Quantum, Atomic and Nuclear Physics

Regular Quantum, Atomic and Nuclear Physics Worksheets and Solutions

Workshop Tutorials for Physics

QR1: **Photons**

A. Qualitative Questions:

1. Light is commonly described in terms of brightness and colour. Copy and complete the following table by filling in the quantities in the wave and particle models of light which relate to colour and brightness.

	Wave Model	Particle Model
Brightness		
Colour		

2. Electrons are ejected from a surface when light of a certain frequency is incident upon the surface. What would happen to the maximum kinetic energy of the individual ejected electrons if
a. the intensity of illumination was doubled?
b. the length of time of exposure to light was doubled?
c. the frequency of the light was doubled?
d. the material of the surface was changed?
Explain your answers.

B. Activity Questions:

1. Photoelectric effect
Draw a series of sketches which show how you can observe the photoelectric effect using this apparatus.
Why do you think the 'Photoelectric Effect" is one of the first topics studied in quantum mechanics?
In the photoelectric effect, why does the existence of a cutoff frequency speak in favour of the photon theory and against the wave theory?

2. Wave and particle nature of light 1- interference pattern
Observe the interference pattern produced by the laser light passing through the slits.
Does this experiment show the wave nature or particle nature of light? Explain your answer.

3. Wave and particle nature of light 2- emission spectra
Use the spectroscope to examine the spectral lines of the hydrogen lamp.
Which model of light does this experiment support? Explain your answer.

C. Quantitative Questions:

1. The photoelectric effect was extremely important in the development of quantum physics. It was Einstein's explanation of the photoelectric effect that won him his Nobel prize, and *not* his theory of relativity which led to the famous "$E = mc^2$" equation.

a. Write down the "photoelectric equation" and explain how this is consistent with the principle of conservation of energy.

b. Do you expect all the ejected electrons to have the same kinetic energy? Explain your answer.

Two units for energy are commonly used in physics, the joule (J) and the electron volt (eV). This problem could be solved using either J or eV.

Ultraviolet light illuminates an aluminium surface. Using the data below determine :

c. the kinetic energy of the fastest emitted photoelectrons,

d. the kinetic energy of the slowest emitted photoelectrons,

e. the stopping potential,

f. the cut-off wavelength for aluminium.

Data: $h = 6.63 \times 10^{-34}$ J.s
$c = 3.00 \times 10^{8}$ m.s
$1 \text{ eV} = 1.60 \times 10^{-19}$ J
$\Phi_{Al} = 4.20$ eV
$\rho_{Al} = 2.75 \times 10^{-8}$ Ω.m
$\lambda_{UV} = 200$ nm

2. A caesium surface is illuminated with 600 nm light from a laser.

a. Calculate the energy of the photons emitted from this laser.

b. Given that the laser has a power of 2.00 mW, calculate the number of photons emitted per second.

Photosensitive surfaces are not always very efficient. Suppose the fractional efficiency of a Cs surface is 1.00×10^{-16} (one in every 1.00×10^{16} photons ejects an electron).

c. How many electrons are released per second?

d. Determine the current if every photoelectron takes place in charge flow.

e. Explain the difference, if any, between an electron and a photoelectron and a current and a photocurrent.

Workshop Tutorials for Physics

Solutions to QR1: **Photons**

A. Qualitative Questions:

1. Light as a wave and particle.

	Wave Model	Particle Model
Brightness	square of wave amplitude	number of photons (flux density)
Colour	frequency or wavelength	energy of photons

2. The photoelectric effect.

a. If the intensity of illumination was doubled the maximum kinetic energy would not change as each electron is ejected by a single photon. Increasing intensity changes the number of photons, not their energy, hence the same energy per photoelectron is still available. However the photocurrent, which depends on the number of photons, would increase.

b. If the length of time of exposure to light was doubled the electron kinetic energy would not change. See **a** for explanation.

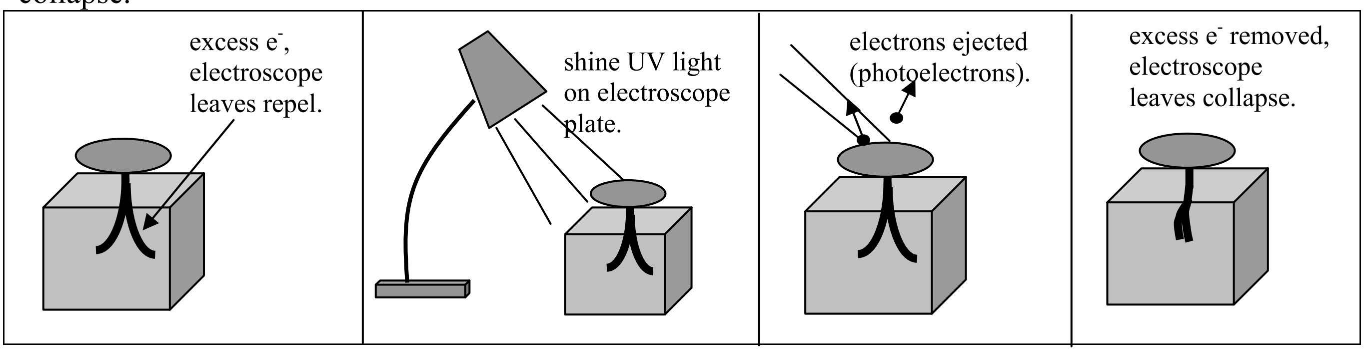
c. If the frequency of the light was doubled then the energy of each photon, $E = hf$, would also be doubled, hence the energy of the ejected electrons would also increase. $K_{max} = hf - \Phi$, if $f \rightarrow 2f$ then the $K_{max} \rightarrow 2hf - \Phi$. (Note that it more than doubles because the work function doesn't change.)

d. If the material of the surface was changed the work function would be different, hence the amount of energy from the photon which becomes kinetic energy would also change. If Φ increases, K decreases and vice versa.

B. Activity Questions:

1. Photoelectric effect

The UV light removes electrons from the negatively charged electroscope, which allows the leaves to collapse.



The Photoelectric Effect is one of the first topics studied in quantum mechanics to introduce experimental evidence of the particle nature of light. This experiment clearly shows the inadequacy of the wave model. The photoelectric effect is dependent on frequency. The wave model predicts that the ejection of electrons will occur at any frequency, given enough intensity. This is not observed. The particle model, which requires that light be absorbed by the electrons in discrete quanta, each with energy hf, accounts for the cut-off frequency. The electron requires at least as much energy as the work function, Φ, to be ejected from the material, hence the lowest frequency which will allow an electron to be ejected is $f_{cut\text{-}off} = \Phi/h$.

2. Wave and particle nature of light 1- interference pattern

This demonstrates the wave nature of light. A particle could only pass through one slit or the other. However, a wave can pass through both slits simultaneously and interfere with itself.

3. Wave and particle nature of light 2- emission spectra

If you accept that the spectral lines result from transitions of electrons from one energy level to another, then the excess energy of an electron when it jumps down from one energy level to another is released as a photon. These lines have discrete colours (frequencies) and correspond to photons of different energies.

C. Quantitative Questions:

1. The photoelectric effect and the photoelectric equation.

a. $hf = \Phi + K_{\max}$

The energy provided by the photon is conserved in the collision, with some being used to overcome the attraction between the electron and the target material, allowing it to escape the material, (the work function) and the remainder being carried off by the electron as kinetic energy. Hence this equation is a statement of conservation of energy.

b. There will be a range of kinetic energies, from zero to K_{max}, as many of the electrons lose some of the energy they have gained from the photon before being ejected, so their kinetic energy is

$K = K_{max} - E_{lost}. = hf - \Phi - E_{lost}.$

These energy losses are usually considered to be due to collisions within the material.

c. using $hf = \Phi + K_{\max}$,

$K_{max} = hf - \Phi$

$= h\,(c/\lambda) - \Phi$

$= 6.63 \times 10^{-34}$ J.s $(3.00 \times 10^{8}$ m.s$^{-1}/200 \times 10^{-9}$ m$) - 4.20$ eV $\times 1.60 \times 10^{-19}$ J.eV^{-1}

$=3.23 \times 10^{-19}$ J or $2.02 \times$ eV

d. $K_{min} = 0$ J. An electron may lose any amount of energy up to $(hf - \Phi)$ and still be ejected. If an electron loses more than this it will not be ejected and the energy will be dissipated as thermal energy (heat) in the material.

e. The stopping potential will be $V_{\text{stop}} = K_{max} / e = 3.23 \times 10^{-19}$ J / 1.60×10^{-19} C = 2.02 V

f. The cut-off wavelength for aluminium is when $hf = \Phi$,

so $\lambda = hc / \Phi$

$= 6.63 \times 10^{-34}$ J.s $\times 3.00 \times 10^{8}$ m.s^{-1} / 4.20 eV $\times 1.60 \times 10^{-19}$ J.eV^{-1}

= 295 nm.

2. A caesium surface is illuminated with 600 nm light from a laser.

a. The energy of the photons emitted from this laser is

$E = hf = hc/\lambda = 6.63 \times 10^{-34}$ J.s $\times 3.00 \times 10^{8}$ m.s^{-1} / 600×10^{-9} m $= 3.31 \times 10^{-19}$ J or 2.07 eV.

b. The laser has a power of 2.00 mW, which is 2.00×10^{-3} J per second. The number of photons emitted per second is therefore 2.00×10^{-3} J.s^{-1} / 3.31×10^{-19} J per photon $= 6.03 \times 10^{15}$ photons.s^{-1}

Photosensitive surfaces are not always efficient. Suppose the fractional efficiency of a Cs surface is 1.00×10^{-16} (one in every 1.00×10^{16} photons ejects an electron).

c. We will get 1.00×10^{-16} electrons per photon, and we have 6.03×10^{15} photons.s^{-1}, so the number of electrons ejected per second is 1.00×10^{-16} electrons per photon $\times$ 6.03×10^{15} photons.s^{-1} = 0.603 electrons per second.

d. If every photoelectron takes place in charge flow, then we have 0.603 electrons per second, which is 0.603 electrons.s^{-1} $\times 1.60 \times 10^{-19}$ C. electron^{-1} $= 9.6 \times 10^{-20}$ C.s^{-1} or 9.6×10^{-20} A.

e. A photoelectron is just an electron which has been ejected from its orbital by a photon, it's exactly the same as any other electron, a photocurrent is a current due to photoelectrons and is the same as the flow of any other electrons.

Workshop Tutorials for Physics

QR2: **Wave Functions I - Particles as Waves**

A. Qualitative Questions:

1. Davisson and Germer accidentally observed electron diffraction in 1927 at Bell laboratories after their vacuum system failed and a sample was exposed to air. The sample oxidized and had to be heated to remove the oxygen. Prior to heating their sample was polycrystalline - it was a single piece of material made up of many tiny crystals. After being heated and allowed to cool the sample formed a single crystal, the atoms were rearranged into regular planes so that constructive interference could occur. When the sample was again exposed to an electron beam they observed maxima and minima at different scattering angles. This discovery led to the invention of the electron microscope.

a. Explain the origin of the maxima and minima observed in the diffraction pattern, and give an expression for their positions in terms of the lattice spacing, d.

b. How would the pattern be different, if at all, if the accelerating voltage used to accelerate the electrons was increased?

c. How would the pattern be different, if at all, if neutrons of the same kinetic energy, rather than electrons, were used? Explain your answer and draw a diagram showing the patterns with electrons and neutrons of the same kinetic energy.

d. Consider two crystal, one with a lattice spacing of d and one with a spacing of $1.5d$. How will the electron diffraction patterns for these two crystals differ? Why?

2. Rebecca and Brent are discussing quantum mechanics over dinner one evening. The subject of Compton scattering comes up, and what a useful technique it is. Brent finds it strange that an X-ray, which is a wave, can impart momentum to an electron. Rebecca explains that "X-rays must have momentum, as they are able to impart some of this momentum to an electron during a collision, therefore X-rays have mass and are particles."

a. Do you agree? Explain why or why not.

"Oh, that's right, X-rays are photons." says Brent. "But don't you find it odd that an X-ray can give up some energy to an electron in Compton scattering, and continue on, but in other circumstances, like atomic transitions, only a whole photon can ever be absorbed or emitted."

"That is odd…" replies Rebecca.

b. Can you solve this mystery for them?

B. Activity Questions:

1. Electron interference

A beam of electrons is directed through two narrowly spaced slits. The emerging beam falls on a sheet of film. These pictures contain clear evidence that the electrons are behaving like ordinary classical particles (tiny billiard balls).

a. State one such piece of evidence in these pictures and explain why that feature suggests that electrons are particles.

These pictures also contain clear evidence that the electrons are behaving like ordinary waves.

b. State one such piece of evidence in these pictures and explain why that feature suggests that electrons are waves.

c. How do physicists describe electrons in order to account for both the observations you have just described?

2. Wave and particle nature of light 1- interference pattern
Observe the interference pattern produced by the laser light passing through the slits.
Does this experiment show the wave nature or particle nature of light? Explain your answer.

3. Wave and particle nature of light 2- emission spectra
Use the spectroscope to examine the spectral lines of the hydrogen lamp.
Which model of light does this experiment support? Explain your answer.

C. Quantitative Questions:

1. Quantum physics tells us that matter has both a wave and particle nature. The wave nature of matter can be described using de Broglie wavelengths.
a. If the following particles all have an energy of 10 keV, which has the shortest wavelength: electron, alpha particle, neutron, proton? Which has the longest wavelength?
In an ordinary colour television set, electrons are accelerated through a potential difference of 25 kV.
b. How much energy does the electron gain as it is accelerated?
c. What is the de Broglie wavelength of such electrons?

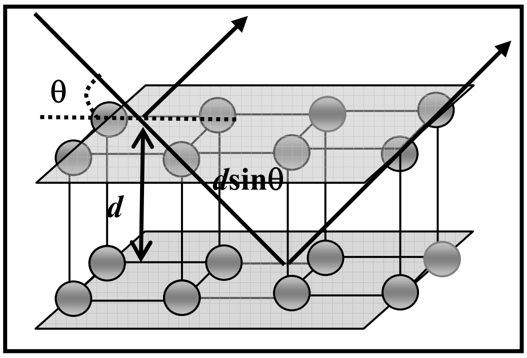
2. In 1923 Compton measured the scattering of X-rays by electrons. Classical wave theory predicts that if an electromagnetic wave of frequency *f* is incident on a material containing charges, the charges will oscillate at the same frequency and reradiate electromagnetic waves of the same frequency. Compton observed that there was a change in frequency, and that the electrons absorbed some energy from the X-rays. He explained this by modeling the interaction between the electron and the photon as a collision.
Prior to colliding with a "stationary" electron, an X-ray has a wavelength of 6.0 pm. The photon collides with an electron head on so that it is scattered at 180^o.
a. What is the wavelength of the scattered photon?
b. What is the difference in energy between the incident and the scattered photons?
c. What is the energy of the scattered electron?

Workshop Tutorials for Physics

Solutions to QR2: **Wave Functions I - Particles as Waves**

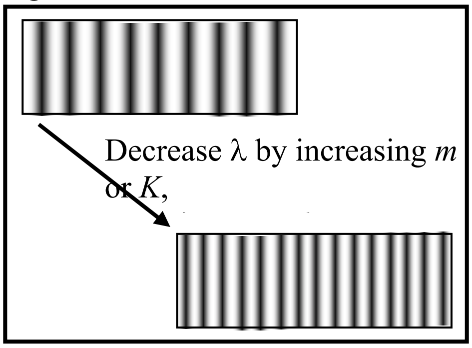
A. Qualitative Questions:

1. Electron diffraction.



a. The electrons are behaving as waves. When they are reflected from different planes of atoms in the crystal there is a path difference between the waves from the different planes. When the path difference, Δl, is equal to an integer number of wavelengths there will be constructive interference, when $\Delta l = n\lambda$ and n = 0, 1, 2…. This corresponds to the condition $n\lambda=2d \sin\theta$, and is known as Bragg's law. When the path difference is equal to $n+ ½\lambda$ there will be complete destructive interference.
The wave function tells us about the probability of the particle being at a particular position, so where there is constructive interference there is a high probability of finding particles, and where there is destructive interference there will be no particles.

b. If the accelerating voltage used to accelerate the electrons was increased then the electrons would have more kinetic energy and hence a greater velocity and greater momentum. The de Broglie wavelength of the particles, $\lambda = h/p$, would be smaller. Using Bragg's law, the angular separation, θ, is proportional to the wavelength, so the diffraction maxima (and minima) will be closer together.



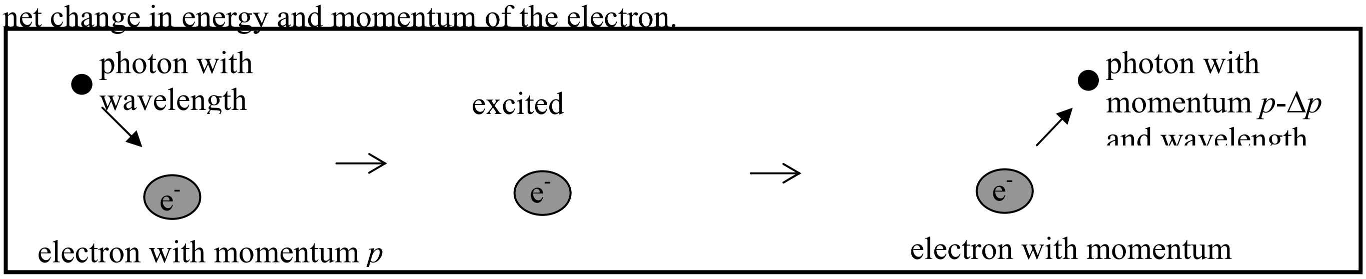
c. Neutrons of kinetic energy, K, will have a de Broglie wavelength of $\lambda = h/p = h/\sqrt{(2m.K)}$, which will be much smaller than the de Broglie wavelength for electrons with energy K, because neutrons have a much greater mass, m. As above, a smaller wavelength gives more closely spaced maxima and minima.

d. If the electrons used for both cases have the same wavelength, the crystal with a lattice spacing of d will give greater separation of diffraction maxima than one with a spacing of 1.5d because the angular separation is inversely proportional to d, i.e. $\theta \propto 1/d$.

2. Mass, momentum and waves.

a. No, Brent should not agree that X-rays have mass. They certainly do not have a rest mass, which is not a problem since photons are never found at rest. Rebecca's argument ignores relativistic considerations. At relativistic speeds the momentum not only depends on the rest mass but on the total energy of the particle. In a nutshell, momentum not only depends on mass, but total energy. As the photons have energy they have momentum, even though they do not have mass. The Compton effect provides experimental evidence of photon momentum, as the target electron gains momentum, which must come from the photon.

b. It is generally true that light is quantised and only a whole photon can be absorbed, not part of a photon. Compton scattering is better modeled as an absorption and re-emission process, than simply a scattering process. The photon is absorbed, and then a second photon is emitted from the electron giving a net change in energy and momentum of the electron.

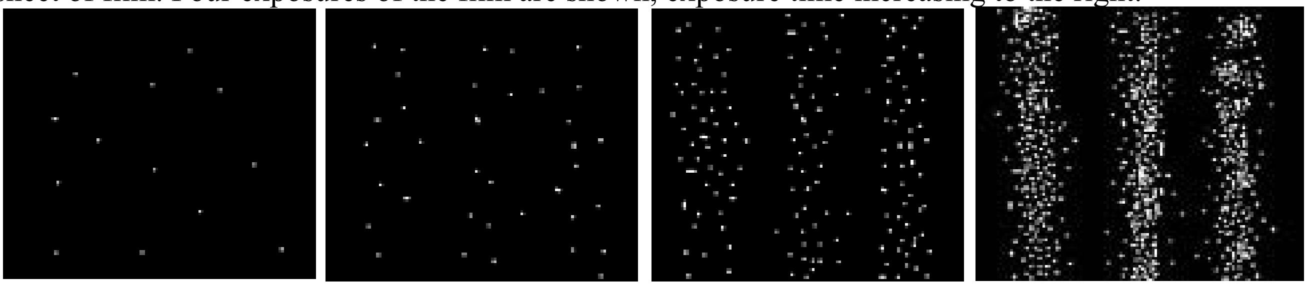


In Compton scattering we treat the process as single scattering event, while in the photoelectric effect no photon is emitted after absorption.

B. Activity Questions:

1. Electron interference

A beam of photons is directed through two narrowly spaced horizontal slits. The emerging beam falls on a sheet of film. Four exposures of the film are shown, exposure time increasing to the right.



a. The pictures are made up of discrete points of light, the electrons are small localised object which are interacting with only a single grain of the film.
b. The later pictures show distinct stripes. Waves passing through twin slits will produce an interference pattern, as is observed here. Hence the electrons are behaving as waves.
c. Quantum mechanics views electrons as both waves and particles. They exhibit particle properties when they interact with matter, and wave properties as they propagate through space, leading to effects such as interference.

2. Wave and particle nature of light 1- interference pattern

This demonstrates the wave nature of light. A particle could only pass through one slit or the other. However, a wave can pass through both slits simultaneously and interfere with itself.

3. Wave and particle nature of light 2- emission spectra

If you accept that the spectral lines result from transitions of electrons from one energy level to another, then the excess energy of an electron when it jumps down from one energy level to another is released as a photon. These lines have discrete colours (frequencies) and correspond to photons of different energies.

