of the bronchiole is narrowed due to inflammation, and has a diameter of only 0.7 mm . We can use continuity to find the flow rate in this region: $A_{1} v_{1}=A_{2} v_{2}$.
$v_{2}=A_{1} v_{1} / A_{2}=\pi r_{1}{ }^{2} \times v_{1} / \pi r_{2}{ }^{2}=\pi\left(\times 1.0 \times 10^{-3} \mathrm{~m}\right)^{2} \times 0.5 \mathrm{~m} . \mathrm{s}^{-1} / \pi\left(\times 0.7 \times 10^{-3} \mathrm{~m}\right)^{2}=1.0 \mathrm{~m} . \mathrm{s}^{-1}$.
b. If the air moves past the surfaces much faster then the oxygen spends less time in contact with the gas exchange surfaces and is less likely to be absorbed, decreasing the oxygen supply to the body. To compensate for this the lungs and heart have to work harder to get more oxygen and move it around more effectively.

## PI4: Solids I - Stress, Strain and Elasticity

## A. Review of Basic Ideas:

## Use the following words to fill in the blanks:

hydraulic, stress, weaker, strain, elastic, shearing, fluid, Young's, rigid, proportional, ultimate.

## Stress and strain of solids

A $\qquad$ is something which can flow and changes shape to match the container that holds it. A solid does not flow but it can change shape. Many solids seem very $\qquad$ , such as bones and bricks, while others we describe as elastic, like rubber bands and skin.
All solids are to some extent $\qquad$ , in that they will change shape when a force is applied to them. When a rubber band or plank of wood is held secure at one end and pulled on from the other, it will stretch and get longer. The amount it stretches is $\qquad$ to the force applied, for some range of force. We use this linear relationship to define the $\qquad$ modulus, $E$, of a material:
$F / A=E \times \Delta L / L$
where $F$ is the force applied along the length of the object, $A$ is the cross sectional area of the object and $L$ is its length. The applied pressure, $F / A$, is also known as the stress, and the response of the material, the change in length, $\Delta L / L$ is called the $\qquad$ —.
If you apply too great a force, either stretching or compressing, the object will break. The breaking force may be different for stretching and compressing. For example concrete can withstand a large compressive force, but will easily break under a tensile (stretching) force. The pressure at which a material will break is called the $\qquad$ strength.
When forces are applied which are not along the same line, bending or $\qquad$ occurs. Most materials are much $\qquad$ under shearing forces than compressing forces, and it is usually easier to break something by bending it rather than compressing it. We can describe the strength of a material to shearing forces by its shearing modulus, $G$. The shearing modulus is again defined using the $\qquad$ applied and the strain of the material:

$$
F / A=G \times \Delta x / L
$$

where $x$ is the amount the material bends from its original (equilibrium) position.
It is also possible to change the volume of a material by subjecting it to $\qquad$ pressure, by applying a uniform pressure, $P$, all over the surface using a fluid. The change in volume, $V$ depends on the bulk modulus, $B$ of the material, which is defined by

$$
P=B \times \Delta V / V
$$



## B. Activity Questions:

## 1. Shoes

Look at your shoes.
While standing still, what forces are acting on the soles of your shoes?
What sort of deformation would you expect to occur?
When you are walking what forces are acting on the soles of your
shoes? What sort of deformation would you expect to occur?
Draw a diagram showing the forces and resulting deformations.

## 2. Rubber bands

Which rubber band has the largest spring constant?


How could you estimate the elastic modulus of the rubber bands?
Cut a rubber band to form a strip, and hang a weight off it. What would happen if you cut the strip in half and hung a weight from it?
What if you joined two strips together in parallel?

## 3. Breaking chalk

Can you break the chalk by compressing or stretching it?
What about by bending or twisting it?
What is the easiest way to break it and why?