C. Quantitative Questions:

1. de Broglie wavelengths.
a. $\lambda = h/p = h/mv = h / \sqrt{(2m.K)}$ If the all the particles all have the same energy, then λ will depend inversely on the square root of the mass. The electron will have the smallest mass and hence the greatest wavelength, the α particle will have the shortest wavelength and the neutron and proton will be in between.
b. The electron, which we assume to have very little kinetic energy initially, is accelerated through 25 kV, hence it will gain 25 keV, or 25×10^3 eV $\times 1.6 \times 10^{-19}$ J.eV^{-1} $= 4 \times 10^{-15}$ J.
c. The de Broglie wavelength of such electrons will be

$$\lambda = h / \sqrt{(2m.K)}$$
$$= 6.63 \times 10^{-34} \text{ J.s} / \sqrt{(2 \times 9.1 \times 10^{-31} \text{ kg} \times 25 \times 10^3 \text{ eV} \times 1.60 \times 10^{-19} \text{ J.eV}^{-1})}$$
$$= 7.8 \times 10^{-12} \text{ m} = 7.8 \text{ pm}.$$

(Note that a 25 keV electron is slightly relativistic and we should really use relativistic mechanics to obtain an accurate answer.)

2. Compton scattering.

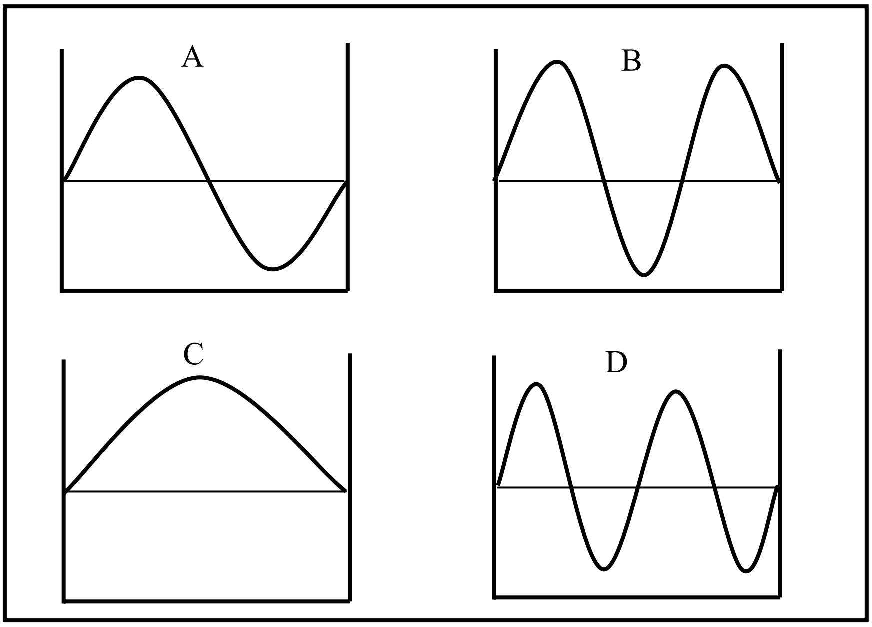
a. $\Delta\lambda = \lambda_2 - \lambda_1 = h(1 - \cos\theta)/m_e c = 2.43 \text{ pm } (1 - \cos 180^o) = 2.43 \text{ pm } (1-(-1)) = 4.86\text{pm}.$
b. Energy of photon $=hc/\lambda$. Difference in energy $E_1 - E_2 = hc/\lambda_1 - hc/\lambda_2$.
$\lambda_2 = \lambda_1 + \Delta\lambda = (6.0 + 4.86) \text{ pm} = 10.86 \text{ pm}.$
So $E_1 - E_2 = 6.63 \times 10^{-34} \text{ J.s} \times 3 \times 10^8 \text{ ms}^{-1}(1/6.0\text{pm} - 1/10.86 \text{ pm}) = 14.8 \times 10^{15}$ J.
c. Since energy is conserved in the collision the kinetic energy of the scattered electron will equal the energy difference above i.e. 14.8×10^{-15}J or 93keV.

Workshop Tutorials for Physics

QR3: **Wave Functions II - Particles in Boxes**

A. Qualitative Questions:

1. Electrons show both wave like and particle like behaviour, and we need to take into account both aspects to understand their behaviour.
Shown below are four wave functions of an electron in an infinite potential well.
a. Rank the wave functions in order of increasing energy of the electron. Justify your answer.



b. Sketch the probability density for the electron in the first excited state.
The electron is now replaced with a proton.
c. Is the proton's zero point energy higher or lower than the electron's?
d. Sketch, to the same scale, the ground state wave functions of the proton and the electron.

2. When an electron is part of an atom it is confined to an orbital. We can model a bound electron as a particle trapped in a potential well.
a. How does confinement of a particle, such as an electron, account for discrete energy levels for that particle?
b. Why is it not possible for the ground state energy of a confined electron to be zero?

B. Activity Questions:

1. Potential Wells and Wave functions
Examine the drawings of the wave functions for particles in potential wells.
a. What do the axes represent?
b. What does the wave function represent?

2. Classical particle in an elastic potential energy well

Send the glider along the air track and allow it to bounce off the spring at the end.

a. Sketch the elastic potential energy of the system (glider and track, including springs) as a function of glider position.

Allow the glider to bounce back and forth.

b. Where does it spend most of its time?

c. Sketch the probability of finding the glider at a position on the track as a function of position.

d. How does this compare to the probability density for an electron trapped in a potential well?

3. Waves on a string

Why are only certain wavelengths of the standing wave possible?

a. Discuss the terms “trapped inside an atom” and “electron in a potential well”. What is the “well”?

b. Use your answers to part **a** to build a simple quantum model of an electron in an atom. (Your model should not be a simple analogy to the planets orbiting the sun in the solar system.)

c. What is the role of standing waves in your model? Why does the existence of standing waves require quantisation of energy?

C. Quantitative Questions:

1. A pollen grain of mass 2.0 mg moves back and forth under a microscope between two glass slides. The slides are separated by 0.05mm and the pollen grain moves so slowly that it takes 90s to move from one slide to the other. Think of this motion as that of a quantum particle trapped in a one dimensional infinite potential well.

a. What energy quantum number (n) describes its motion?

Quantum mechanics says that the wave function describing the motion will be positive at some points and negative at others. Furthermore, if n is an even number (n = 2,4,6…), the wave function will be negative as many times as it is positive.

b. It could be argued therefore that that the average probability of being able to see the pollen grain with the microscope at any point is zero. Is this argument correct? (yes or no only)

c. If you answered yes, explain why your answer is apparently in contradiction to the classical result that the pollen grain must be seen somewhere. If you answered no, explain why you think this apparently straight forward argument is wrong.

2. One of the puzzles of early models of atomic structure was why the electrons didn’t simply go into the nucleus, to which they are attracted by electrostatic (Coulomb) force.

a. Calculate the smallest allowed energy of an electron were it trapped inside an atomic nucleus (diameter about 1.4×10^{-14}m).

b. Calculate the smallest allowed energy of a proton were it trapped inside an atomic nucleus.

c. Comparing these energies, should we expect to find electrons inside nuclei?

Workshop Tutorials for Physics

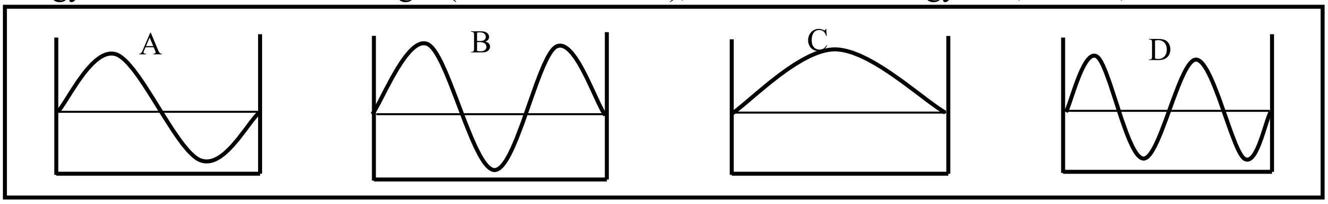
Solutions to QR3: **Wave Functions II - Particles in Boxes**

A. Qualitative Questions:

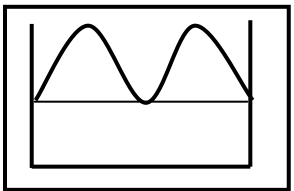
1. Shown below are four wave functions of an electron in an infinite potential well.

a. In order of increasing energy of the electron: C, A, B and D.

The wave function goes like $\psi = A \sin(n\pi/Lx)$, so C is the ground state with $n = 1$, A has $n = 2$ etc. The energy increases with increasing n (it increases as n^2), so the lowest energy is C, then A, B and D.



b. Diagram A above shows the first excited state, the probability density, shown opposite, is the square of this wave function

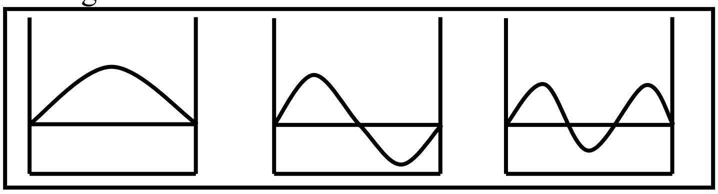


The electron is now replaced with a proton.

c. The energy is given by $E_1 = h^2 / 8mL^2$ where L is the length of the well and m is the mass of the trapped particle.

The proton's zero point energy will be around 2000 times lower than the electron's, because a proton has mass 1.7×10^{-27} kg and an electron has mass 9.1×10^{-31} kg.

d. The wave function will be the same for both, $\psi = A \sin(n\pi x/L)$, which does *not* depend on energy or mass. The wave functions for the first three energy levels are shown.



2. Confinement of particles.

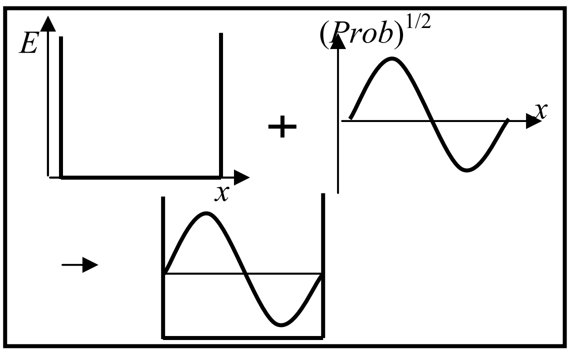
a. When a particle is confined to an infinite well (for example our simple model of an electron trapped in the electric field due to a nucleus) then the probability of finding the particle outside the well must be zero. In our quantum mechanics model a wave is used to represent the probability of finding the particle at a given place. The wave must be zero, i.e. have nodes, at the walls of the well (and be zero outside the well). A standing wave pattern, such as can be observed on a plucked guitar string, fits this model. Since there can only be certain modes of vibration for the standing wave, this results in certain fixed energy levels (discrete rather than continuous) for the trapped particle.

b. The total energy of a confined electron is $E_e = n^2h^2 / 8mL^2$ and cannot be zero unless n is zero. If n is zero then the wave function, $\psi = A \sin(n\pi x/L) = 0$ would be zero, if the wave function is zero then the probability density (which is the square of the wave function) must also be zero. Hence there is zero probability of finding an electron in a potential well with zero energy!

B. Activity Questions:

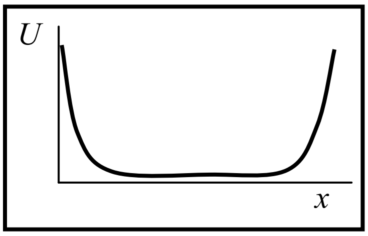
1. Potential Wells and Wave functions

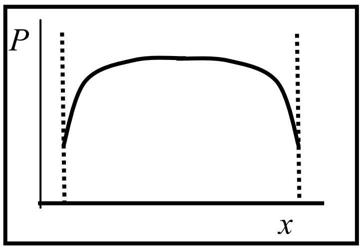
a. The horizontal axis represents distance. The vertical axis represents potential energy, overlaid on top of this is the wave function. This is really two diagrams in one, a potential well and a wave function on top of that.



b. The wave function represents a standing wave in the potential well. An electron in a potential well can be modelled as a standing wave, and the square of the wave function at a given point gives the probability of finding the electron at that point.

2. Classical particle in an elastic potential energy well





a. The elastic potential energy, U, of a spring-mass system is proportional to $(\Delta l)^2$ where Δl is the compression (or extension) of the spring away from its equilibrium length. The spring is compressed by the glider, and is otherwise at equilibrium. The potential energy as a function of glider position, x, is therefore zero except where the glider compresses the springs at either end of the track, at which positions it is proportional to $(\Delta l)^2$.
b. For a given interval in space, the glider spends more time near the ends of the track as it must be slowed down and change direction, then be accelerated away again at the track ends. The surface is approximately frictionless so the glider travels at constant speed once it is no longer in contact with the end springs.
c. The probability density, P, is shown opposite, the dotted lines show the ends of the air track.
d. The probability density for an electron in a well is the opposite to this for the electron's ground state, and in general is a minimum (zero) at the edges of the well.

3. Waves on a string
a. When an electron is bound to an atom it behaves as if it is trapped in a potential well. There is a coulomb attraction between the nucleus and the electron, and the electron will have lower potential energy closer to the nucleus. The electron becomes trapped in the well and is bound to the atom. However it is not an infinite potential well because it is possible to give an electron enough energy to free itself from a nucleus (ionisation), thus exiting the potential well.
b. The electrons can be modelled as standing waves in a potential well. The well is curved (Coulombic) rather than square and the size of the well gives the size of the orbitals. This is analogous to being trapped in a gravitation potential well, for example falling into a hole.
c. The electron is both a particle and a wave, and for it to fit into the potential well it must have the right wavelength as a standing wave. This restricts the possible energies of the electrons to discrete values, hence the energy is quantised.

C. Quantitative Questions:
1. A pollen grain of mass 2.0 mg moves back and forth under a microscope between two glass slides.
a. The pollen grain travels 0.05×10^{-3} m in 90 s, hence it has a speed of
$v = d/t = 0.05 \times 10^{-3} /90 = 5.6 \times 10^{-7}$ m.s^{-1}
It's kinetic energy is therefore $K = ½\, mv^2 = ½ \times 2.0 \times 10^{-6}$ kg $\times (5.6 \times 10^{-7}$ m.s$^{-1})^2 = 3.1 \times 10^{-19}$ J.
This energy is equal to $E = h^2n^2/8mL^2$ which we can rearrange to solve for n:
$n = \sqrt{(E\times 8m)}L / h = \sqrt{(3.1 \times 10^{-19} \text{ J} \times 8 \times 2.0 \times 10^{-6} \text{ kg})}\; 0.05 \times 10^{-3} \text{ m} / 6.63 \times 10^{-34} \text{ J.s} = 1.7 \times 10^{17}$.
b. No. The average probability of being able to see the pollen grain with the microscope at any point is NOT zero.
c. The probability is proportional to the wave function squared, and hence must always be positive. The total probability is usually set to 1 (normalised), as the grain must be seen somewhere.

2. A particle which is confined has quantised energy.
a. using $E_n = \dfrac{h^2 n^2}{8\, mL^2}$
with $L = 1.4 \times 10^{-14}$m and $m = m_e$, we get $E_1 = 3.07 \times 10^{-10}$ J = 1920 MeV.
b. Repeat using same L, but $m = m_p$, where $m_p \sim 2000\, m_e$, which gives $E_1 \sim 1$ MeV.
c. Given that it takes ~2000 MeV to bind an electron into the nucleus, compared to ~1 MeV for a proton, we would not expect to find electrons in the nucleus. (Although an electron can *come out* of the nucleus when a neutron transforms into a proton and an electron, this is called β^- decay.)

Workshop Tutorials for Physics

QR4: The Uncertainty Principle

A. Qualitative Questions:

1. The idea of uncertainty is used in many contexts; social, economic and scientific. People often talk about uncertain times, and when you perform a measurement you should always estimate the uncertainty (sometimes called the error). In physics the Heisenberg Uncertainty relation has a very specific meaning.

a. Write down the Heisenberg uncertainty relation for position and momentum.

b. Explain its physical significance.

c. Does the Heisenberg uncertainty principle need to be considered when calculating the uncertainties in a typical first year physics experiment? Why or why not?

d. Discuss the following statement: the uncertainty principle places a limit on the accuracy with which a measurement can be made. Do you agree or disagree, and why?

2. Brent and Rebecca are discussing quantum mechanics over dinner one evening. Rebecca says that the thing she really likes about quantum mechanics is that it brings the romance and passion back to physics. After all, a deterministic or clockwork universe in which everything is predictable is not as exciting as one in which the uncertainty principle applies. Brent strongly disagrees with her, and explains that "The uncertainty principle is merely a result of the fact that when you make a measurement of a particle's position or momentum, you interact with it, thus changing its state."

"Ah," says Rebecca, "but Heisenberg's original statement of the uncertainty principle translates more accurately as the indeterminacy principle. You should read his work in the original German. It doesn't matter how little the measurement effects the particle, the quantum world is still fundamentally unpredictable."

Who do you agree with? Discuss your answer.

B. Activity Questions:

1. Measuring momentum and position I

When the marble is released from the top of the slide it rolls down, gathering momentum as it falls. As it leaves the end of the slide it has horizontal velocity *v*. It is accelerated vertically due to gravity, and hits the floor at a time $t = (\frac{2h}{g})^{1/2}$ after it leaves the end of the slide, where *h* is the height of the end of the slide above floor level.

Use the apparatus to find the horizontal momentum of the marble during its flight.

How has your measurement affected the position and momentum of the marble?

Do you know both position and momentum simultaneously? If so, does this contradict the uncertainty principle?

2. Measuring momentum and position II

Use the apparatus to measure the time at which the marble passes a given point.

What effect has your measurement had on the marble's position and momentum?

Do you need to take the uncertainty principle into account in this experiment?

How would it be different if you were measuring the momentum of an electron using this sort of apparatus?

C. Quantitative Questions:

1. Brent is measuring the velocity of a cricket ball as part of a laboratory exercise on uncertainties. In the meantime his lab partner, Rebecca, has been messing with the structure of space-time and much to her surprise opened a wormhole into another universe. Brent and the cricket ball are both sucked into the wormhole and disappear into the other universe where Planck's constant has a value of 0.6 J.s.

Brent fails to notice this and continues with the experiment. He measures the velocity of the 0.50 kg ball to be 20.0 ± 1.0 m.s$^{-1.}$

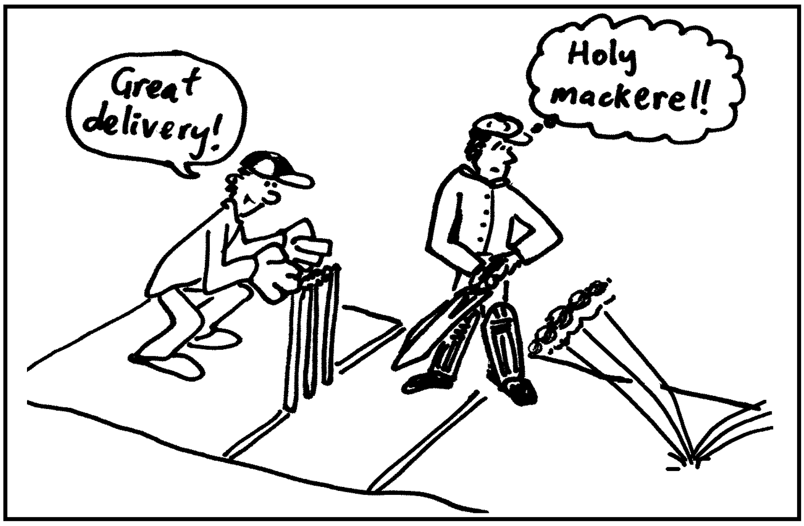
a. Explain qualitatively how the limits on the uncertainty in the ball's velocity and position will be different in the two universes.

b. What would be the uncertainty in the position of the moving cricket ball in the second universe?

Imagine playing cricket in this universe.

c. What would it be like trying to catch a ball?

d. Calculate the de Broglie wavelength for this ball. What sort of effects might you observe?



2. The x, y and z components of the velocity of an electron are measured to be :

$v_x = (4.00 \pm 0.18) \times 10^5$ m.s^{-1} ,

$v_y = (0.34 \pm 0.12) \times 10^5$ m.s^{-1} and

$v_z = (1.41 \pm 0.08) \times 10^5$ m.s^{-1}

a. Find the uncertainties in the x, y, z components of the momentum, p.

b. The measurements described above are consistent with the electron being localised in some volume. What is the smallest volume possible?

Workshop Tutorials for Physics

Solutions to QR4: **The Uncertainty Principle**

A. Qualitative Questions:

1. The Uncertainty principle.

a. $\Delta p \Delta x \geq h/2\pi$.

b. The Heisenberg uncertainty principle says that no matter how precise your measurements, the more you know about one variable, the less it is possible to know about the other, and the product of the two uncertainties is always greater than or equal to Planck's constant, h, on 2π.

c. All experiments involve some uncertainty due to inaccuracies in measurements, these uncertainties are also often called "errors". In a first year physics experiment these uncertainties are enormous compared to that from the uncertainty relation, so it can be ignored in the first year laboratory.

d. The uncertainty principle places no limit on how accurately you can measure the position *or* velocity of an object. It limits how much you can know about position *and* momentum simultaneously, the more you know about one, the less you can know about the other.

2. Both Rebecca's and Brent's points of view are quite reasonable, given our current knowledge of the quantum world. Many physicists believe that the uncertainty principle is entirely due to the fact that you cannot measure something without in some way interacting with it. Einstein said that *"God does not play dice"*, meaning that the world is still inherently deterministic.

Many other physicists believe that the universe is not deterministic, and even if you knew everything about all particles in the universe, you would still not be able to predict the future. Heisenberg wrote that *"In the sharp formulation of the law of causality-- "if we know the present exactly, we can calculate the future"-it is not the conclusion that is wrong but the premise."* Thus there are at least two opposing viewpoints on what exactly the uncertainty principle means about the universe, and neither has as yet been shown to be right.

This has led to a great many philosophical debates on the nature of the universe, free will and the existence of God. Physicists are still working on the answer!

B. Activity Questions:

1. Measuring momentum and position I

When the marble is released from the top of the slide it rolls down, gathering momentum as it falls. As it leaves the end of the slide it has horizontal velocity v. It is accelerated vertically due to gravity, and hits the floor at a time $t = (\frac{2h}{g})^{1/2}$ after it leaves the end of the slide, where h is the height of the end of the slide above floor level.

The horizontal momentum of the marble during its flight is given by $p=mv = mx/t = m\,x(\frac{2h}{g})^{-1/2}$.

Your measurement *has* changed the momentum of the marble significantly, in fact it has reduced it to zero.

You do *not* know both position and momentum simultaneously; you have found the momentum of the marble just *before* it hit the floor. At the time of your measurement you knew its position, but by finding this you changed the momentum. You know momentum before, and position during the measurement, but you do not know both simultaneously at the time of the actual measurement. This does not contradict the uncertainty principle which states that you cannot precisely know both momentum and position simultaneously.

2. Measuring momentum and position II

Your measurement has had very little effect on the marble. The scattering of photons from the marble to the detector will have no measurable effect on the momentum as the change in momentum due to the measurement is negligible compared to the momentum of the marble.

You do not need to take the uncertainty principle into account in this experiment, as the effects are tiny compared to the experimental uncertainties involved.

If you were measuring the momentum of an electron using the scattering of light, the momentum transfer from a photon to the electron, before the photon arrives at the detector, may be significant compared to the initial momentum of the electron. In this case the uncertainty principle would need to be considered.

C. Quantitative Questions:

1. Life, and cricket, with a large value of Planck's constant.

a. In both universes there will be an uncertainty due to the accuracy of the equipment Brent is using to measure the velocity. For example, if he is using a stop-watch to time the ball's motion he will be limited by his own reflexes and by the uncertainty in the device, which may only read in seconds or milliseconds. There will also be an inherent uncertainty due to the wave nature of the ball. The uncertainty principle tells us that we cannot know both the position and momentum of an object at the same time, $\Delta p \Delta x \geq h/2\pi$. In this universe Planck's constant is small enough that this will make a negligible difference. However in the new universe Planck's constant is very large. There will be a much greater uncertainty in either the momentum or position (or both) when he attempts to measure the velocity due to the wave nature of the ball.

b. Assuming no uncertainty in mass, we have $\Delta p = m\,\Delta v$ = 1.0 m.s^{-1} × 0.5 kg = 0.5 kg.m.s^{-1},

c. Using $\Delta p \Delta x \geq h/2\pi$ we get $\Delta x \geq h/(2\pi \times 0.5$ kg.m.s^{-1}) ≥ 0.2 m. It would be quite difficult trying to catch a ball whose position you only know to within 20 cm!

d. The de Broglie wavelength will be $\lambda = h/p$ = 0.6 J.s / 10 kg.m.s^{-1} = 0.06 m = 6 cm.

You might see the ball diffract from the cricket bat and go around it, or form an interference pattern as it goes through the wicket, as the wavelength is of similar size to these objects.

2. The x, y and z components of the velocity of an electron are measured to be :

$v_x = (4.00 \pm 0.18) \times 10^5$ m.s^{-1} ,

$v_y = (0.34 \pm 0.12) \times 10^5$ m.s^{-1} and

$v_z = (1.41 \pm 0.08) \times 10^5$ m.s^{-1}

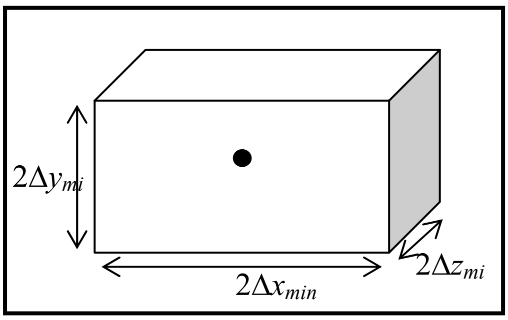
a. The momentum is $p = mv$, assuming that there is no uncertainty in m, then $\Delta p/p = \Delta v/v$, which we can rearrange to give $\Delta p = p \times \Delta v/v = m\Delta v$.

$\Delta p_x = m\Delta v_x = 9.11 \times 10^{-31}$ kg × 0.18 × 10^5 m.s^{-1} = 1.64×10^{-26} kg.m.s^{-1}.

$\Delta p_y = m\Delta v_y = 9.11 \times 10^{-31}$ kg × 0.12 × 10^5 m.s^{-1} = 1.10×10^{-26} kg.m.s^{-1}.

and $\Delta p_z = m\Delta v_z = 9.11 \times 10^{-31}$ kg × 0.08 × 10^5 m.s^{-1} = 7.29×10^{-27} kg.m.s^{-1}.

b. Using $\Delta p \Delta x \geq h/2\pi$ we can find the smallest uncertainties in the x, y, and z positions of the electron.



$\boldsymbol{\Delta x_{min} = h/(2\pi \times \Delta p_x)}$

= 6.64 × 10^{-34} J.s /(2π × 1.64 × 10^{-26} kg.m.s^{-1})

= 6.44 × 10^{-9} m = 6.44 nm.

$\boldsymbol{\Delta y_{min} = h/(2\pi \times \Delta p_y)}$ **= 6.64 × 10^{-34} J.s /(2π × 1.10 × 10^{-26} kg.m.s^{-1})**

= 9.60 × 10^{-9} m= 9.60 nm

$\boldsymbol{\Delta z_{min} = h/(2\pi \times \Delta p_z)}$ **= 6.64 × 10^{-34} J.s /(2π ×7.29 × 10^{-27} kg.m.s^{-1})**

= 1.45 × 10^{-8} m = 14.5 nm.

The smallest volume to which we can localize the electron is:

$\boldsymbol{V = 2\Delta x_{min} \times 2\Delta y_{min} \times 2\Delta z_{min}}$ **= 2 × 6.44 × 10^{-9} m × 2 × 9.60 × 10^{-9} m × 2 × 1.45 × 10^{-8} m = 7.17 × 10^{-24} m^3**

Workshop Tutorials for Physics

QR5: **Atomic Structure**

A. Qualitative Questions:

1. The Bohr model of the atom was the first quantum mechanical model of the atom.
a. Bohr postulated that a hydrogen atom could only exist without radiating in one of a set of stationary states. Explain what is meant by this postulate.
b. Bohr related his postulate to the classical picture of a hydrogen atom by quantising which quantity?
c. Bohr introduced a second postulate to explain how an atom emits radiation. Explain this postulate and show how it can be used to calculate the frequency of the radiation.
d. Is the Bohr model useful? What are its limitations?

2. The currently accepted model of the atom is the quantum model, which overcomes some of the limitations of the Bohr model. While this model is bound to change a bit, it works extremely well. In fact, all modern electronics is based on it. Everything that has a transistor in it, from a digital watch to a computer, is based on quantum mechanics and the quantum model of the atom. This model uses the wave function, ψ, which obeys the Schroedinger equation, to describe the electron.
a. Compare the Bohr model to the quantum model of the atom, what parallels can you draw?
b. What is normalization and what is its physical significance?
The requirements that the wave function be normalisable and continuous introduce three quantum numbers, n, l and m.
c. What do these quantum numbers represent?
There is a fourth quantum number, m_s.
d. What property of an electron does this fourth quantum number represent?

B. Activity Questions:

1. Hydrogen Spectrum

Spectroscopes separate photons of different wavelengths and can be used to observe the photons produced by electronic transitions.
Describe what you see when you look at the lamp.
Now look at the lamp through the spectroscope. What do you see? Draw a sketch of the spectrum you observe.

2. Emission spectra

Use the spectroscope to observe light from other sources, including the lamps, fluorescent tubes and sunlight. Why are the spectra from these sources different?
How is this useful?

C. Quantitative Questions:

1. For an electron in the ground state of the hydrogen atom, according to Bohr's theory, what are

a. the principle quantum number,
b. the electron's orbit radius,
c. its angular momentum,
d. its linear momentum,
e. its angular velocity,
f. its linear speed,
g. the force on the electron,
h. the acceleration of the electron,
i. the electron's kinetic energy,
j. the potential energy, and
k. the total energy.

Data: $1\text{eV} = 1.60 \times 10^{-19}$ J,
electron mass = 9.11×10^{-31} kg,
elementary charge; $e = 1.60 \times 10^{-19}$ C
$h = 6.63 \times 10^{-34}$ J.s or $h = 4.14 \times 10^{-15}$ s,
speed of light; $c = 3.00 \times 10^{8}$ m.s^{-1},
$r_B = 0.0529$ nm
$k = (4\pi\varepsilon_0)^{-1} = 9.0 \times 10^{9}$ N.m^{2}C^{-2}.

2. Consider a hydrogen atom, ${}^{1}_{1}H$, which has one electron. In Bohr's model the electron can be in any one of many discrete energy levels. An electron in the ground state (n=1) energy level of hydrogen has an energy of -13.6eV. The next energy level (n=2) corresponds to an electron energy of –3.4eV, and the next two levels have –1.5eV (n=3) and -0.85eV(n=4), and in general $E = -13.6\text{eV}/n^2$.

a. Draw an energy level diagram using the information given above.
b. Where is the energy of the electron zero?
c. What transitions take place if an electron is hit by a photon with energy 12.1eV?
d. What if it is hit by a photon of 12.5eV?
e. Describe what happens when an excited electron relaxes back to the ground state.
f. What wavelength photon is emitted when an electron relaxes from the $n = 4$ state to the $n = 2$ state?
g. What is the shortest wavelength photon a hydrogen atom can emit due to an electron transition?

By convention there is color,
By convention sweetness,
By convention bitterness,
But in reality there are atoms and space.
-Democritus (c. 400 BCE)

Workshop Tutorials for Physics

Solutions to QR5: **Atomic Structure**

A. Qualitative Questions:

1. The Bohr model of the atom.

a. Atoms could exist with certain fixed values of energy (in certain energy states) without radiating energy i.e. where their energy remained constant, these states are called stationary states.

b. Bohr stated that the angular momentum of the orbiting electron was quantised, i.e. $l = mvr = nh/2\pi$, where n has values 1,2,3,etc

c. The atom emitted radiation when it moved from one energy state to another. The energy difference between the states was equal to the energy of the photon emitted. $E_n - E_{n-1} = hf$.

d. The Bohr model is useful when describing atoms with only one electron. It cannot be extended to multi electron atoms.

2. The quantum mechanical model of the atom.

a. The concept of the electron orbiting the nucleus as, for example, the earth around the sun, is replaced by a concept of a cloud around the nucleus where the probability of finding the electron at a certain point depends on the square of the wave function at that point. The wave function is obtained from the solution of the Schrodinger equation in the quantum mechanical model of the atom. In both models there are stationary states i.e. certain fixed values of energy for the atom where it does not radiate energy.

b. Normalisation is the process whereby a constant factor is introduced into the wave function (the solution of the Schrodinger equation) such that the probability of finding the electron anywhere at all in space is 1.

c. n represents the radial position quantisation and is known as the principal quantum number. For example the lowest energy level known as the ground state is represented by $n = 1$. l represents the angular momentum quantum number and can have values $0, \pm 1, \pm 2, \quad \pm n\text{-}1$. m is called the orbital magnetic quantum number and can have values $0, \pm 1, \pm 2, ... \pm l$.

d. The spin of the electron.

B. Activity Questions:

1. Hydrogen Spectrum

a. You should see a blue-ish coloured light.

b. You should have seen lines of different colours, due to different electronic transitions. Discrete energies mean that electrons can only make distinct transition, hence they can only change energy by fixed amounts, hence they can only emit (or absorb) photons of particular energy.

2. Other Spectra

The spectrum of any given element is unique, hence by observing the spectrum of a source, we can tell what elements are present. This is used to identify what elements are in all sorts of things, including stars.

C. Quantitative Questions:

1. For an electron in the ground state of the hydrogen atom, according to Bohr's theory:

a. the principle quantum number is n = 1,

b. the electron's orbit radius is $r = r_B = 0.0529$ nm,

c. its angular momentum is $L = nh/2\pi = h/2\pi$,

d. its linear momentum is $p = L/r_B = h/(2\pi.r_B) = 1.99 \times 10^{-24}$ kg.m.s^{-1},

e. its angular velocity, $\omega = v/r = p/mr_B = 4.14 \times 10^{16}$ rad.s^{-1},

f. its linear speed, $v = p/m = 2.19 \times 10^{6}$ m.s^{-1},

g. the force on the electron, $F = (4\pi\varepsilon_o)^{-1}e^2 / r_B^2 = 8.26 \times 10^{-8}$ N,

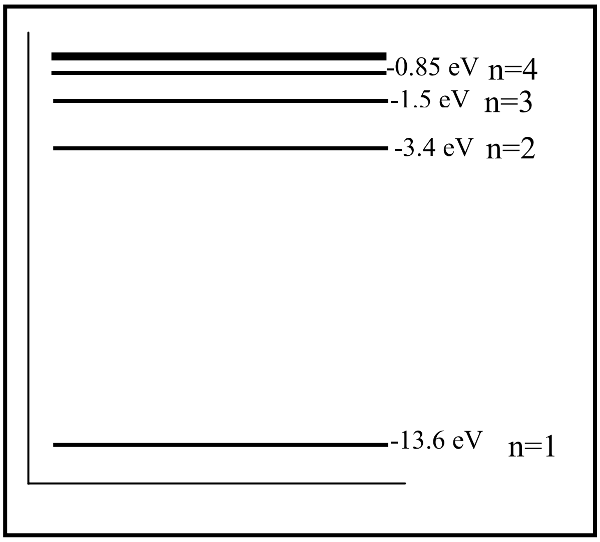
h. the acceleration of the electron, $a = v^2/r_B = 9.07 \times 10^{22}$ m.s^{-2},

i. the electron's kinetic energy, $KE = ½ mv^2 = 13.6$ eV,

j. the potential energy, $U = -(4\pi\varepsilon_o)^{-1}e^2 / r_B = -27.2$ eV and finally,

k. the total energy = $U + KE = -13.6$ eV.

2. An electron in the ground state (n=1) energy level of hydrogen has an energy of -13.6eV. The next energy level (n=2) corresponds to an electron energy of –3.4eV, and the next two levels have –1.5eV (n=3) and -0.85eV(n=4), and in general E = -13.6eV/n^2.



a. The energy level diagram is shown opposite.

b. The energy of the electron is zero infinitely far away from the nucleus. This is an arbitrary but convenient choice of a zero point, as we can only measure changes in energy, not absolute energies. Similarly, the surface of the Earth is usually chosen as the zero for gravitational potential energy.

c. If an electron in the ground state is hit by a photon with energy 12.1 eV it can jump to the n = 3 level. If an electron above the ground state is hit by a photon with energy 12.1 eV it can leave the atom which is ionised.

d. If an electron in the ground state is hit by a photon with energy 12.5 eV no transition will take place because there is no energy level at (-13.6 eV + 12.5 eV =) 1.1 eV. If an electron above the ground state is hit by a photon with energy 12.5 eV it can leave the atom which is ionised.

e. When an excited electron relaxes back to the ground state from energy level n = x, a photon with energy equal to $(E_x - E_1)$ is released.

f. When an electron relaxes from the n = 4 state to the n = 2 state it has a change in energy of
$\Delta E = –0.85 - -3.4 = 2.55$ eV = 4.1×10^{-19}J.
The frequency is then $f = E/h = 4.1\times10^{-19}$J / 6.63×10^{-34} J.s = 6.2×10^{14} s^{-1}.
Which is a wavelength of $\lambda = c/f = 3 \times 10^8$ m.s^{-1}/ 6.2×10^{14} s^{1}= 490 nm.(Blue-ish/green light)

g. The shortest wavelength possible corresponds to the highest energy possible, which is from $n = \infty$, with energy 0 eV to n =1, with -13.6 eV, a change of 13.6 eV = 2.18×10^{-18}J.
Again using $f = E/h = 2.18\times10^{-18}$J / 6.63×10^{-34} J.s = 3.28×10^{15} s^{-1}
Which gives a wavelength of $\lambda = c/f = 3 \times 10^8$ m.s^{-1}/ 3.28×10^{15} s^{-1}= 91nm.
This is in the ultraviolet region of the spectrum.

Workshop Tutorials for Physics

QR6: **Atomic Structure II**

A. Qualitative Questions:

1. In the 19th century Dmitri Mendeleev organized the known elements by their atomic weights into what has since become known as the periodic table. Originally, because the ordering was done by mass only, the chemical properties didn't quite fit into a pattern. They have since been re-ordered somewhat.

a. How are the elements in the periodic table ordered now, and why?

b. What is it that characterises a row in the periodic table?

c. What is it that characterises a period (column)?

A small section of the periodic table is shown below.

d. Which of the elements shown shaded would you expect to behave in a similar way? Explain your answer.

e. Which would you expect to behave differently, and why?

5 B boron	6 C carbon	7 N nitrogen
13 Al aluminium	14 Si silicon	15 P phosphorus
31 Ga gallium	32 Ge germanium	33 As arsenic

2. Chemists spent a lot of time trying to work out why the characteristics of the elements followed a pattern and looking for the missing elements to fill in the table. In 1925 Wolfgang Pauli solved the mystery by proposing an exclusion principle which is now known as the Pauli exclusion principle.

a. What is this exclusion principle?

b. How does it explain the relationships between the elements in the periodic table?

B. Activity Questions:

1. Periodic table

Examine the chart of the periodic table.

a. Locate some common metals, including iron, copper and lead. What do you notice about their positions in the table?

b. Locate some radioactive elements, including plutonium, uranium and radium. What do you notice about their positions in the table?

c. Why do elements in a given column have similar characteristics?

d. Why are the noble gases (the last column) so inert? They are called noble because they don't usually associate (form bonds) with other atoms.

e. Why are elements in the first column so reactive?

2. Molecular models

Examine the ball and stick models of the atoms.

Can you group them according to period?

What determines the bonding behaviour of the atoms?

C. Quantitative questions:

1. The electron states for an atom can be described using quantum numbers, each of which is related to the orbit or spin of the electron. The table below gives the names, symbols and possible values for these quantum numbers.

a. Copy and complete this table:

Quantum number	**Symbol**	**Possible values**	**related to:**
	n	1, 2, 3, …	
orbital		0, 1, 2, …, (n-1)	orbital angular momentum
orbital magnetic	m_l		orbital angular momentum (z component)
	m_s		spin angular momentum (z component)

An electron in a multi-electron atom has the quantum number $l = 3$.

b. What are the possible values of n for this electron?

c. What are the possible values of m_l and m_s?

2. The ionisation energies of the elements tend to increase along a given row.

a. What is the ionisation energy and why does it vary like this?

For atoms heavier than helium the ionisation energy is much less than you would expect by considering only the coulomb attraction between the nucleus and the outer shell electron.

b. Explain why this is the case.

c. If the outer electron in a lithium atom is in the $n = 2$ state, what would you expect the effective nuclear charge as seen by the outer electron to be?

d. Using the Bohr model of the atom, what would you expect the energy of this electron to be? Take the radius of the electron to be $4a_0$.

e. The ionisation energy of lithium is 5.39 eV. Using the Bohr model again, what is the effective nuclear charge seen by the outer shell electron?

f. Comment on your answers to **d** and **e.**

Workshop Tutorials for Physics

Solutions to QR6: **Atomic Structure II**

A. Qualitative Questions:

1. The periodic table.

a. The elements are ordered according to the number of protons in the nucleus (the atomic number, Z). The number of protons determines the number of electrons, which determines how an element will behave chemically.

b. A row in the periodic table all have their outermost electrons in the same shell, and this shell is the row number, and they have filled shells at lower energies. For example, Li, Be and B have a filled $n = 1$ shell and partly filled $n = 2$ shell.

c. A period or column of the periodic table contains elements with the same number of electrons in their outer shell, for example H, Li and Na all have one electron in their outer shell

d. C, Si and Ge have similar chemical properties because they all have 4 electrons in their outer shell.

e. Al, Si and P behave quite differently because they have different numbers of electrons in their outer shell.

5 B boron	6 C carbon	7 N nitrogen
13 Al aluminium	14 Si silicon	15 P phosphorus
31 Ga gallium	32 Ge germanium	33 As arsenic

2. The exclusion principle.

a. The Pauli exclusion principle says that no two electrons may occupy the same quantum state; in other words they can't have the same quantum numbers n, l, m_l and m_s. (The exclusion principle applies to a large group of particles, including protons and neutrons, collectively known as fermions).

b. Atoms which have the same number of electrons in their outer shell (described by the n quantum number) have similar characteristics. For example, the noble gases (down the last column of the periodic table) have a filled outer shell and are inert, they do not react easily with other atoms. The first row corresponds to the n =1 shell. The possible l numbers for any shell are 0 to n–1; in this case, for n = 1, the only possible l number is zero, which is called the s sub-shell. There is only one possible value of m_l, and 2 possible values of the spin number, m_s. Hence there can be only 2 electrons in the n =1 shell, and only 2 elements in the first row of the periodic table: H with only one outer shell electron and He with a filled outer shell, which is put at the opposite end of the row above the other elements with filled outer shells. In the n = 2 shell there are 8 possible combinations of quantum numbers, hence there are eight elements in the next row. After this it gets a little more complicated as the shells don't quite fill in order because some high l sub-shells have higher energy than the low l sub-shells of the next shell.