## C. Qualitative Questions:

1. A squash match never begins until the ball is warm, because a cold squash ball bounces about as well as a cold sausage.
a. Why does hitting the squash ball around for a few minutes warm it up?
b. Why does the squash ball bounce better when it is warm?
c. Explain in terms of elasticity why a well inflated basketball bounces better than a flat one?
2. Examine the stress-strain diagram below for bone and tendon.
a. Which is the closer to Hookean?
b. What does the graph tell you about the behaviour of bone and tendon under stress?
c. Which has the larger Young's modulus? Explain your answer.

## D. Quantitative Question:



A tibia (shin bone) is approximately 40 cm long with an average cross sectional area of $3.0 \mathrm{~cm}^{2}$. Bone has a Young's modulus of approximately $1.8 \times 10^{10} \mathrm{Nm}^{-2}$, and an ultimate compression strength of $17 \times 10^{7} \mathrm{~N} . \mathrm{m}^{-2}$.
a. What is the total weight that the legs can support?
b. If these were the legs of an 85 kg man, by how much would the tibias shorten when he gets out of bed in the morning and stands up?
c. Why should you bend your knees when landing from a fall or jump?

## Workshop Tutorials for Introductory Physics

## Solutions to PI4: Solids I - Stress, Strain and Elasticity

## A. Review of Basic Ideas:

## Stress and strain of solids

A fluid is something which can flow and changes shape to match the container that holds it. A solid does not flow but they can change shape. Many solids seem very rigid, such as bones and bricks, while others we describe as elastic, like rubber bands and skin.

All solids are to some extent elastic, in that they will change shape when a force is applied to them. When a rubber band or plank of wood held secure at one end and pulled on from the other, it will stretch and get longer. The amount it stretches is proportional to the force applied, for a range of force. We use this linear relationship to define the Young's modulus, $E$, of a material:
$F / A=E \times \Delta L / L$
where $F$ is the force applied along the length of the object, $A$ is the cross sectional area of the object and $L$ is its length. The applied pressure, $F / A$, is also known as the stress, and the response of the material, the change in length, $\Delta L / L$ is called the strain.

If you apply too great a force, either stretching or compressing, the object will break. This may be different for stretching and compressing. For example concrete can withstand a large compressive force, but will easily break under a tensile (stretching) force. The pressure at which a material will break is called the ultimate strength.

When a force is applied which is not along the length of an object, but is at an angle to it, bending or shearing occurs. Most materials are much weaker under shearing forces than compressing forces, and it is usually easier to break something by bending it rather than compressing it. We can describe the strength of a material to shearing forces by its shearing modulus, $G$. The shearing modulus is again defined using the stress applied and the strain of the material:
$F / A=G \times \Delta x / L$
where $x$ is the amount the material bends from its original (equilibrium) position.
It is also possible to change the volume of a material by subjecting it to hydraulic pressure, by applying a uniform pressure, $P$, all over the surface using a fluid. The change in volume, $V$ depends on the bulk modulus, $B$ of the material, which is defined by
$P=B \times \Delta V / V$.


## 1. Shoes

When standing you exert a compressive force on the soles of the shoes. Most materials are very strong to compressive forces. When walking the shoes are also subject to shearing forces, you apply a force at the upper surface of the sole, and the ground applies a force at the lower surface. The shoes are also subject to bending, and some shoes develop cracks in the soles. It is these shearing and bending forces that wear the shoes out.


## 2. Rubber bands

The rubber band that stretches the least for a given weight (applied force) has the greatest spring constant. If you had rubber bands of the same cross sectional area then the one that stretches the least
also has the greatest elastic modulus. The area is the width times the thickness, so if the thickness is similar the area can be estimated from the width of the band.
If you cut a strip of rubber in half it would stretch less for a given weight, so its spring constant will have decreased, but it will stretch by the same proportion. The modulus of elasticity depends on the material, and will not have changed.
If you joined two rubber strips or bands together in parallel they would also stretch less, but again the modulus of elasticity has not changed. Effectively you will have doubled the spring constant by doubling the cross sectional area.

## 3. Breaking chalk

The chalk is much easier to break by twisting or bending, as it has a greater ultimate compression strength than either torsional or shearing strength. This is also the case for bones. Bending, twisting and stretching the chalk opens up micro-cracks in the material, allowing it to "come apart", compression tends to close these cracks.