B. Activity Questions:

1. Periodic table

a. Most common metals, including iron, copper and lead are located around the middle of the periodic table, four or more rows down, or to the left hand side in rows 2 and 3.

b. The radioactive elements, such as plutonium, uranium and radium are at the bottom end of the periodic table, with high atomic numbers.

c. Elements in a given column have similar characteristics because they have the same outer electron configuration, and when atoms interact (at normal energies) it is via their outer shell electrons.

d. The noble gases (the last column) are so inert because they have a complete outer shell of electrons. This is a very stable configuration, and it is hard for them to either gain or lose electrons, so they do not associate with other atoms.

e. The elements in the first column are so reactive because they have only a single electron in their outer shell and a low ionisation energy. This means it is easy for them to form bonds by losing that outer electron to another atom.

2. Molecular models

Atoms with the same number of outer electrons have similar characteristics, and it is also their outer electron number that determines their bonding behaviour.

C. Quantitative questions:

1. Quantum numbers.

a.

Quantum number	Symbol	Possible values	related to:
principal	n	1, 2, 3, …	distance from nucleus, energy
orbital	l	0, 1, 2, …, (n-1)	orbital angular momentum
orbital magnetic	m_l	0, ±1, ±2, …, ±l	orbital angular momentum (z component)
spin magnetic	m_s	± ½	spin angular momentum (z component)

If the quantum number l is 3, then:

b. The principal quantum number must be greater than 3, as l must be less than n. Hence possible values of n are n = 4, 5, 6, …..

c. The possible values of m_l are 0, ±1, ±2, ±3, and the possible values of m_s are ± ½.

2. The ionisation energies of the elements tend to increase along a given row.

a. The ionization energy is the energy required to separate an electron from the atom, thus ionizing it. Filled shells are very stable, so it is hard to remove an electron from a filled shell. In general, the closer to full that a shell is, the harder it is to remove electrons from it. Elements such as lithium and sodium have only a single electron in their outer shell, and a filled shell at lower energy than this, so they are easily ionized. Noble gases have a filled outer shell so they have very high ionization energies.

b. For atoms heavier than helium the ionisation energy is much less than you would expect by considering only the coulomb attraction between the nucleus and the outer shell electron because of "shielding" due to the other electrons. The attraction to the nucleus is less due to the presence of other electrons in lower energy states, hence the ionization energy is lower.

c. The nucleus of the lithium atom (Z = 3) would be shielded by two electrons in the n = 1 shell, so the electron in the n = 2 shell would see an effective nuclear charge of $Z'e = 1e$.

d. The energy of this outer shell electron would hence be

$$E = -\frac{1}{2}\frac{kZe^2}{r} = -\frac{1}{2}\frac{kZe^2}{4a_0} = -\frac{Z}{4}\ (13.6\text{ eV}) = -\tfrac{1}{4}\ (13.6\text{ eV}) = -3.4\text{ eV}.$$

e. In fact, the energy E = 5.39 eV,

$$E = -\frac{1}{2}\frac{kZe^2}{4a_0} = -\frac{Z}{4}\ (13.6\text{ eV}) = -5.39\text{ eV},$$ which we can rearrange to find Z' :

$Z = 4 \times (-5.39\text{ eV}/-13.6\text{eV}) = 1.59$.

f. Comparing the answers to **d** and **e**, we can say that the outer electrons see a reduced nuclear charge but the nucleus is only partly shielded by the inner electrons.

Workshop Tutorials for Physics

QR7: **Band Structure and Conductivity**

A. Qualitative Questions:

1. At room temperature a given applied electric field will generate a drift speed for the conduction electrons of silicon that is about 40 times as great as that for the conduction electrons of copper.
a. Why isn't silicon a better conductor of electricity than copper?
Digital thermometers measure temperature by using the change in resistance of the sensor with changing temperature. The sensors may be thermocouples which are made of metal, or thermistors which are made of a semiconductor material.
b. If the sensor of a digital thermometer is placed in a hot oven and the resistance of the sensor decreases, is it a thermistor or thermocouple that is being used?
c. Explain using diagrams why the resistance decreased.

2. Metals are characterised by their shininess and by their thermal and electrical conductivities.
a. What is it that gives a metal these properties?
b. What is the Fermi energy of a material? Does it change with temperature?
When two pieces of metal with different Fermi energies are brought into contact a potential difference is produced.
c. Explain how this contact potential is produced. What determines the magnitude of the contact potential?

B. Activity Questions:

1. Band structure model
Examine the band structure models.
What is that distinguishes a conductor (a metal) from an insulator?
How can a material be made more conductive?
How can a "hole" act as a charge carrier?

2. Molecular models of semiconductors
Examine the ball and stick models of the atoms.
What happens when you put P impurities into Si? What sort of semiconductor results?
What happens when you put Al impurities into Si? What sort of semiconductor results?

C. Quantitative questions:

1. The density and molar mass of sodium are 971 $kg.m^{-3}$ and 23 $g.mol^{-1}$ respectively; the radius of the ion Na^+ is 98 pm.

a. What fraction of the volume of metallic sodium is available to its conduction electrons?

b. Carry out the same calculations for copper. Its density, molar mass, and ionic radius are, respectively, 8960 $kg.m^{-3}$, 63.5 $g.mol^{-1}$, and 135 pm.

c. For which of these two metals do you think the conduction electrons behave more like a free electron gas?

d. Which of these metals do you think is a better conductor?

2. Carbon and silicon are both very common elements on earth, and both have four outer shell electrons. Sand and many rocks are composed mainly of silicon, and organic material (including people) is largely composed of carbod. Computers and other electronic devices use silicon chips, which contain transitors. The production of these chips is a multibillion dollar industry, and requires very pure silicon grown into crystals and made into thin wafers. Silicon has a band gap of 1.1 eV.

a. What is the probability of an electron at the top of the valence band jumping across into the conduction band at room temperature?

Physicists are starting to look at using organic molecules, such as DNA, as a semiconductor. Carbon can form crystals with very different structures, for example graphite and diamond. The energy gap for diamond is 5.5 eV.

b. What is the probability of an electron at the top of the valence band in diamond jumping across into the conduction band at room temperature?

c. Comment on your answers to **a** and **b**. Why is silicon used for making semiconductors rather than diamond?

Workshop Tutorials for Physics

Solutions to QR7: **Band Structure and Conductivity**

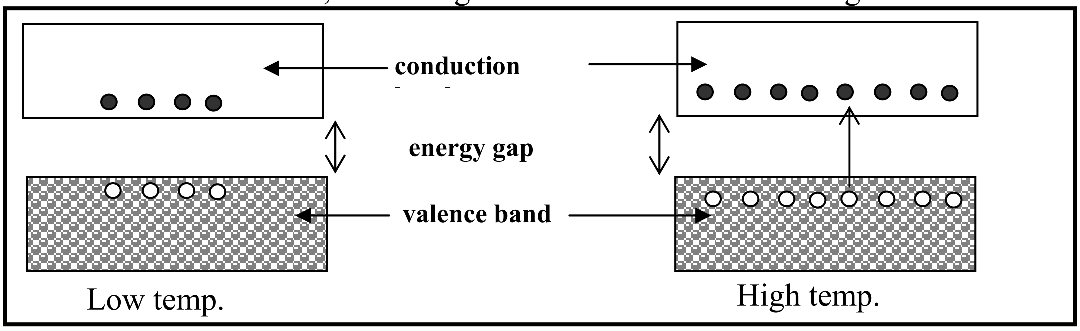
A. Qualitative Questions:

1. Conductivity of copper and silicon.

a. Why isn't silicon a better conductor of electricity than copper? The drift speed is much greater, however, the number of electrons in the conduction band at room temperature is very low. Silicon has a band gap of 1.14 eV, and so there are few electrons available to conduct electricity. Copper has a lower drift speed, but many more electrons in the conduction band.

b. Resistance of metals increases with increasing temperature, the resistance of semiconductors decreases. As the resistance has decreased, this must be a semiconductor sensor, hence a thermistor is being used.

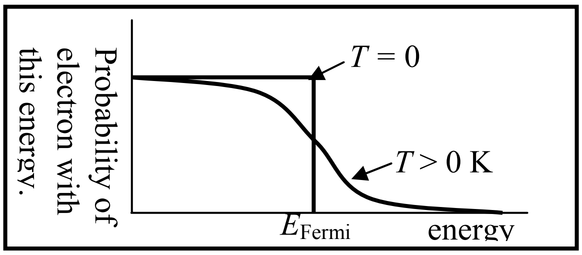
c. The resistance of a semiconductor decreases because more electrons are able to move from the valence band to the conduction band, increasing the number of available charge carriers. See below.



2. Metals are characterised by their shininess and by their thermal and electrical conductivities.

a. Metals have a large number of free conduction electrons per m^3, which give them a high electrical conductivity. These free electrons can reflect electromagnetic radiation (light), making metals shiny. Insulators conduct heat only via vibrations of atoms since they have no free electrons, but metals also conduct heat by free electron movement.

b. The Fermi energy is the energy corresponding to that of the highest occupied band at 0 K. The Fermi energy is characteristic of a material and does not depend on temperature. Above 0 K there will be some electrons with higher energy than the Fermi energy, and the higher the temperature the more electrons will be above the Fermi level.



c. If the Fermi energy of one of the metals is higher then electrons will migrate to the other metal where they can be in a lower energy band. However this creates a potential difference, which will eventually balance the difference in the Fermi energies, at which point there will be no net electron migration. The magnitude of the potential difference is determined by the difference in the Fermi energies of the two metals.

B. Activity Questions:

1. Band structure model

A conductor (a metal) has a much smaller band gap than an insulator, and has some electrons in the conducting band.

A material can be made more conductive by giving it additional charge carriers, for example by doping. Either electrons or holes can be added. Holes act as if they are mobile, positive charges with a charge of +e. Holes are an invention of physicists, but understanding the operation of many important electronic devices depends on our the concept of holes.

2. Molecular models of semiconductors

Putting P impurities into Si gives you an *n* type semiconductor because you have extra electrons which become charge carriers.

Putting Al impurities into Si leaves you with too few outer electrons, so you end up with holes, giving a *p* type semiconductor.

C. Quantitative questions:

1. Density and molar mass of Na are 971 kg.m^{-3} and 23 g.mol^{-1} respectively; the radius of Na^+ is 98 pm.

a. The number of sodium atoms per cubic metre is

$$n_{Na} = \frac{\rho}{M}.N_A = \frac{971}{23\times10^{-3}}.6.023\times10^{23} = 2.54\times10^{28}\,\text{atoms}$$

where N_A is Avogadro's number (number of atoms per mole).

Each sodium ion has a radius of 98 pm = 98 × 10^{-12} m. The volume of each ion is

$V = (4/3)\,\pi r^3 = (4/3)\,\pi(98\times10^{-12})^3 = 3.94\times10^{-30}$ m^3.

The volume of the *n* ions is $n \times V = 3.94\times10^{-30}$ m^3 /atoms × 2.54×10^{28} atoms = 0.100 m^3.

This only 10% of the volume, 1m^3, so 90% is available to the conduction electrons.

b. The number of copper atoms per cubic metre is

$$n_{Cu} = \frac{\rho}{M}.N_A = \frac{8960}{63.5\times10^{-3}}.6.023\times10^{23} = 8.50\times10^{28}\,\text{atoms}$$

Each copper ion has a radius of 135 pm

The volume of each ion is $V = (4/3)\,\pi r^3 = 1.03\times10^{-29}$ m^3. The volume of the *n* ions is $n \times V = 0.88$ m^3.

This 88% of the volume, so only 12% is available to the conduction electrons.

c. We would therefore expect sodium's conduction electrons to behave most like a free electron gas, since the conduction electron density and volume of ions is lower.

d. Copper would be expected to be the better conductor because of its higher conduction electron concentration.

2. Silicon has a band gap of 1.1eV [= 1.1 × 1.6 × 10^{-19} J = 1.8 × 10^{-19} J].

a. Take room temperature to be 27ºC = (27+273) K = 300K. The probability of occupation of the conduction band at energy εis given by the Fermi-Dirac function, *f*(ε),

$$f(\varepsilon) = \frac{1}{1+\exp\left(\frac{\varepsilon-\varepsilon_F}{k_BT}\right)} \approx \exp\left(-\frac{\varepsilon-\varepsilon_F}{k_BT}\right) = \exp\left(-\frac{1.1\text{eV}-0.5\text{eV}}{0.025\text{ eV}}\right) = 3.7\times10^{-11}$$

[Note that we can make this approximation since 1 is negligible compared to the expontential].

This means only about 1 in 10^{11} electrons is found in the conduction band.

b. A similar calculation for diamond gives us, approximately,

$$f(\varepsilon) \approx \exp\left(-\frac{\varepsilon-\varepsilon_F}{k_BT}\right) = \exp\left(-\frac{5.5\text{eV}-2.75\text{eV}}{0.025\text{ eV}}\right) = 1.7\text{x}10^{-48}$$

which is vanishingly small compared to the probability in case **a.** for silicon.

c. Comparing the probability of finding an electron in the conduction band for silicon and for diamond at the same (room) temperature, we see that there is negligible probability of there being any conduction electrons in the case of diamond – the probability is many orders of magnitude less than for silicon at the same temperature. Silicon is used for making semiconductor devices because it can be conveniently 'doped', that is impurity atoms can be intentionally added to control the number of electrons in the conduction band. Since it is these conduction electrons that dictate the electrical properties of the material, the presence of conduction electrons, plus control over this parameter makes silicon technologically and commercially attractive – hence the 'silicon chip'. (It is worth noting that silicon is also cheap, plentiful, and easy to work with. Notwithstanding this, there has recently been quite a lot of research interest in semiconductor devices fabricated from thin diamond films – these would operate at very high temperatures, where silicon chips might behave unpredictably, or even melt!)

Workshop Tutorials for Physics

QR8: **Semiconductors**

A. Qualitative Questions:

1. The concept of an energy band is important in understanding why some materials are conductors and others are insulators. Semiconductor characteristics depend on their band structure and the band structure is manipulated, for example by doping, to produce the desired characteristics.

a. What is an energy band, and how does it arise?

b. Draw a band structure diagram for a metal, an insulator and a semiconductor. Label the energy levels and show where the Fermi energy is.

c. What does a "hole" refer to in semiconductors?

d. How can having "holes" increase the conductivity of a material?

2. Semiconductors can be produced by doping a material, such as silicon, to increase the number of charge carriers.

a. Explain how doping silicon with phosphorous increases the number of charge carriers.

b. What sort of semiconductor results?

c. How could you produce the other sort of semiconductor, and what are its majority charge carriers?

d. Does a piece of *p*-type semiconductor have a net negative, positive or zero charge?

e. Explain how a contact potential difference is generated when an *n* type and a *p* type semiconductor are brought into contact.

B. Activity Questions:

1. Thermocouples and Thermistors

Measure the resistance of the two samples.

Now heat them using the hot water and measure the resistance again.

Which one uses a metal and which uses a semiconductor? Explain your answer.

2. Semiconductor diodes

Explain what a diode does.

What are some uses for a diode in an electrical circuit?

3. Light emitting diodes

Light emitting diodes (LEDs) are also made from semi-conductor material.

Find the voltage just needed to get the LED to produce light.

How does the voltage depend on colour?

How does the LED produce photons?

What determines the colour of the light produced by a LED?

4. Solar cells

Explain how the solar cell converts light energy into electrical energy.

How is this different to an LED? How is it similar?

C. Quantitative Questions:

1. Draw a carefully labeled diagram of the idealized energy band structure of
a. an intrinsic semiconductor
b. an n-type doped semiconductor
c. a p-type doped semiconductor.
In each case clearly show which energy levels are full, empty or partially occupied and describe any other important features.
d. The electrical conductivity of undoped silicon can be increased by irradiating it with photons. This has the effect of exciting valence electrons into the conduction band. Given that the energy band gap of silicon is 1.14 eV, calculate the lowest energy of a photon which can excite a valence electron to the conduction band.
e. What is the wavelength of this photon, and to which part of the spectrum does it belong?

2. Pure silicon at room temperature has an electron density in the conduction band of approximately 1×10^{16} m^{-3} and an equal density of holes in the valance band. Suppose that one of every 10^7 silicon atoms is replaced by a phosporus atom.
a. Which type of semiconductor will this doped silicon be, *n* or *p*?
b. What charge carrier density will phosporus add?
c. What is the ratio of the charge carrier density in the doped silicon to that in the pure silicon?

Data for Silicon:
Z = 14, Molar mass 28.086 $g.mol^{-1}$, density = 2.33×10^3 $kg.m^{-3}$.

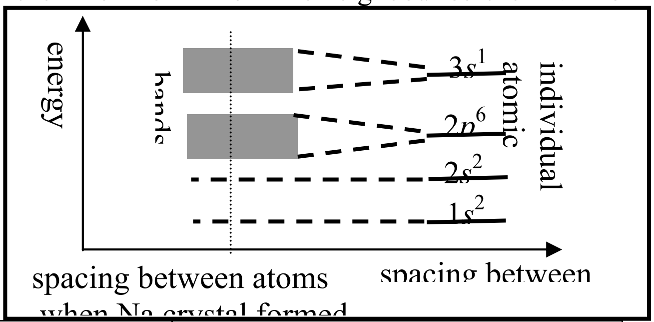
Workshop Tutorials for Physics

Solutions to QR8: **Semiconductors**

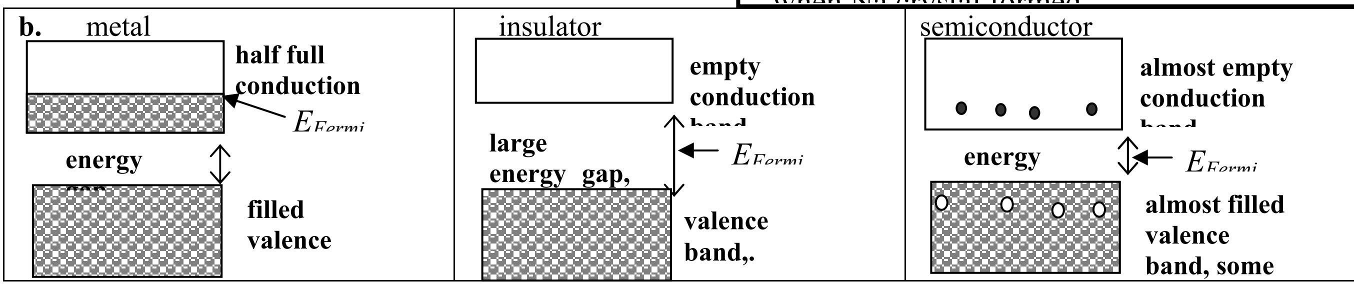
A. Qualitative Questions:

1. a. An energy band is the range of energies in a crystal (solid) spanning those energy values that electrons can take in that material. Why is this a characteristic of a solid in particular? Taking sodium as an example, sodium has 11 electrons in the configuration $1s^2$, $2s^2$, $2p^6$, $3s^1$, so the single 3*s* electron is in the outermost, part-filled shell (or 'orbital'). Take some very hot sodium, which is vapourised - each atom is far from it's neighbour so each Na atom has its own individual $1s^2$, …$3s^1$electron configuration.

Now let the sodium cool until it solidifies: as the atoms condense the outermost electron shells approach and begin to overlap, causing the electron shells to 'mix' and change in character (hybridise) from individual atom-like orbitals to electron orbitals characteristic of a solid. In the solid, electron's are 'allowed' to have a range of energies – called a band – rather than the single, sharply defined energies (the $1s^2$, …$3s^1$) of the individual, single sodium atoms.



b.



c. The conduction band (c.b.) for the semiconductor is almost empty and the valence band (v.b.) is almost full. The few electrons in the c.b. have 'jumped' across the band gap from the v.b. leaving behind a 'hole' in what would otherwise be a complete, full energy band. Holes act as if they are mobile, positive charges with a charge of +e.

d. The conductivity of a material, σ, is given by the expression $\sigma = ne\mu$ where n is the concentration (number per m^3) of charges (electrons or holes) with a mobility of μ (velocity divided by field strength, v/E) and e is the value of the charge ($\pm e$). In a typical semiconductor we have *both* mobile electrons and holes so the total conductivity is $\sigma_{total} = n^+e\mu + n^-e\mu$..

2a. Doping means adding 'impurities' to the semiconductor. The impurities added to the semiconductor are chosen to have a 'spare' electron, e.g. phosphorous, P. The spare electron is only weakly attached (~0.01 eV to remove it) to the P impurity atoms and can easily jump into the conduction band. This means that at room temperature there are many electrons in the conduction band supplied by the impurity atoms.

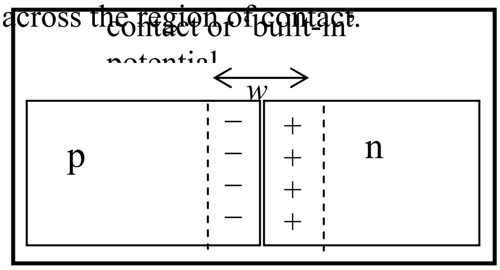
b. This is an *n*-type semiconductor, because the phosphorus has donated electrons (***n***egatively charged charge carriers) to the conduction band.

c. Each dopant atom removes one electron from the otherwise completely full valance band leaving behind a positive 'hole'. The resulting semiconductor is p-type meaning the free charges are positive. There will be only very few conduction electrons in the conduction band of *p*-type material since these can only get to the conduction band by jumping the large energy across the band gap.

d. A *p*-type semiconductor has no net charge, the nuclear charge still equals the number of electrons.

e. When pieces of n-type and p-type semiconductor are brought into electrical contact the free charges near the junction (holes in the p-type, electrons in the n-type) diffuse (like one gas mixing into another) so that electrons move to the p-side and holes move to the n-side. The electrons and holes move due to a charge concentration gradient and this establishes an electric field *E* across the region of contact.

After a short time the diffusion process is balanced by the electric field. The resulting depletion region contains no mobile charges; an electric field E_d exists across the depletion region, width w. The electric field E_d results from the contact potential due to the separation of charges (n and p) that has occurred and is related by $E_d = -dV/dx$ where x is distance measured along the p-n junction (perpendicular to the area of the p-n contact).



B. Activity Questions:

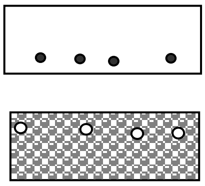
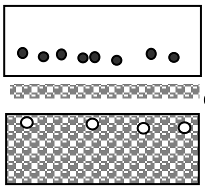
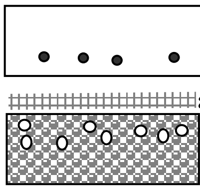
1. Thermocouples and Thermistors
Metals have increasing resistance with increasing temperature, due to thermal agitation of the atoms in the lattice. The atoms move about more, shortening the path length of the moving electrons. Semiconductors have decreasing resistance with increasing temperature as more electrons are able to jump to the conduction band.

2. Diodes
Diodes act like valves, allowing current to flow in only one direction. They are used as rectifiers, to remove half of the sine wave of an AC power supply, such as the 240V mains supply. Diodes are also used in rectifying circuits in radios to get the information (speech, music, etc) from the carrier waves.

3. Light Emitting Diodes
Light is emitted from an LED when an electron falls from the conduction band to the valence band. This requires a lot of electrons in the conduction band and a lot of holes in the valence band for them to fall into. This is done using a *p-n* junction. The colour produced by an LED depends on the band gap, which depends on the type of dopant used and the degree of doping. The shorter the wavelength, the greater the band gap and the greater the voltage required to produce a transition and emit a photon.

4. Solar cells
In the solar cell an incident photon excites an electron across the band gap, and causes a current to flow. This is the opposite of what happens in an LED, where the current (movement of electrons) causes a photon emission.

C. Quantitative Questions:

1. a. intrinsic semiconductor	**b.** n-typed doped semiconductor.	**c.** p-type doped semiconductor.
		

a. Intrinsic semiconductor: Small gap, so at room temperature some electrons will jump the band and be free to conduct in the conduction band**.**
b. n-typed doped semiconductor: The *n* type dopant donates electrons, negative charge carriers, which are in bands just below the conduction band of the silicon, and some electrons are in the conduction band. The majority charge carriers are electrons in the conduction band.
c. p-type doped semiconductor: Dopant has fewer outer shell electrons, it contributes holes, positive charge carriers. The energy levels of the dopant are just above silicon's. Electrons from the silicon readily jump to these levels, leaving holes in the valence band, these holes are the majority charge carriers. The dopant is called an acceptor because it accepts electrons.
d. The energy band gap of silicon is 1.14 eV, therefore the minimum photon energy required to move an electron across the band gap is 1.14 eV.
e. $E = hf = hc/\lambda$,
so $\lambda = hc/E = (6.63 \times 10^{-34} \times 3.00 \times 10^{8})/(1.14 \times 1.60 \times 10^{-19}) = 1.09 \times 10^{-6}$m = 1090 nm. infrared.

2.a. Silicon doped with phosphorus is an *n*-type semiconductor, the phosphorus has one more electron than the silicon and contributes negative charge carriers.
b. The phosphorus will add one electron for every 10^7 atoms. The molar mass of silicon is 28.086 g per mol, so 10^7 atoms has a mass of $(10^7/ 6.02 \times 10^{23}) \times 28.086$ g $= 4.66 \times 10^{-16}$ g $= 4.66 \times 10^{-19}$ kg.
The density of silicon is 2.33×10^3 kg.m^{-3}, so this is equivalent to
4.66×10^{-19} kg / 2.33×10^3 kg.m^{-3} = 2.00×10^{-22} m^{-3}, an extra charge carrier per 2.00×10^{-22} m^{-3}
or 1/ 2.00×10^{-22} m^{-3} = 5.0×10^{21} extra charge carriers per m^3.
d. The ratio of the charge carrier density in the doped silicon to that in the pure silicon is
5.0×10^{21} m^{-3}/1×10^{16} m^{-3} = 5.0×10^5 or five hundred thousand.

Workshop Tutorials for Physics

QR9: **Lasers**

A. Qualitative Questions:

1. LASER stands for Light Amplification by Stimulated Emission of Radiation.
a. Explain the steps involved in producing laser light. Use diagrams to illustrate your explanation.
b. Why does a helium-neon (HeNe) laser use helium instead of just neon, when it is the neon that emits the coherent photons which make up the laser beam?

2. Lasers have many applications in manufacturing, such as precision cutting and welding. The gas inlet in the torches used in the Olympic torch relay for the Sydney 2000 Olympic games was cut using a laser. They are also used for targeting and lining up objects at a distance, and can be used for missile defense. Lasers also have medical applications, such as laser eye surgery to correct short or long sightedness.
Why is focused laser light inherently better than light from a tiny incandescent lamp filament for such delicate jobs as spot welding detached retinas?

B. Activity Questions:

1. Laser light I - focus
Look at the point of light from the laser on the screen.
Compare this to the incandescent light.
Does the size of the point change much as you move the laser closer to or further from the screen?
Look at the beam from the side. Can you see the beam in between the laser and the screen?
Can you see a beam from the incandescent light?

2. Laser light II – spectrum
Shine the light from the incandescent light through a prism to observe its spectrum.
What do you observe?
Now shine the laser beam through the prism.
What do you observe this time?
Why are the spectrums of the two light sources so different?

C. Quantitative Questions:

1. There are many different types of lasers, and they are generally named according to the type of lasing material used. The lasing material may be gas, liquid or solid. The most common type of gas laser is the helium-neon laser. The pocket sized laser pointers use a semiconductor diode to produce a beam of laser light.

The first type of laser invented was the ruby laser. The lasing medium is a synthetic ruby crystal of aluminium oxide and chromium atoms, which is excited by flash lamps. Ruby lasers have many applications including cosmetic ones such as unwanted hair removal, tattoo removal and various skin treatments for freckles, sun spots and wrinkles. Ruby lasers are good for tattoo removal as the light is strongly absorbed by black, blue and green inks.

Consider a ruby laser which emits light at a wavelength of 694.4 nm. This laser emits pulses which last for 1.20×10^{-11} s and the energy released per pulse is 0.150 J.

a. What is the length (distance in m) of the pulse?

b. How many photons are in each pulse?

c. What is the power delivered in each pulse?

2. An atom has two energy levels with a transition wavelength of 580 nm.

At 300 K, there are 4.0×10^{20} atoms are in the lower state.

a. How many occupy the upper state, under conditions of thermal equilibrium?

Suppose, instead, that 7.0×10^{20} atoms are pumped into the upper state, with 4.0×10^{20} in the lower state.

b. How much energy could be released in a single laser pulse?

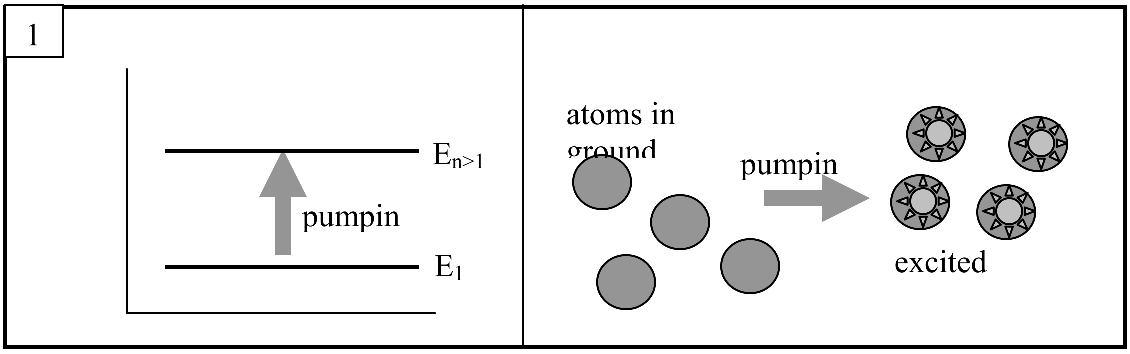
Workshop Tutorials for Physics

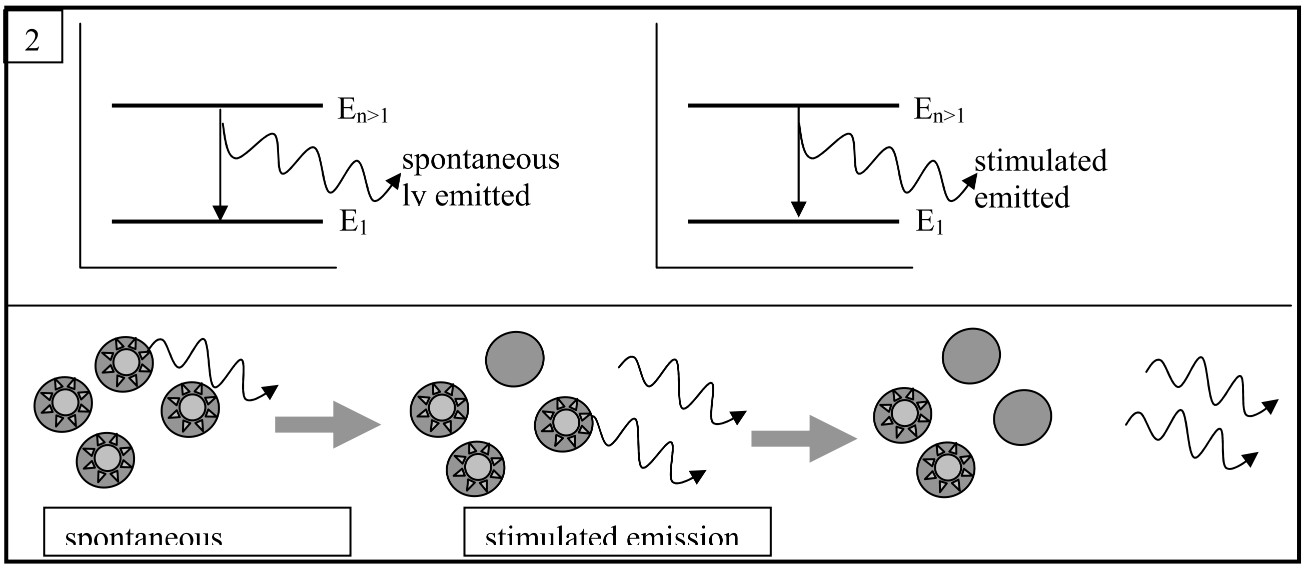
Solutions to QR9: **Lasers**

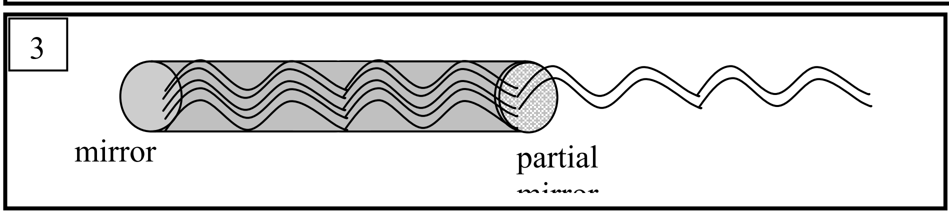
A. Qualitative Questions:

1. LASER stands for Light Amplification by Stimulated Emission of Radiation.

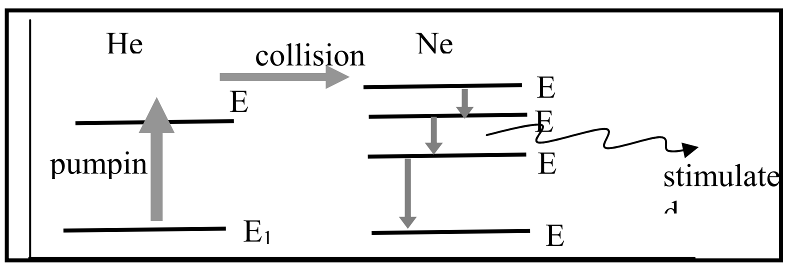
a. Atoms or ions of a suitable material are excited by the pumping process, usually electrical discharge, so that a population inversion is achieved, giving more atoms in a higher energy state than in the ground state (1). Excited atoms release photons as they relax. This is spontaneous emission. These photons stimulate other excited atoms to release photons in phase with the stimulating photon. This is stimulated emission (2). All these photons are contained in a resonant cavity, in which only waves with particular wavelengths can exist as standing waves. The resonant cavity consists of a mirror and a partial mirror which allows some photons through (3). These photons are the laser beam.







b. Helium is necessary because it is the collisions between the helium atoms and neon atoms which produce the population inversion of the neon atoms.



2. Laser light is highly monochromatic, highly directional and can be focused sharply. The advantage of a "laser knife" is that laser light cuts and coagulates at the same time, leading to substantial reduction in blood loss. An infrared laser can cut through muscle tissue by heating and vaporising the water in the cells.

B. Activity Questions:

1. Laser light I - focus

The point of light from the laser is much smaller than that from the incandescent light, and does not change much you move the screen close or further away.

You cannot see a laser beam from the side, as all the photons are going straight ahead in the beam, and none are coming to your eyes. If you put some chalk dust in the beam you will be able to see some of the laser light scattered by the dust.

The incandescent light emits light in a wider angle, so you can see light from it when you look from the side.

2. Laser light II – spectrum

An incandescent light source produces a continuous spectrum of light, which can be broken up into separate colours by a prism. A laser produces almost monochromatic light, so when shone through a prism it bends, but still comes out as a single wavelength beam.

The laser produces light by a process known as stimulated emission, the light is the energy lost by electrons when they move from one energy level to a lower energy level, hence it is monochromatic. An incandescent light works by passing current through a filament which heats the filament. Hot objects emit a spectrum of radiation, with a peak emission at a frequency which is determined by the temperature. The incandescent globe emits all frequencies from the infrared into the visible range.

C. Quantitative Questions:

1. A ruby laser emits light at wavelength 694.4 nm. A laser pulse is emitted for 1.20×10^{-11} s and the energy release per pulse is 0.150 J.

a. The length of the pulse is $d = vt = 1.20 \times 10^{-11}\text{ s} \times c = 3.6 \times 10^{-3}$ m.

b. Each photon in the pulse has $E = hf = hc/\lambda = 2.7 \times 10^{-19}$ J.

The total number in each pulse is therefore $0.150\text{ J} / 2.7 \times 10^{-19}\text{ J. photons}^{-1} = 5.55 \times 10^{17}$ photons.

c. Power is energy, in joules, per second so in a 1.20×10^{-11} s pulse the power is

$P = 0.15\text{ J} / 1.20 \times 10^{-11}\text{ s} = 1.25 \times 10^{10}$ W, or 12.5 giga-watts! This is a lot.

2. An atom has two energy levels with a transition wavelength of 580 nm. At 300 K, 4.0×10^{20} atoms are in the lower state.

a. Label the upper level 1 and the lower level 2. Then use

$$n_1/n_2 = e^{-(E_1 - E_2)/kT}$$

rearrange for n_1:

$$n_1 = n_2\, e^{-(E_1 - E_2)/kT} = n_2\, e^{-hc/\lambda kT} = 5.0 \times 10^{-16} << 1.$$

Approximately zero electrons occupy the upper state, under conditions of thermal equilibrium.

b. 7.0×10^{20} atoms are pumped into the upper state, with 4.0×10^{20} in the lower state. The energy released is:

$$E = n_1 E_{photon} = \frac{n_1 hc}{\lambda} = 240\text{ J}.$$

Workshop Tutorials for Physics

QR10: **Quantum Technology**

A. Qualitative Questions:

1. The invention of the transistor changed the world of electronics. Virtually every electronic device, from a digital watch, to a supercomputer, and everything in between, contains transistors.
The most common type of transistor used in digital electronics is the field effect transistor, or FET. There are MISFETS, MESFETS, and so on. Technologically the most important FET is the MOSFET or metal oxide semiconductor.

a. What does a transistor do?

b. Draw a diagram showing the main components of a MOSFET and give a simple description of how it can work as a switch.

2. Quantum tunneling has many important applications, from the tunneling diode to the scanning tunneling microscope (STM) and even radioactive decay.
The fraction of particles of mass *m* and energy *E* which will tunnel through a barrier of average potential energy height $<U>$ and width *a* is approximately

$$P_{tun} \cong e^{-\frac{4\pi a}{h}\sqrt{2m<U>-E}}$$

a. Inspect this formula and describe which factors will favour tunneling.
In a semiconductor, the electron moves *as if* its mass were less than the actual free electron mass. In silicon, $m_{e\ eff} = 0.4m_e$ and in gallium arsenide $m_{e\ eff} = 0.067m_e$.

b. Explain why this reduced mass has consequences for the speed of electronic devices made from semiconductors.

B. Activity Questions:

1. Compact Discs

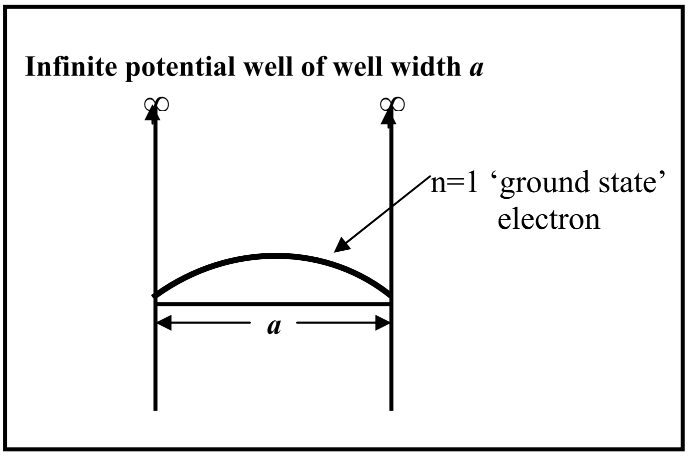
Examine the CD under the microscope. Can you see the pattern of pits which digitally stores the information?
How is this different to the way information was stored on vinyl records?
How is this different to the way information is stored on magnetic tape?

2. Transistors

Examine the circuit.
What does the transistor do?
Investigate the behaviour of the transistor by varying the voltage supplied.
What happens when you increase or decrease the voltage?

C. Quantitative Questions:

1. In the quantum well, a confined electron is described as a wave of wavelength λ according to the the de Broglie relationship: $\lambda = h/p$ where p is the particle (electron) momentum. The lowest energy, or ground state wavfunction, n=1, is shown in the diagram. Since the electron energy is quantized, i.e. only certain energy values are allowed, the electron can't exist at energies between these values (unless we change something, like the well width), it is represented by certain allowed 'standing wave' patterns. We can think of these by analogy with the allowed modes of vibration of a stretched string. (But beware! That is a classical mechanics situation, the electron and the well containing it are *very small* so the electron needs to be described by *quantum* mechanics!)

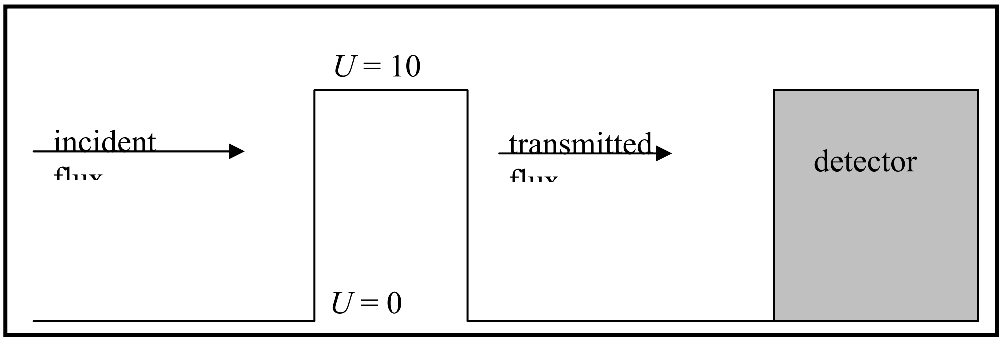


a. Consider the potential well of width a with infinite walls, as shown. Calculate the zero point energy for an electron in a well of width 0.1nm (10^{-10}m) and 10nm.
b. Using the expression for the energy levels in an infinite well, estimate the width of a 'quantum well' such that a transition from the n = 2 to n = 1 level will emit infra-red radiation of wavelength λ = 750 nm. [Remember $E = hf = hc/\lambda$].

2. The probability of a particle tunneling through a barrier can be approximated to

$$P_{tun} \cong 4e^{-\frac{4\pi a}{h}\sqrt{2mU}}$$

if the energy of the particle is much less than the barrier height.
Electrons are incident on a barrier of height 10 keV manufactured from a semiconductor as shown below. The detector on the other side can measure a flux of electrons as low as 100 electrons cm^{-2}.s^{-1}.



a. Draw the wave function of an incident electron which passes through the barrier.
b. If the incident flux is 10^{12} electrons cm^{-2}.s^{-1}, what is the maximum barrier width for electrons to be detectable on the right hand side of the barrier?
c. If this device were made from gallium arsenide, in which $m_{e\ eff} = 0.067m_e$, how would the current on the right hand side of the barrier be different to your estimate in **b** ?