## C. Qualitative Questions:

1. Bounciness of balls.
a. When the ball is hit it deforms, and then springs back into shape. There is internal friction as the ball changes shape and this increases the temperature of the ball.
b. As the temperature of the ball increases the air inside it expands and stretches the surface of the ball. On impact a taut surface converts kinetic energy into elastic potential energy, and is able to efficiently convert it back again. A soft surface converts the kinetic energy into a rearrangement of molecules and is unable to reverse the conversion, and thus the kinetic energy is absorbed and the ball doesn't bounce.
c. As above, a taut surface is an efficient energy converter.
2. Bones and tendons.
a. The behaviour of the bone is closer to Hookean, as Hooke's law, $\Delta F=k \Delta x$, predicts a linear response (strain) to an applied force (stress).
b. Bones are more rigid than tendons, and do not stretch as much for a given force, tendons are "stretchier", and will also break at a lower applied stress.
c. Using the relationship: Stress $=$ Young's modulus $\times$ strain, bone has the greater Young's modulus as it has the steeper slope on the stress-strain graph.


## D. Quantitative Question:

a. The maximum weight the legs can support standing vertically is
$17 \times 10^{7} \mathrm{~N} . \mathrm{m}^{-2} \times 3.0 \mathrm{~cm}^{2}=17 \times 10^{7} \times 3.0 \times 10^{-4}=51 \mathrm{k} \mathrm{N}$ or 5100 kg ( 5.1 tonnes!) each.
For two legs this is $100,000 \mathrm{~N}$ or 10 tonnes.
b. We can use $F / A=E \Delta l / l$ where $F$ is the force applied, $A$ is the cross sectional area of the bone, $E$ is the Young's modulus and $l$ is the length of the bone. Rearranging this equation gives:
$\Delta l=F l / A E=85 \mathrm{~kg} \times 9.8 \mathrm{~ms}^{-2} \times 0.40 \mathrm{~m} /\left(2 \times 3.0 \times 10^{-4} \mathrm{~m}^{2}\right) \times 1.8 \times 10^{10} \mathrm{Nm}^{-2}=3.0 \times 10^{-5} \mathrm{~m}$.
c. You should bend your knees when landing from a fall so that the impact is spread over time, de-acceleration is minimised, and the force is less.
d. Most broken bones are due to twisting or bending of bones, not compression. Skiing in particular tends to cause twisting of leg and ankle bones.

# Workshop Tutorials for Introductory Physics <br> PI5: Solids II - Bonding and Crystals 

## A. Review of Basic Ideas:

## Use the following words to fill in the blanks:

sea, hydrogen, lose, strong, opposite, solids, gain, electrostatic, covalent, diamond, salt

## Bonding in solids

Bonds can be divided into two main types, primary or $\qquad$ bonds and secondary or weak bonds. The strong bonds are what holds $\qquad$ together, these are ionic bonds, covalent bonds and metallic bonds. The two types of weak bonds are $\qquad$ bonds and van der Waals bonds.
The three types of primary bonding reflect the ways in which atoms can group together by gaining or losing or sharing electrons.

Atoms near the left or right sides of the periodic table can lose or gain 1 (or 2) electrons to form charged ions. For example, a sodium atom can $\qquad$ one electron and become a positively charged cation. A chlorine atom can $\qquad$ one electron to become a negatively charged anion. These two ions then will be attracted to each other by non-directional $\qquad$ force and form an ionic bond. When large numbers of such ion pairs come together an ionic solid is formed, such as table $\qquad$ , NaCl . In ionic solids there is a charge requirement for stacking atoms. Each ion must have nearest neighbours of
$\qquad$ charge. There are no directional requirements, so stacking depends on meeting charge and size requirements and the bonding can be at any angle. However there are long range requirements because they attract or repel other ions beyond the nearest and next-nearest neighbours.

Atoms at the centre of the periodic table find it difficult to lose or gain electrons and end up sharing. These atoms form $\qquad$ bonds. When large numbers of such atom pairs come together, all sharing some of their electrons, a covalent solid is formed, for example $\qquad$ . In covalent bonding there are no charge requirements - each atom does not have to have nearest neighbours of opposite charge, and there are no long range requirements. The bonds act only between those nearest-neighbour atoms sharing electrons. However there are strong directional requirements which determine structural geometries.

Metals have atoms that release some electrons to be shared by all the atoms of the solid, often referred to as a bed of atoms with a " $\qquad$ of electrons". Metallic bonding occurs between the positive atom cores and the "free" electrons. In metallic bonding there are no charge requirements, or directional requirements, but there are long range effects. This means that the atoms pack together as closely as possible.

## B. Qualitative Questions:

1. The three types of strong bonds that form between atoms are ionic, covalent and metallic bonds.
a. In which regions of the periodic table are the elements that form ionic bonds? Why?
b. Why do covalent solids not have long range interactions, but ionic solids do?
2. The properties of a solid, such as its thermal and electrical conductivity and strength depend on the bonding between the atoms that make up the solid.
a. Why is it that solids which have ionic bonds, like salt, tend to be brittle, but metals are usually quite plastic?
b. Why are metals like copper such good conductors compared to ionic and covalent solids like salt and chalk?

## C. Activity Questions:

## 1. Lenard - Jones potential

The graph shows a plot of potential as a function of inter-atomic distance.
Where is the potential energy positive and where is it negative for this pair of atoms?
At what distance is the force between them repulsive?
At what distance is it attractive? How do you know?

## 2. Crystal structures

Can you identify the basis cell for the crystal structures shown?
What is the coordination number for each lattice?

## 3. Bend and Stretch

What sort of bond holds the atoms together in chalk?
Can you break the chalk by compressing or stretching it?
What about by bending or twisting it? What is the easiest way to break it and why?
Now try to break the piece of metal.
How do the metallic bonds affect its elastic properties?

## D. Quantitative Question:

A face centred cubic crystal has atoms arranged as shown.
Each face of the cubic unit cell has an atom at its middle, and one at each corner.


Modelling the atoms as rigid spheres sitting up against their nearest neighbours, we can draw a single face of the cubic unit cell like this:
a. Write an expression for the atomic radius, $r$, in terms of the cell length, $a$.
b. How many complete atoms are contained in each unit cell?
c. What volume of atoms are contained in each unit cell?
d. What is the volume of the unit cell?
e. What is the packing fraction for the face centred cubic crystal when modeled this way?


# Workshop Tutorials for Introductory Physics Solutions to PI5: Solids II - Bonding and Crystals 

## A. Review of Basic Ideas:

## Bonding in solids

Bonds can be divided into two main types, primary or strong bonds and secondary or weak bonds. The strong bonds are what holds solids together, these are ionic bonds, covalent bonds and metallic bonds. The two types of weak bonds are hydrogen bonds and van der Waals bonds.

The three types of primary bonding reflect the ways in which atoms can group together by gaining or losing or sharing electrons.

Atoms near the left or right sides of the periodic table can lose or gain 1 (or 2 ) electrons to form charged ions. For example, a sodium atom can lose one electron and become a positively charged cation. A chlorine atom can gain one electron to become a negatively charged anion. These two ions then will be attracted to each other by non-directional electrostatic force and form an ionic bond. When large numbers of such ion pairs come together an ionic solid is formed, such as table salt, NaCl . In ionic solids there is a charge requirement for stacking atoms. Each ion must have nearest neighbours of opposite charge. There are no directional requirements, so stacking depends on meeting charge and size requirements and the bonding can be at any angle. However there are long range requirements because they attract or repel other ions beyond the nearest and next-nearest neighbours.

Atoms at the centre of the periodic table find it difficult to lose or gain electrons and end up sharing. These atoms form covalent bonds. When large numbers of such atom pairs come together, all sharing some of their electrons, a covalent solid is formed, for example diamond. In covalent bonding there are no charge requirements - each atom does not have to have nearest neighbours of opposite charge, and there are no long range requirements. The bonds act only between those nearest-neighbour atoms sharing electrons. However there are strong directional requirements which determine structural geometries.