Workshop Tutorials for Physics

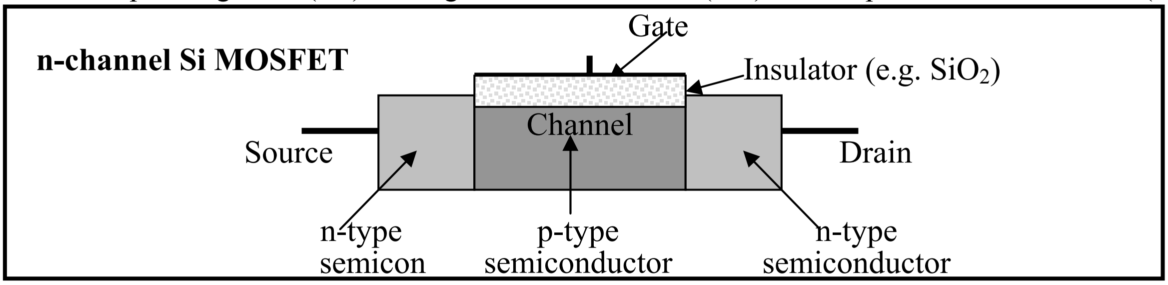
Solutions to QR10: **Quantum Technology**

A. Qualitative Questions:

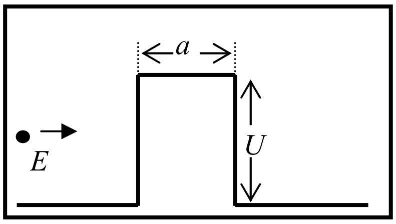
1. Transistors

a. A transistor can be used to amplify signals (e.g. in mobile phones transistor amplifiers amplify (increase the amplitude of) small voltage signal (typically ~ microvolts) from the antenna to produce a signal large enough (~tens of millivolts) in amplitude to drive the earpiece. Transistors are also used to rectify signals (up to high frequencies, for example in satellite communication), and as the basic switching elements in digital electronics.

b. A transistor is a '3-terminal device' (has three major electrical connections) that allows a small amount of charge (applied to the Gate electrode) to control much larger currents conducted between the Source and Drain contacts. The diagram below shows the main components of a MOSFET. Conduction between the source and drain contacts occurs through a narrow sheet of electrons at the interface of the insulator and the p-type semiconductor. In simple terms, voltage bias on the gate modifies the conduction band in the p-type semiconductor causing an electron inversion layer to form – this is the channel shown in the diagram below. The gate can be biased postive or negative with respect to ground (0V) causing the channel to form (ON) or be depleted of free electrons (OFF).



2. Quantum tunneling: The tunneling formula is: $P_{tun} \approx \exp\left[-\frac{4\pi a}{h}\sqrt{2m\langle U\rangle - E}\right]$



This gives the probability of tunnelling, or alternatively, may be interpreted as the fraction of particles of energy E that will tunnel through a barrier of height U and width a metres. The barrier is shown diagrammatically on the left.

Note: it is important to remember that tunnelling is a purely quantum mechanical process. For any appreciable tunnelling probability, the barrier has to be thin, that is, a ~ nanometres.

From the formula we see that $P_{tun} \approx \exp(-2a)$, or, $P_{tun} \approx 1/e^{2a}$. The tunnelling probability falls exponentially (very strongly) with increasing barrier width. The factor most strongly favouring tunnelling will be a thin barrier. The tunnelling probability also goes exponentially with the square roots of the particle mass and barrier height, and incident particle energy.

b. The reduced mass for electrons in semiconductors is an 'effective mass' – but we do not mean that the electron's actual mass is less inside the semiconductor crystal, rather, that the electron *moves as if* it has smaller mass because it experiences both the force from an electric field (if a voltage is applied) *and* the internal field due to the crystal lattice of fixed (+ve) ions. The smaller the effective mass, the faster a device will be since the electrons are accelerated more quickly. GaAs is used at GHz frequencies (satellites and mobile phones), Si is the material of choice for nearly all digital electronics.

B. Activity Questions:

1. Compact Discs

You should be able to see a pattern of pits, which is how the information is stored. The information is stored digitally on a CD, unlike a vinyl record or magnetic tape which are analog devices.

2. Transistors

A transistor allows you to control a large voltage with a small applied voltage. They can be used in many ways, for example as amplifiers. Small adjustments to the input (gate) give a large change to the output. See qualitative question above for more detail.

C. Quantitative Questions:

1. a. Infinite wells of width 0.1 nm and 10 nm. The formula for the energy levels in an infinite square well is $E_n = n^2 \frac{\hbar^2}{8m_e L^2}$, where n is the principal quantum number (n=1,2,3…), m_e =9.1×10^{-31}kg (or, in the case of a semiconductor, we use m^* the effective mass) and L is the well width.

0.1nm well: (ground state energy (n=1) of 0.1nm infinite well)

$$E_1 = n^2 \frac{\hbar^2}{8m_e L^2} = (1)^2 \frac{(1.05\,x10^{-34})^2}{8(9.1x10^{-31})(0.1x10^{-9})^2} = \frac{1.1x10^{-68}}{7.28\,x10^{-50}} = 1.51\,x10^{-19}\text{ J} = 0.94\,\text{eV}$$

10nm well: $E_1 = (1)^2 \frac{(1.05x10^{-34})^2}{8(9.1x10^{-31})(10^{-8})^2}$ = 9.4 × 10^4 eV = 94 keV

b. The expression for an infinite well is $E_n = n^2 \frac{\hbar^2}{8m_e L^2}$

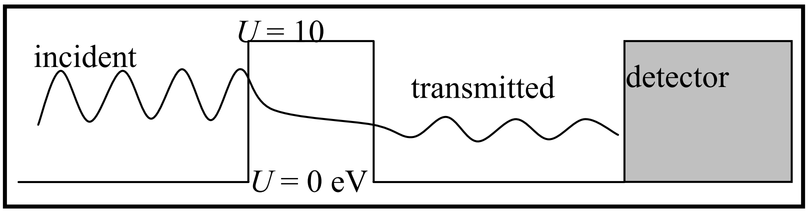
The n = 2 and n = 1 levels are E_2, E_1. The energy change in the transition n=2 to n =1 is

$$\Delta E = E_2 - E_1 = \frac{\hbar^2}{8m_e L^2}[(2)^2 - (1)^2] = \frac{1.05\,x10^{-68}}{8(9.1x10^{-31})L^2}[(3)] = \frac{4.32\,x10^{-39}}{L^2}$$

The transition will emit photons of energy ΔE and wavelength $\lambda_{2-1} = \frac{hc}{\Delta E}$, so

$750x10^{-9} = \frac{hcL^2}{4.32x10^{-39}}$, rearranging for L gives $L = [\frac{(4.32x10^{-39})(750x10^{-9})}{(6.6x10^{-34})(3x10^8)}]^{1/2}$ = 1.28 × 10^{-10} m = 0.13 nm.

2. a.



b. The formula for barrier tunnelling simplifies, if E<<U, to $P_{tun} \cong 4\exp\left[-\frac{2a}{\hbar}\sqrt{2mU}\right]$

We need to be able to detect at least 1.0 × 10^2 electrons cm^{-2}s^{-1} on the other side of a barrier height of 10keV with an incident flux of electrons 1.0 × 10^{12} electrons cm^{-2}s^{-1}, or $\frac{1.0\times10^2}{1.0\times10^{12}} \cong 4\exp\left[-\frac{2a}{\hbar}\sqrt{2mU}\right]$.

The barrier height is 10^4 eV =1.6 × 10^{-15} J, $\hbar = \frac{h}{2\pi}$ = 1.05 × 10^{-34} J.s and m_e= 9.1 × 10^{-31} kg,

so $\frac{1.0\times10^2}{1.0\times10^{12}} \cong 4\exp\left[-\frac{2a}{1.05\times10^{-34}}\sqrt{2(9.1\times10^{-31})(1.6\times10^{-15})}\right] \cong 4\exp[-1.02\times10^{-12}a]$

or 2.5 × 10^{11} ≈exp[1.02 × 10^{12} a].

Taking (base e) logs of both sides, $a = \frac{24.41}{1.02\times10^{12}} = 2.39\times10^{-11}\text{ m} = 0.024\text{ nm}$

So, 0.024nm is the maximum barrier thickness for the output to be detectable with this detector.

c. For gallium arsenide, the effective mass is m^* = 0.067m_e = 0.067 ×9.1 × 10^{-34} kg.

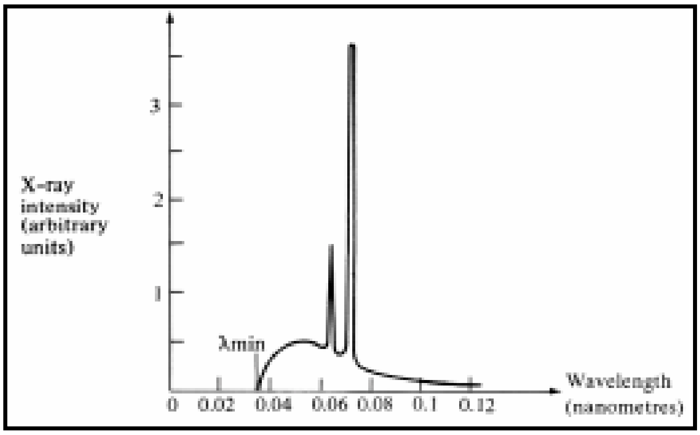
$$\frac{10^2}{10^{12}} \cong 4\exp\left[-\frac{2a}{1.05\times10^{-34}}\sqrt{2(0.067)(9.1\times10^{-31})(1.6\times10^{-15})}\right]$$

The barrier thickness a becomes, $a = \frac{24.41}{2.6\times10^{11}} = 9.38\times10^{-11}\text{ m} = 0.094\text{ nm}$ = 0.1 nm or 1 × 10^{-10} m.

Workshop Tutorials for Physics

QR11: **X-rays I**

A. Qualitative Questions:



1. A plot of x-ray intensity as a function of wavelength for a particular accelerating voltage and a particular target is shown.

a. There are two main components of this x-ray spectrum: a broad range of x-ray energies and a couple of sharp peaks. Explain how each of these arises.

b. What is the origin of the cut-off wavelength λ_{min} of the figure shown below? Why is it an important clue to the photon nature of x-rays?

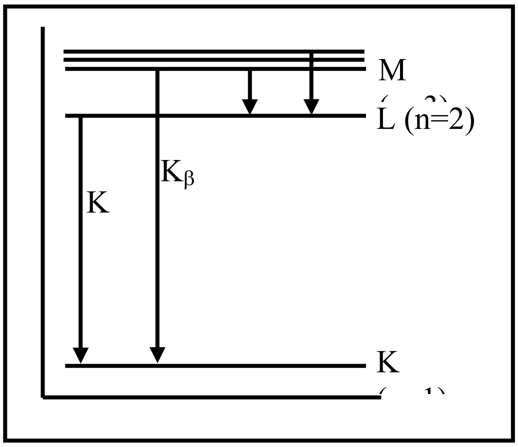
c. What would happen to the cut-off wavelength if the accelerating voltage was increased? What would happen to the characteristic peaks? Use a sketch to show how this spectrum would look if the accelerating voltage was increased.

d. What would happen to the cut-off wavelength if the target was changed, keeping the same accelerating voltage? What would happen to the characteristic peaks? Use a sketch to show how the spectrum would look if some other target material was used, but the accelerating voltage was kept the same.

2. X-rays have many uses, from investigating crystal structures to medical imaging. The use to which the x-rays will be put determines the energy of the desired x-ray radiation. It is important to able to produce x-rays of suitable energy to the application.

a. Draw diagrams which show the process leading to the emission of an x-ray.

b. Which part of this process is represented in the diagram below?



Often this diagram is shown the other way up, with the *K* atomic energy level at the top and the *L* and *M* levels below, with arrows pointing from lower to higher levels.

c. What process is represented by the arrows when the diagram is drawn the other way?

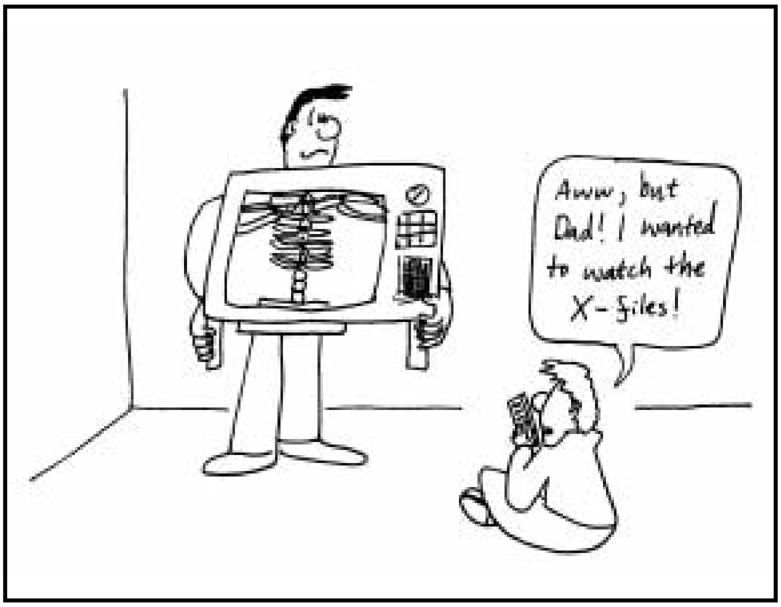
B. Activity Questions:

1. X-ray tube
Examine the x-ray tube and draw a diagram showing the main components.
Where are the electrons emitted?
How and where are they accelerated?

2. Cathode ray tube
The cathode ray tube, one of the main components of cathode ray oscilloscope and Television sets, is very like the x-ray tube. Identify the main components. How is the cathode ray tube different to the x-ray tube?

C. Quantitative Questions:

1. You have decided to build your own x-ray machine out of an old television set. The electrons in the TV set are accelerated through a potential difference of 20kV. What will be the λ_{min} for this accelerating potential?



2. A tungsten target (Z=74) is bombarded by electrons in an x-ray tube. The *K*, *L*, and *M* atomic x-ray energy levels for tungsten are -69.5, -11.3 and -2.30 keV, respectively.
a. Why are the energy levels given as negative values?
b. What is the minimum kinetic energy of the bombarding electrons that will permit the production of the characteristic K_α and K_β lines of tungsten?
c. What is the minimum value of the accelerating potential that will give electrons this minimum kinetic energy?
d. What are the K_α and K_β wavelengths?

Workshop Tutorials for Physics

Solutions to QR11: **X-rays I**

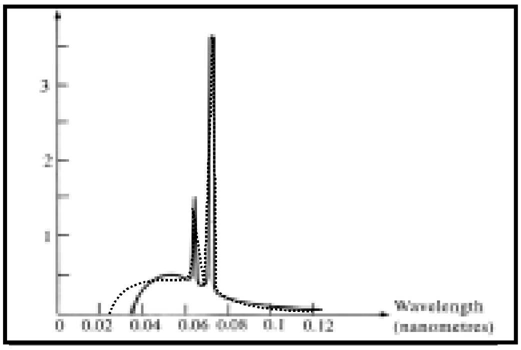
A. Qualitative Questions:

1. X-ray production – characteristic radiation and bremstrahlung.

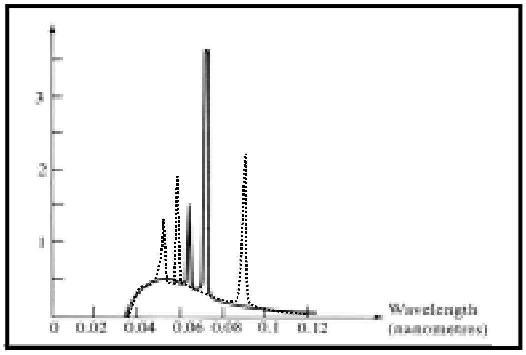
a. X-rays are produced by bombarding a target material with high energy electrons. If the incident electron interacts or collides with an atom in the target then it will lose some of its kinetic energy. This energy can be emitted as an x-ray. The broad range of x-ray wavelengths is the bremstrahlung ("braking radiation"). It arises from the sudden decelerations of the electrons as they strike the target. Since there will be a range of magnitudes of accelerations, there will be a range of x-ray energies. The sharp spikes in the spectrum are the characteristic radiation. These x-ray wavelengths are characteristic of the particular atoms in the target. Some of the bombarding electrons cause electrons within the target atoms to be promoted to higher energy levels. When these electrons drop down again to lower levels, they release energy in the form of photons. The photons have an energy which is the difference in energy between two electron shells in the target atoms.

b. If the incident electron gives up all its kinetic energy in a single interaction a photon with the highest possible energy will be produced. This maximum energy corresponds to the minimum wavelength, λ_{min}. It is impossible to get an x-ray with higher energy (shorter wavelength) than that originally possessed by the incident electron. This is an important clue to the photon nature of x-rays, more collisions will produce more photons, but not higher energy photons, in the same way that increasing the intensity of the incident light in the photoelectric effect will increase the photo-current, but not the stopping voltage.

c. If the accelerating voltage was increased, the cut-off wavelength would decrease (dotted line in figure) as each incident electron would carry more energy allowing higher energy x-rays to be produced. The characteristic peaks would not change as these correspond to x-rays emitted when electrons move from one energy level to another in the target atom. These energy levels will not change, hence the characteristic peaks will not change. The characteristics x-rays are characteristic of the target material.

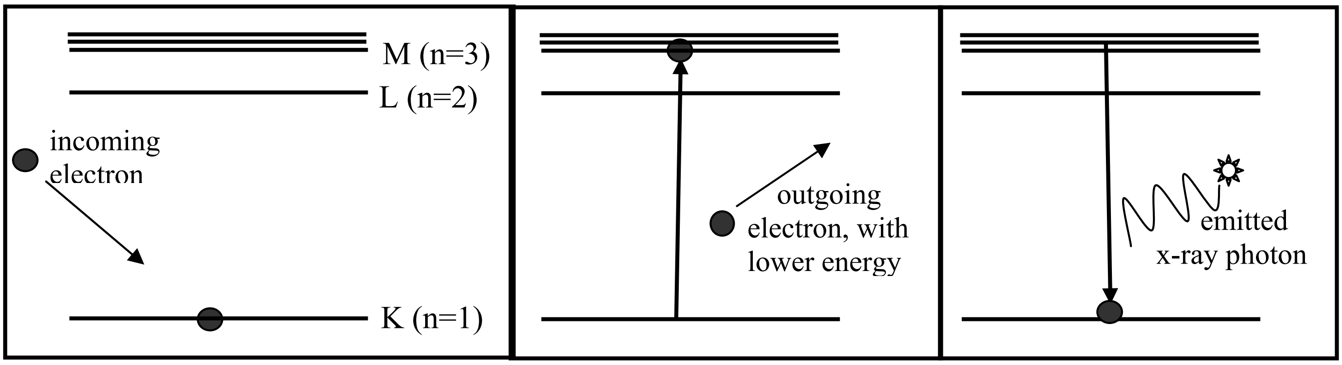


d. If the target was changed the cut-off wavelength would remain the same. The characteristic peaks would change as these depend on the electron energy levels of the target material, see figure opposite.

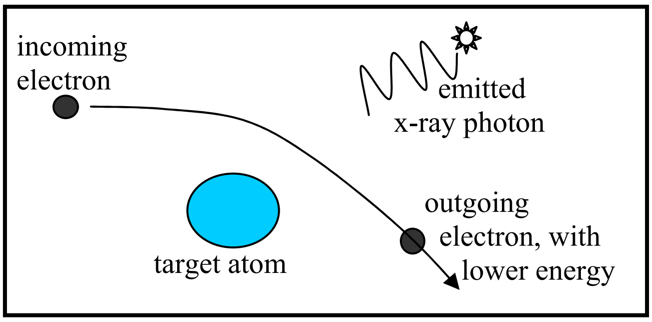


2. Production of characteristic and bremstrahlung x-rays.

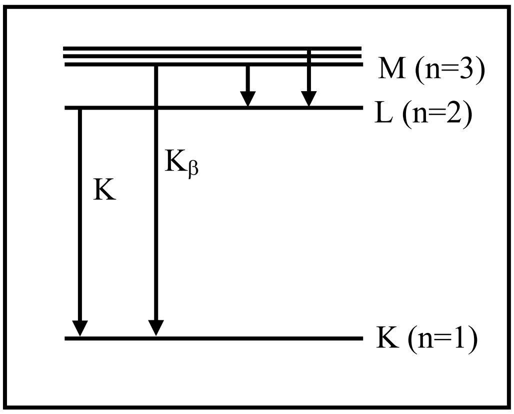
a. Production of a characteristic x-ray:



a. continued- bremstrahlung production**.**
The bremstrahlung is due to the rapid deceleration of the incident electrons as they hit the target. The change in kinetic energy of the incident electron is given off as an x-ray photon.



b. The diagram opposite represents the electrons in the target atoms dropping to lower energy levels, during this process the characteristic x-ray photons are emitted. These x-rays are characteristic of the *target* atoms.
c. Often this diagram is shown the other way up, with the *K* atomic energy level at the top and the *L* and *M* levels below, with arrows pointing from lower to higher levels. In this case the process represented by the arrows is the absorption of energy by inner shell electrons and their resultant promotion to higher energy levels.



B. Activity Questions:

1. X-ray tube
The electrons are emitted by the filament, and are accelerated as they pass between the plates or coils along the tube.

2. Cathode ray tube
The cathode ray tube contains an electron source, and coils to accelerate the electrons. The electrons are deflected by a magnetic field produced by more coils. The electrons are incident on a phosphorus coated screen where they cause the emission of a visible photon. The only difference between this and the x-ray tube is that the electrons are more precisely "steered" to points on the target, and the target emits visible photons rather than x-ray photons.

C. Quantitative Questions:

1. The electrons in the TV set are accelerated through a potential difference of 20kV.
$\lambda_{min} = hc/K = 6.63 \times 10^{-34}$ J.s $\times 3.00\times10^{8}$ m.s^{-1} / 20×10^{3} eV $\times$ 1.6 $\times 10^{-19}$ J.eV^{-1} = 6.2×10^{-11}m = 62 pm.

2. A tungsten target (Z=74) is bombarded by electrons in an x-ray tube. The *K*, *L*, and *M* atomic x-ray energy levels for tungsten are -69.5, -11.3 and -2.30 keV, respectively.
a. The energy levels are given as negative values because these are the values of electrical potential energy when a free electron is taken as the reference at 0 eV. In other words, they are the energies required to totally remove the electron from that energy level. It is rather like the gravitational potential energy down the bottom of a hole when the surface of the earth is taken as the reference of zero.
b. The minimum kinetic energy of the bombarding electrons is the energy required for the transition:
$K\alpha$ line is from the transition from n = 2 to n = 1 energy level, ΔE = (69.5 – 11.3) = 58.2 keV
$K\beta$ line is from the transition from n = 3 to n = 1 energy level, ΔE = (69.5 – 2.3) = 67.2 keV
c. The minimum values of the accelerating potential are 58.2 keV and 67.2 keV, respectively.
d. $K\alpha : E = hf = hc/\lambda$
so $\lambda = hc/E$ = (6.63 x 10^{-34} Js x 2.98 x 10^{8} ms^{-1})/(58.2 x 10^{3} eV x 1.6 x 10^{-19} J eV^{-1}) = 2.1 x 10^{-11} m
$K\beta : \lambda = hv/E$ = (6.63 x 10^{-34} Js x 2.98 x 10^{8} ms^{-1})/(67.2 x 10^{3} eV x 1.6 x 10^{-19} J eV^{-1}) = 1.8 x 10^{-11} m.

Workshop Tutorials for Biological and Environmental Physics

QR12B: **X-rays II- X-ray Interactions and Applications**

A. Qualitative Questions:

1. Most people have an x-ray at some point in their life, usually when something goes wrong. X-rays are a very valuable diagnostic tool as they allow doctors to look inside people without having to cut them open.

a. Why is it possible to use x-rays to see inside things that we can't see into with visible light?

b. When you have an x-ray taken, for example if you break a bone, why are some bits of the picture light and others dark? Which areas have the higher absorption of x-rays, and why?

X-rays are also widely used in medical research, particularly by microbiologists and biochemists to gain information about the structure of molecules. This is because x-rays can be used to "see" things that are smaller than we could see with visible light

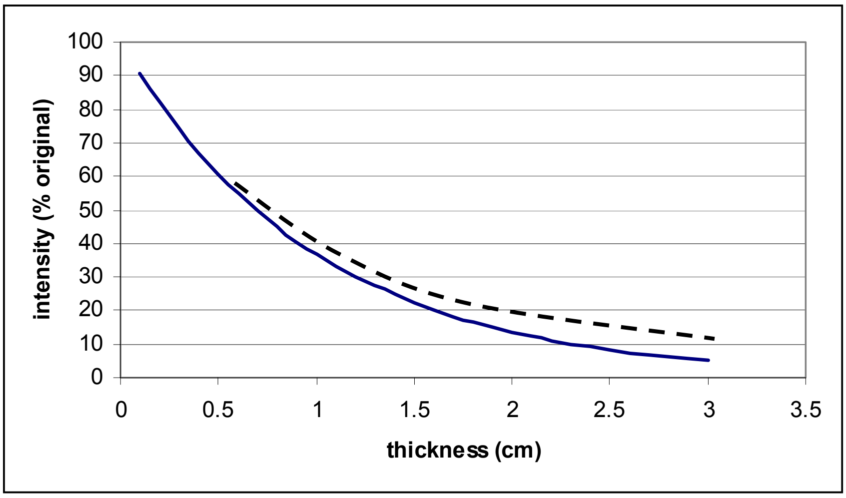
c. Explain why this is possible.

2. The diagram below shows the intensity versus thickness of attenuator for an x-ray beam from a clinical x-ray machine, and a beam of mono-energetic x-rays.

a. Which line represents a mono-energetic x-ray source?

b. Which line represents a clinical x-ray source?

c. Why are they different?



B. Activity Questions:

1. X-ray pictures

Examine the x-ray films.

Why are some areas light and others dark? In which areas are more x-rays absorbed?

Why are more x-rays absorbed in these regions?

2. Diffraction patterns

When a microbiologist or biochemist wants to find out the structure of a protein, for example an enzyme, they will almost always use x-ray diffraction. In this technique the x-rays are shone onto a crystallised sample of the protein and the diffraction patter of the reflected beam gives information about the structure of the protein. We can see how diffraction patterns are formed using the laser and piece of fabric.
Shine the laser light through the fabric.
What sort of pattern do you see?
How does the pattern change when you stretch the fabric horizontally?
What about when you stretch it vertically?

3. X-ray diffraction pattern

Examine the x-ray diffraction pattern and try to match the peaks to the lattice spacings in the crystal.
How many peaks can you match?
What would you expect the pattern to look like if you had a more complex crystal structure, such as that of a crystallised protein?

C. Quantitative Questions:

1. When x-rays scatter from a material we may observe Compton scattering.
a. What is the Compton effect? Use diagrams to illustrate your answer.
An x-ray photon with a wavelength of 100 pm collides with an electron at rest and is scattered at an angle of 90^o.
b. What is the change in wavelength of the photon?
c. What is the kinetic energy of the electron?
d. How many head on ($\phi = 180^o$) scattering events would be necessary to double the wavelength of a 100 pm x-ray?

2. The depth of tissue which x-rays will penetrate can be calculated using the mass attenuation coefficient for that tissue. The mass attenuation coefficient, μ/ρ, is the attenuation coefficient divided by the density of the tissue. For x-rays with energies of some keV, (typical diagnostic x-rays) the mass attenuation coefficient for most body tissues is $\mu/\rho \sim 10^3$ $cm^2.g^{-1}$. The intensity at a depth t through a tissue is given by $I = I_o\, e^{-\mu t}$.where I_o is the incident intensity.

The table below lists some typical densities for different body tissues.

tissue type	density (ρ), $g.cm^{-3}$
adipose (fat)	0.95
blood	1.06
bone (skull)	1.9
muscle	1.05

a. Which of the above tissues is the best absorber of x-rays? Which is the worst?
b. What are the values of μ for the tissues you gave in part **a** ?
c. What thickness of bone will absorb 50% of incident x-rays?
d. What proportion of incident x-rays will be absorbed by 4.0 cm of muscle?

Workshop Tutorials for Biological and Environmental Physics

Solutions to QR12B: **X-rays II- X-ray Interactions and Applications**

A. Qualitative Questions:

1. Using x-rays for imaging.

a. The photons which make up x-rays are at a much higher energy than those from visible light. This means they can penetrate substances which would not be possible for lower energy photons.

b. Typically x-rays are directed to the part of the body and the x-ray film will be placed underneath. X-rays are like a picture "negative". Lighter regions in an x-ray picture indicate areas which have been exposed less to the x-rays. So, for instance, denser substances like bone absorb more of the x-rays and allow less to fall on the x-ray film and it is less exposed. Soft tissues like muscle do not absorb or attenuate the x-rays as much, allowing more to fall on the film and show up as darker patches.

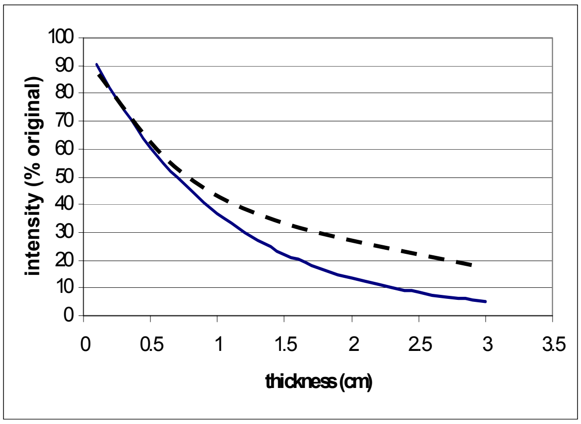
c. When an object is smaller than a wavelength, the waves just diffract around the object and keep travelling as if nothing was in their path. In order to "see" something with waves, the object needs to disturb the waves in some ways. This means that objects can only be seen if they are at least as big (or bigger) than a wavelength. The wavelengths of x-rays are many orders of magnitude less than the wavelengths of visible light. Thus visible light diffracts around small objects while x-rays will not.

2. Intensity versus thickness of attenuator for clinical and mono-energetic x-ray beams.

a. The solid line shows a mono-energetic x-ray source.

b. The dashed line represents a clinical x-ray source.

c. The clinical beam is polychromatic (ie has many "colours" or wavelengths or energies). Higher energy x-rays are absorbed less than lower energy x-rays through the same substances. So as a polychromatic beam travels through a substance, the lower energy x-rays are absorbed to a greater extent at first – this leaves a higher fraction of the higher energy x-rays. The beam becomes "harder" and less prone to absorption as it travels further.



B. Activity Questions:

1. X-ray images

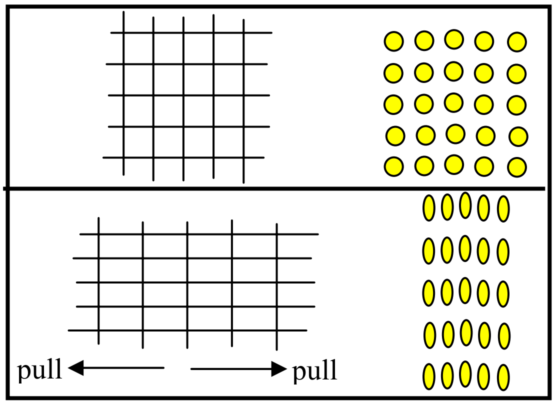
Because of their short wavelength, X- rays can pass through objects which are opaque to visible light. They can penetrate most tissue, but are absorbed well by bone and can be recorded on photographic film. The different levels of 'greyness' is due to the different abilities of the various tissues to absorb x-rays, the greater the absorption the fewer x-rays get through, and the lighter the film.

2. Diffraction patterns

The network of fine threads in the fabric forms a grating. When you shine the laser light through the fabric you see a diffraction pattern.

The spacing between the maxima in the pattern (bright spots) is inversely proportional to the grid spacing.

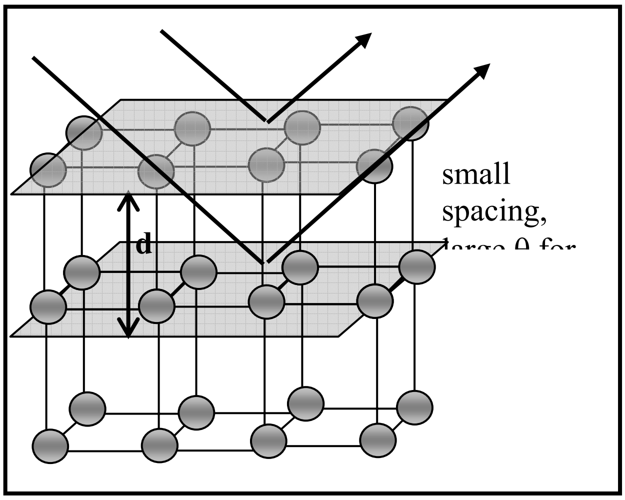
The diagrams show the fabric to the left and the diffraction pattern to the right. When you stretch the fabric horizontally it also squeezes in vertically, the pattern will do the reverse of this, squeezing in horizontally and stretching vertically.



X-ray diffraction

The atoms in the crystal form planes. The spaces between the planes are like the spaces between the lines of a diffraction grating. The larger spacings give smaller angles, because for constructive interference $2d\sin\theta = n\lambda$, so the peaks at larger angles are for the smaller lattice spacings, such as the planes which make the sides of the unit cells for the crystal shown. The smaller angles are for large lattice spacings.

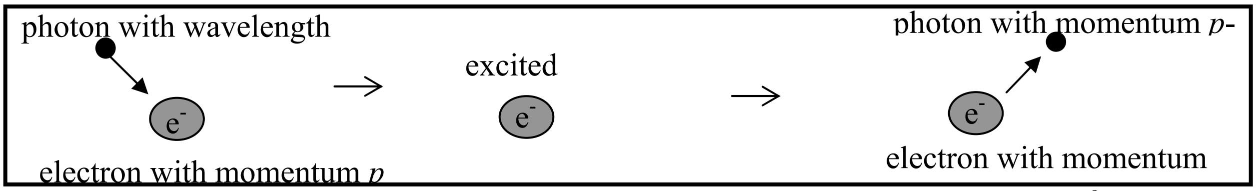
A protein crystal has very complex symmetry, hence will have a very complex diffraction pattern, and they can be extremely difficult to interpret.



C. Quantitative Questions:

1. When x-rays scatter from a material we may observe Compton scattering.

a. The Compton effect is the scattering of a photon of wavelength λ from an electron. The electron takes some energy and momentum from the photon, which continuous with increased wavelength, $\lambda + \Delta\lambda$. Compton scattering is better modeled as an absorption and re-emission process, than as a simple scattering process. The photon is absorbed, and then a second photon is emitted from the excited electron giving a net change in energy and momentum of the electron. In Compton scattering a photon is emitted from the electron after absorption, and we usually treat the process as single scattering event, while in the photoelectric effect no photon is emitted after absorption.



b. The change in wavelength of the photon is $\Delta\lambda = h/mc\,(1-\cos\phi) = h/mc$ where the scattering angle is 90°

so $\Delta\lambda = h/mc = 6.63 \times 10^{-34}$ J.s $/(9.11 \times 10^{-31}$ kg $\times$ 3.00×10^{8} m.s$^{-1}) = 2.43 \times 10^{-12}$ m = 2.43 pm

c. The kinetic energy of the electron, which was initially at rest, will be the energy lost by the photon:

$$\Delta E = hf - hf' = hc/\lambda - hc/\lambda' = \frac{hc}{\lambda} - \frac{hc}{\lambda+\Delta\lambda} = \frac{hc\Delta\lambda}{(\lambda+\Delta\lambda)\lambda}$$

$$= \frac{6.63\times10^{-34}\ \text{J.s}\times3.00\times10^{8}\ \text{m.s}^{-1}\times2.43\times10^{-12}\ \text{m}}{(100\times10^{-12}\ \text{m}+2.43\times10^{-12}\ \text{m})100\times10^{-12}\ \text{m}} = 4.72 \times 10^{-17}\ \text{J} = 295\ \text{eV}.$$

d. Each head on ($\phi = 180°$) scattering event will give a

$\Delta\lambda = h/mc\,(1-\cos\phi) = 2h/mc = 2 \times 6.63 \times 10^{-34}$ J.s $/(9.11 \times 10^{-31}$ kg $\times$ 3.00×10^{8} m.s$^{-1}) = 4.85 \times 10^{-12}$ m

This will be the change regardless of the photons initial wavelength.We require a change of 100 pm = 100 $\times 10^{-12}$ m to double the wavelength, which corresponds to 1.00×10^{-10} m / 4.8×10^{-12} m per collision = 21 collisions.

2.

a. Bone is the best absorber of x-rays because it is the most dense, fat is the worst.

b. see table below:

tissue type	density (ρ), g.cm^{-3}	$\mu=\mu/\rho \times \rho$
adipose (fat)	0.95	0.257
blood	1.06	0.286
bone (skull)	1.9	0.513
muscle	1.05	0.284

c. To absorb 50% of incident x-rays, we want: $\frac{I}{I_0} = ½ = e^{-\mu t}$ which we rearrange for t, and putting in $\mu = 0.513$ for bone gives $-\mu t = \ln(½)$, so $t = \frac{\ln\frac{1}{2}}{-\mu} =$ 1.35 cm.

d. The proportion of incident x-rays absorbed by 4.0 cm of muscle is: $\frac{I}{I_o} = e^{-\mu t} = e^{-4.0 \times 0.284} = 0.32 = 32\%$.

Workshop Tutorials for Technological and Applied Physics

QR12T: X-rays II- X-ray Interactions and Applications

A. Qualitative Questions:

1. X-rays are widely used in industry and in medicine for imaging. They can be used to find bone damage and flaws in engine parts, and to investigate the structure of things too small to see with visible light.
a. Why is it possible to use x-rays to see inside things that we can't see into with visible light?
b. X-rays can also be used to see things that are smaller than we could see with visible light. Explain why this is possible.
c. When you have an x-ray taken, for example if you break a bone, why are some bits of the picture light and others dark? Which areas have the higher absorption of x-rays, and why?

2. When x-rays scatter from a material we may observe Compton scattering.
a. What is the Compton effect? Use diagrams to illustrate your answer.
b. Why don't we observe Compton scattering with visible light?
c. How is it possible for the photons to give up part of their energy when we know from the photoelectric effect and other experiments that photons are particles and only a whole photon can be absorbed, not just part of a photon?
d. During Compton scattering the x-ray transfers some of its momentum to an electron. If the x-ray has momentum, does this mean that it has mass?
e. What is the difference between the Compton effect and the photoelectric effect?

B. Activity Questions:

1. X-ray pictures
Examine the x-ray films.
Why are some areas light and others dark? In which areas are more x-rays absorbed?
Why are more x-rays absorbed in these regions?

2. Diffraction patterns
Shine the laser light through the fabric.
What sort of pattern do you see?
How does the pattern change when you stretch the fabric horizontally?
What about when you stretch it vertically?

3. X-ray diffraction
Examine the x-ray diffraction pattern and try to match the peaks to the lattice spacings in the crystal.
How many peaks can you match?
What would you expect the pattern to look like if you had a more complex crystal structure?

C. Quantitative Questions:

1. An x-ray photon with a wavelength of 100 pm collides with an electron at rest and is scattered at an angle of 90^o.

a. What is the change in wavelength of the photon?

b. What is the kinetic energy of the electron?

c. How many head on ($\phi = 180^o$) scattering events would be necessary to double the wavelength of a 100 pm x-ray?

2. X-ray diffraction is a widely used technique in science and industry. In industry it is used to look at stress and strain of materials, particularly during manufacture, for example on turbine blades for aircraft. It is also being used by researchers in Australia to look at changes to the structure of hair with breast cancer, as a possible diagnostic tool.

a. Explain briefly how x-ray diffraction can give information about the structure of a material.

b. Why does stress on a material cause a broadening of the x-ray diffraction peaks?

An x-ray beam is incident on a salt (NaCl) crystal, which has an inter-planar spacing of 0.281 nm. The second order maximum in the reflected beam is found when the angle between the beam and the surface is 21^o.

c. What is the wavelength of the x-rays?

Workshop Tutorials for Technological and Applied Physics

Solutions to QR12T: **X-rays II- X-ray Interactions and Applications**

A. Qualitative Questions:

1. "Seeing" with x-rays.

a. The photons which make up x-rays are at a much higher energy than those from visible light. This means they can penetrate substances which would not be possible for lower energy photons.

b. When an object is smaller than a wavelength, the waves just diffract around the object and keep travelling as if nothing was in their path. In order to "see" something with waves, the object needs to disturb the waves in some ways. This means that objects can only be seen if they are at least as big (or bigger) than a wavelength. The wavelengths of x-rays are many orders of magnitude less than the wavelengths of visible light. Thus visible light diffracts around small objects while x-rays will not.

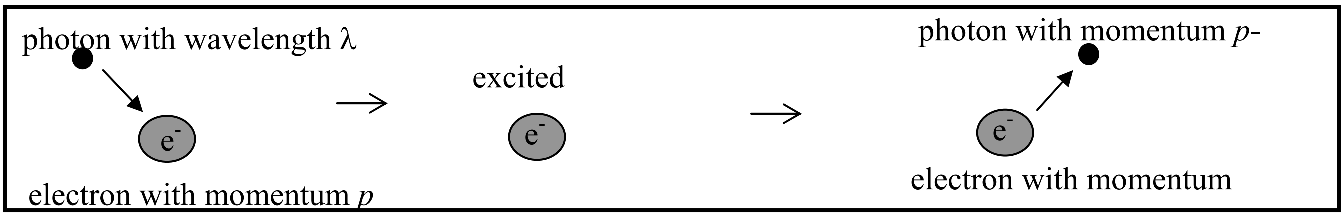
c. Typically x-rays are directed to the part of the body and the x-ray film will be placed underneath. X-rays are like a picture "negative". Lighter regions in an x-ray picture indicate areas which have been exposed less to the x-rays. So, for instance, denser substances like bone absorb more of the x-rays and allow less to fall on the x-ray film and it is less exposed. Soft tissues like muscle do not absorb or attenuate the x-rays as much, allowing more to fall on the film and show up as darker patches.

2. When x-rays scatter from a material we may observe Compton scattering.

a. The Compton effect is the scattering of a photon of wavelength λ from an electron, the electron absorbs some energy and momentum from the photon, which continuous with a longer wavelength, $\lambda + \Delta\lambda$. (See diagram below).

b. The Compton shift, $\Delta\lambda$, depends only on the scattering angle of the photon, and not on the wavelength. This maximum shift is for scattering at 180^o, which gives a $\Delta\lambda$ of 4.8×10^{-12} m. This shift is tiny compared to the wavelength of visible light, around 10^{-6} m, and hence the shift is not detectable for visible light.

c. It is generally true that light is quantised and only a whole photon can be absorbed, not part of a photon. Compton scattering is better modeled as an absorption and re-emission process, than simple a scattering process. The photon is absorbed, and then a second photon is emitted from the excited electron giving a net change in energy and momentum of the electron.



d. According to classical wave theory the momentum of a wave is $p = E/v$, the energy divided by the velocity, which is E/c for a photon. Photons have energy, but do not have rest mass.

e. In Compton scattering a photon absorption by an electron, and then another photon is emitted from the electron, however we treat the process as single scattering event. In the photoelectric effect no photon is emitted after absorption.