Metals have atoms that release some electrons to be shared by all the atoms of the solid, often referred to as a bed of atoms with a "sea of electrons". Metallic bonding occurs between the positive atom cores and the "free" electrons. In metallic bonding there are no charge requirements, or directional requirements, but there are long range effects. This means that the atoms pack together as closely as possible.

## B. Qualitative Questions:

1. The three types of strong bonds that form between atoms are ionic, covalent and metallic bonds.
a. Elements that form ionic bonds are those which will easily lose or gain an electron. These are those to the far left and right of the periodic table (but not the last column). Those to the left have only one or two electrons in their outer shells, which can easily be removed, and those to the far right have an almost full outer shell and will easily accept an extra electron. (The last column has a full outer shell and will not readily form bonds at all.)
b. In an ionic solid some atoms are negatively charged from gaining electrons while others are positively charged from donating electrons. Charges interact via an electric field, and the strength of interaction decreases with the square of the distance between them, hence all the atoms in an ionic solid interact with all the others (within a reasonable distance), giving long range Coulomb interactions. In a covalent solid the atoms share the electrons, so they are not charged, and there is no long range interaction.
2. The properties of a solid depend on the bonding between the atoms that make up the solid.
a. Metals form bonds by sharing many electrons with many atoms, hence the electrons are approximately free. The metallic bonds have neither charge nor direction restrictions, so the atoms can move easily relative to each other, making metals very malleable. Ionic solids, such as salt, have nearest neighbour and charge restrictions on their bonds, so small movements tend to break down many bonds, making these solids brittle.
b. Metals like copper have free electrons, which can move in response to an electric field, making them good conductors. Ionic and covalent solids have electrons localised and bound so that they are not free to move about, hence they are poor conductors.

## C. Activity Questions:

## 1. Lenard - Jones potential

a.


b. Jee uagram avove rigit. nile iorce is me negative of the gradient of the potential, $F=-$ $d P / d r$, so where the slope is positive the force is negative, ie towards $r=0$. If you look at the graph it's easy to tell which way the force is because the particle will move down hill on the potential plot, in the direction of the force.

## 2. Crystal structures

Some typical cubic crystal structures are shown. The coordination number, CN , is the number of nearest neighbour atoms for each atom.


## 3. Bend and Stretch

Chalk is held together by covalent bonds, so it is brittle. Chalk is strong to compression, but breaks easily when stretched, bent or twisted because the atoms cannot move easily relative to each other. Stretching, bending and twisting the chalk also opens up micro-cracks in the material, which compression tends to close.
The bonds in ionic solids have charge requirements, and those in covalent solids have directional requirements, making them both generally brittle.
Metal is much more plastic, it can be bent without breaking. This is because the metallic bonds have fewer restrictions than ionic or covalent bonds, they have neither charge nor direction requirements.

## D. Quantitative Question:

a. Write an expression for the atomic radius, $r={ }_{\bar{\alpha}}$ the diagonal across the square. By trigonometry we can say that $a^{2}+a^{2}=r^{2}$. We can rearrange this for $r=a / 2 \sqrt{ } 2$.
b. Each unit cell encloses 4 complete atoms, $\quad$ an atom from each face plus $\frac{1}{8}$ from each corner.
c. The volume of atoms contained in each unit cell is
$V_{\text {atoms }}=4 \times \frac{4}{3} \pi r^{3}$.

d. The volume of the unit cell is $a^{3}$.
e. To find the packing fraction we can write the atom volume using $r=a / 2 \sqrt{ } 2$ as
$V_{\text {atoms }}=4 \times \frac{4}{3} \pi\left(\frac{a}{2 \sqrt{2}}\right)^{3}=\frac{16}{3} \pi \frac{1}{16 \sqrt{2}} a^{3}=\frac{\pi}{3 \sqrt{2}} a^{3}=\frac{\pi}{3 \sqrt{2}} V_{\text {cell }}=0.74 V_{\text {cell }}$.
Hence the packing fraction is $74 \%$.