B. Activity Questions:

1. X-ray images

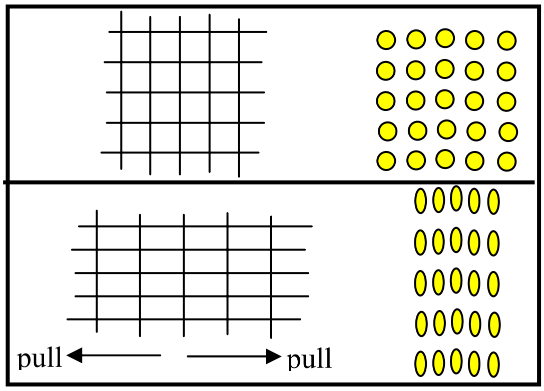
Because of their short wavelength, X- rays can pass through objects which are opaque to visible light. They can penetrate most tissue, but are absorbed well by bone and can be recorded on photographic film. The different levels of 'greyness' is due to the different abilities of the various tissues to absorb x-rays, the greater the absorption the fewer x-rays get through, and the lighter the film.

2. Diffraction patterns

The network of fine threads in the fabric forms a grating. When you shine the laser light through the fabric you see a diffraction pattern.

The spacing between the maxima in the pattern (bright spots) is inversely proportional to the grid spacing.

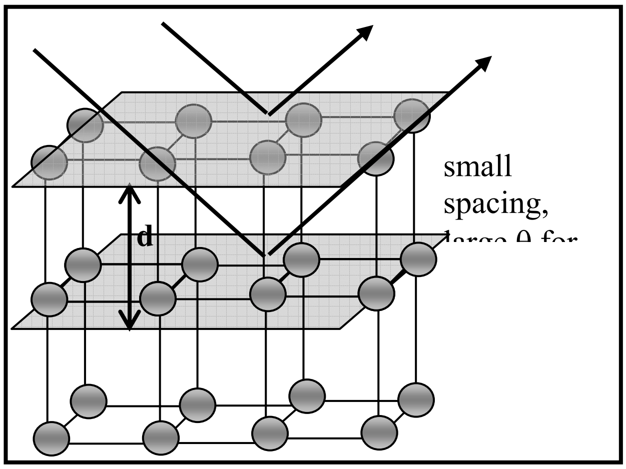
The diagrams show the fabric to the left and the diffraction pattern to the right. When you stretch the fabric horizontally it also squeezes in vertically, the pattern will do the reverse of this, squeezing in horizontally and stretching vertically.



3. X-ray diffraction

The atoms in the crystal form planes. The spaces between the planes are like the spaces between the lines of a diffraction grating. The larger spacings give smaller angles, because for constructive interference $2d\sin\theta = n\lambda$, so the peaks at larger angles are for the smaller lattice spacings, such as the planes which make the sides of the unit cells for the crystal shown. The smaller angles are for large lattice spacings.

This is a reflected beam technique, in contrast to the way x-rays are used in the first activity, in which the attenuated transmitted beam is used to get information about the material.



C. Quantitative Questions:

1. An x-ray photon $\lambda = 100$ pm collides with an electron at rest and is scattered at an angle of 90^o:

a. The change in wavelength of the photon is $\Delta\lambda = h/mc\,(1-\cos\phi) = h/mc$ where the angle $\phi = 90^o$.

so $\Delta\lambda = h/mc = 6.63 \times 10^{-34}$ J.s $/(9.11 \times 10^{-31}$ kg $\times\ 3.00 \times 10^{8}$ m.s$^{-1}) = 2.43 \times 10^{-12}$ m = 2.43 pm

b. The kinetic energy of the electron, which was initially at rest, will be the energy lost by the photon:

$\Delta E = hf - hf' = hc/\lambda - hc/\lambda' = hc/\lambda - hc/(\lambda + \Delta\lambda) = hc\,(1/\lambda - 1/(\lambda + \Delta\lambda))$

$= 6.63 \times 10^{-34}$ J.s $\times\ 3.00 \times 10^{8}$ m.s^{-1} [1/ 100 $\times 10^{-12}$ m - 1/((100 + 2.43) $\times 10^{12}$ m)]

$=1.989\times10^{-25}$ J.m [10^{10} m^{-1} – 9.763×10^{9} m^{-1}] = 4.72×10^{-17} J = 295 eV.

c. Each head on ($\phi = 180^o$) scattering event will give a

$\Delta\lambda = h/mc\,(1-\cos\phi) = 2h/mc = 2 \times 6.63 \times 10^{-34}$ J.s $/(9.11 \times 10^{-31}$ kg $\times\ 3.00 \times 10^{8}$ m.s$^{-1}) = 4.8 \times 10^{-12}$ m

This will be the change regardless of the photons initial wavelength.

We require a change of 100 pm = 100×10^{-12} m to double the wavelength, which corresponds to 1.00×10^{-10} m / 4.8×10^{-12} m per collision = 21 collisions.

2. X-ray diffraction.

a. When x-rays diffract from a material they behave as if they were being reflected from surfaces within the material. At a given angle, called the Bragg angle, the waves reflecting from different planes will interfere constructively to give a peak in intensity. This angle is given by $\sin\theta = m\lambda/2d$ where d is the distance between the planes of atoms. At other angles the x-rays will interfere destructively. Hence by measuring the angles at which peaks occur, the lattice spacing d can be found.

b. Stress on a material causes a broadening of the x-ray diffraction peaks because when the material is stretched or squashed the crystal structure can become distorted, and the spacing between planes can vary. If the spacing d varies slightly then the angle at which a maximum is observed will also vary slightly and a broad peak representing the range of d will be observed.

c. Using $2d\sin\theta = m\lambda$ with $d = 0.281$ nm, $\theta = 21^o$ and $m = 2$:

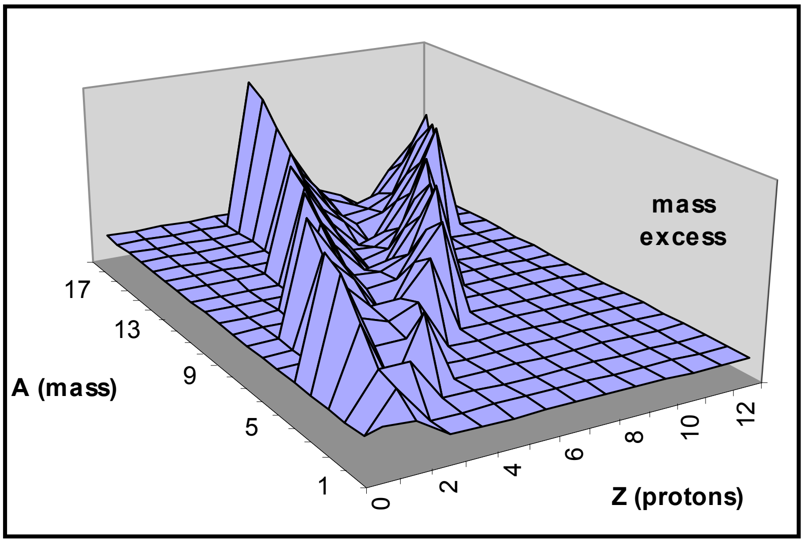
$\lambda = 2 \times 0.281 \times 10^{-9}$ m $\times \sin(21^o) / 2 = 1.0\times 10^{-10}$ m = 0.10 nm.

Workshop Tutorials for Physics

QR13: **The Nucleus**

A. Qualitative Questions:

1. The figure shows the mass excess as a function of A and Z for the first 12 elements.



a. In which region are the nuclei stable?

There are two regions of unstable nuclei, one to the left of the valley, and one to the right.

b. Why are the nuclei to the left unstable?

c. Why are the nuclei to the right unstable?

(Hint: Look at the A and Z numbers.)

The nuclei from each of these regions become stable by different processes.

d. Describe these processes.

e. What happens to N and Z during these processes?

2. Captain Picard and Data have just completed a successful mission on the planet Zog and are beaming back aboard the starship Enterprise (mark III) when there is an error in the transporter circuits!

The transporter de-materialises all their atoms on the planet surface and converts them into information in the circuits of the ship's transporter, and then reconstructs the atoms and puts them back in the right places. However the computer mixes up its N's and Z's so the codes for protons and neutrons are mixed up. When the transporter in the ship reconstructs them all their protons have been exchanged for neutrons and vice versa.

a. What is the effect of this on the carbon, oxygen, nitrogen and hydrogen which make up Captain Picard?

b. What will be the likely effect of this on Captain Picard?

c. What is the likely effect on Data, who is mostly metal (such as copper and iron) beneath his plastic simulated skin?

Hint : you may want to look at a periodic table.

B. Activity Questions:

1. Binding energies

Examine the chart of binding energies.

a. What does the diagram represent?

Fission and fusion are opposite processes, when fission occurs a nucleus breaks apart and when fusion occurs two nuclei fuse to form a larger one.

b. How can both these processes release energy?

c. Which nuclei are more likely to undergo fusion? Which will undergo fission? Explain your answer.

2. Coolite Balls

Charge the coolite balls so that they have opposite charges. What happens?

Now charge them so they have like charges and observe what happens. What would happen to nuclei if there wasn't a strong nuclear force to hold them together?

C. Quantitative Questions:

1. The effective radius of a nucleus can be calculated using $R = R_o A^{1/3}$ where $R_o = 1.2$ fm $= 1.2 \times 10^{-15}$m, and A is the atomic mass number of the nucleus, 197 for gold.

a. Calculate the size of a gold nucleus.

b. If a 5.3MeV α particle is headed directly for a gold nucleus, how close will it get to the centre of the nucleus before it is deflected and scattered back?

(Hint: when does the potential energy due to Coulomb repulsion become equal to the kinetic energy?)

c. Using your answers to **a** and **b**, comment on the likelihood of hitting the nucleus with a neutron compared to an alpha particle.

A thought experiment, or something to try on an oval: imagine making a model of a sheet of atoms with nuclei 1 cm in diameter (marbles, for example), and spacing them so that the atoms were just touching. How hard would it be to hit the nuclei with thrown marbles from several atomic radii away?

2. A deuteron (a proton and a neutron) has a binding energy of 2.22 MeV = 3.55×10^{-13}J.

a. What is the binding energy *per nucleon* for a deuteron?

b. By how much is a deuteron lighter than a proton plus a neutron?

c. How much energy is released when two hydrogen nuclei and two neutrons fuse to form a helium nucleus in the sun ?

d. How much energy is released when uranium232 decays into a thorium228 nucleus and a helium nucleus?

e. Comment on your answers to **c** and **d**.

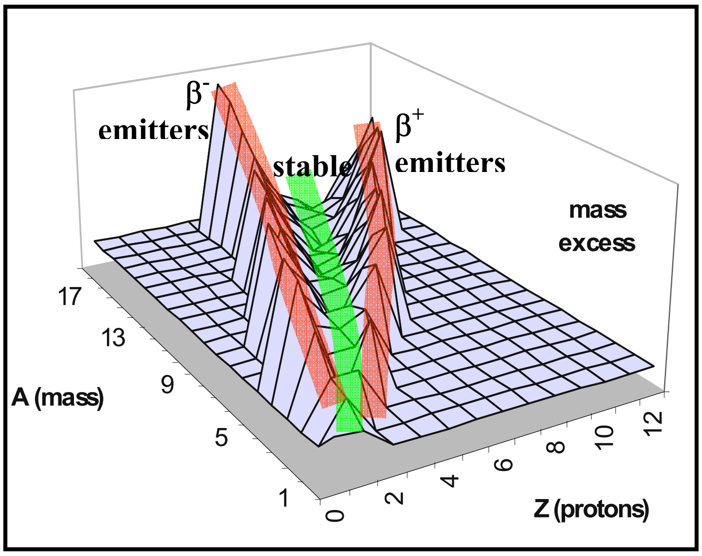
Some useful masses:

Particle	proton	neutron	H	He	^{232}U	^{228}Th
Mass (amu)	1.007276	1.008665	1.0107276	4.002603	232.0371	228.0287

1amu = 1.66054×10^{-27} kg = 931.3 MeV/c^2.

Workshop Tutorials for Physics
Solutions to QR13: **The Nucleus**

A. Qualitative Questions:



1. The valley of stability

a. The region in which the nuclei are stable is in the 'valley' of the graph, called the "valley of stability", where the number of protons and the number of neutrons are approximately equal.

b. The nuclei to the left are unstable because they have too many neutrons for the number of protons, these are called neutron-rich isotopes.

c. The nuclei to the right are unstable because they have too many protons for the number of neutrons, these are called proton-rich isotopes.

d. The neutron rich light elements decay via: [1] $n \rightarrow p + e^-$ [β^- decay - an electron]
The proton rich light elements decay via: [2] $p \rightarrow n + e^+$ [β^+ decay - a positron]
Heavy proton-rich elements decay by alpha emission.

e. In [1] the atomic number, Z, increases by 1 and the number of neutrons decreases by 1. In [2], Z decreases by one and the number of neutrons increases by 1. In both cases the number of nucleons, A, stays the same.

2. The transporter error.

a. Captain Picard is composed mostly of carbon, oxygen, nitrogen and hydrogen. The most common isotopes of C, N and O all have the same number of protons as neutrons, so it won't make any difference to these atoms. However hydrogen has one proton, one electron and no neutron, so the transporter error which swaps protons for neutrons will leave Picard with lots of extra neutrons, a huge deficit of protons but the same number of electrons. Heavier elements, such as calcium in his bones, have more neutrons than protons, and these will become unstable when they become positively charged due to the error.

b. This will be a very negative experience for Captain Picard, and his chemical structure will fall apart as he emits neutrons (from the converted hydrogen atoms) and the excess electrons disperse over him. He will also emit other radiation from the heavier elements, such as calcium which will have become too positive. This may help balance some of the excess electrons, but will still leave these atoms unstable as they will not have enough neutrons, and will become β^+ emitters.

c. Data is mostly metal and most metals such as aluminium, copper and iron have more neutrons than protons to prevent the repulsive Coulombic force from breaking apart the nucleus. Hence the swap will leave the metals with too many protons, which will decay until the metals have stable nuclei again. With too many protons, Data is likely to emit β^+ radiation to convert protons to neutrons.

B. Activity Questions:

1. Binding energies

a. The diagram shows the amount of energy per nucleon that you would need to pull that nucleus into its component protons and neutrons. This is sometimes also expressed as a mass defect, which is the difference between the mass of the nucleus and the sum of the masses of the same number of neutrons and protons.

b. Both fission and fusion can release energy by increasing the mass defect or binding energy of an atom. The lower the binding energy, the less stable the atom.

c. Small nuclei such as hydrogen are more likely to undergo fusion, moving them along the binding energy chart to the right, with increasing binding energy. Large nuclei with small binding energies, those to the far right of the peak, will undergo fission to produce smaller nuclei with higher binding energies. Remember that the binding energy is how much you have to put in to break the nuclei, not how much energy the nuclei have, hence higher is more favourable.

2. Coolite Balls

When the coolite balls have the same charge they repel each other, and when they have opposite charges they attract. If the only force acting on the protons in the nucleus was the Coulomb force, they would repel each other and the nucleus would fall apart.

C. Quantitative Questions:

1. The Size of an atom and its nucleus.

a. $R = R_o A^{1/3} = 6.98\times10^{-15}$m

b. As the α approaches the nucleus it's kinetic energy is converted to electric potential energy. Set the initial kinetic energy equal to the final potential energy, which is where it stops, before reversing and scattering back: K.E = $kq_{\alpha}q_{Au\ nucleus}/d$ rearrange to find d:

$d = kq_{\alpha}q_{Au\ nucleus}$/K.E.= $8.99\times10^{9} \times (2\times1.6 \times 10^{-19}) \times (79\times1.6 \times 10^{-19})/(5.3\times10^{6}\times1.6\times10^{-19}) = 4.29\times10^{-14}$m.

this is almost 10× the size of the nucleus!

c. Your chances of hitting the nucleus with a neutron are less than 1/10 that of hitting it with an α particle, because not only does it have to hit the nucleus itself, it's also a lot smaller than an α.

Thought experiment: An atom is around $10^{-10}/10^{-15}=10^{5}$ × bigger than a nucleus. So if the nucleus is 1cm in diameter, the atom should be $10^{5}\times$ 1 cm = 1 km across! Imagine trying to hit marbles spaced a km apart, from a few kms back!

2. Energy from nuclear reactions.

a. The binding energy per nucleon for a deuteron: Deuteron $B.E.$ = 2.22 MeV and there are two particles, hence $B.E.$ = 2.22 MeV / 2 nucleons = 1.11 MeV / nucleon

b. Mass of proton = 1.007276 amu, Mass of neutron = 1.008665 amu, total = 2.015941 amu.

Mass of deuteron = 2.014102 amu

Δm = 2.015941 - 2.014102 = 0.001849 amu

c. Fusion of 2 H nuclei and 2 neutrons to form a He nucleus:

2 H = 2.014552 amu and 2 n = 2.017330 amu; total = 4.031882 amu

mass of He atom =4.002603 amu

$\Delta m = [2m_p + 2m_n]$ - [mass He] = 0.030377 amu

And we can use $E = \Delta mc^2$, and as Δm is in amu and ΔE is in MeV we can use the conversion factor between amu and MeV- (1 amu = 931.3 MeV):

E = [0.030377 u] c^2 [931.3 MeV/u c^2] = 28.3 MeV.

d. The amount of energy released in the fission of ^{235}U to ^{228}Th + ^{4}He:

[U] 232.0371 → [Th] 228.0287 + [He] 4.0026 = 232.0313 amu.

Δm = 0.0058 amu so ΔE = [0.0058 amu] c^2 [931.3 MeV/amu.c^2] = 5.4 MeV

e. The answers to **c** and **d** illustrate that more energy is released in the process of fusion than fission.

Workshop Tutorials for Biological and Environmental Physics

QR14B: **Radioactivity**

A. Qualitative Questions:

1. A chemist at a large detergent manufacturing company is using radioactively labeled dirt to test the effectiveness of their new enzyme powered super strong no scrub floor cleaner.
a. How can you use radioactively labeled dirt to test the effectiveness of a detergent?
b. What is the advantage of using this method to test the effectiveness compared to other methods?

2. The government has approved funding for a new reactor to replace the existing HiFAR research reactor at Lucas Heights. The reactor supplies isotopes for medicine and industry.
a. Describe how radioisotopes are used in medicine, giving examples of specific isotopes.

This is the reactor building on the ANSTO site at Lucas Heights. This building contains the reactor and much of the instrumentation used for research at the site. The building has airlocks for people and vehicles, and is well insulated.
b. Why are the levels of radiation in the reactor building lower than those outside?
After much debate, it was decided to build it on the existing site to save on infrastructure costs and because of its proximity to Sydney International Airport.
c. Why is it important that the reactor be situated close to the airport?
The new reactor will be a "swimming pool" type, with the reactor core immersed in a pool of water. The water will be a mix of H_2O and D_2O.
d. Why is heavy water used rather than the local tap water?
e. Explain the purpose of the control rods and how they will be used.

B. Activity Questions:

1. Colleens Cubes
Shake the bag containing the nuclei (cubes) and pour them into the tray. Write down the number of cubes with dots showing on top, remove those cubes and replace the rest in the bag. How many cubes are left of the original 100?
Repeat 10 times. Sketch the number of cubes removed (the activity) as a function of number of throws. Sketch the number of cubes remaining in the bag as a function of number of throws.

2. Smoke detector

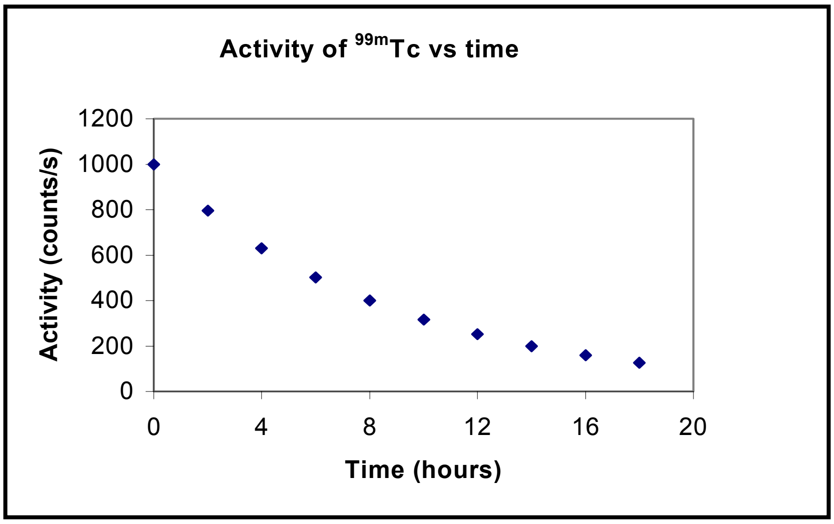
Examine the smoke detector. It contains a radioactive source, ^{241}Americium, an α emitter. The α particles ionize air molecules between two charged plates. The positive ions go to the negative plate, the negative ions to the positive plate, which gives a current.
Use the circuit diagram to locate the main components of the detector.
How does smoke disrupt the current?

C. Quantitative Questions:

1. Technetium is used to image the brain, thyroid, lungs, liver, spleen, kidney, gall bladder, skeleton and blood pool. It is the most commonly used radioisotope in medical imaging. The graph below is for a sample of ^{99m}Tc .
Using the graph find:

a. What is the half life of this sample?

b. What is the decay constant for the sample ?



2. $^{133}_{54}Xe$ is a radioactive gas which is used in studies of lung function. It has a decay constant $\lambda = 1.51 \times 10^{-6}$ s^{-1}. You have just taken delivery of a 4.0 MBq sample for an experiment you wish to conduct.

a. What is the half life of $^{133}_{54}Xe$ in seconds and in days?

b. What will be the activity of your sample in 15.93 days? In 18 days?

The experiment you wish to conduct requires an activity of at least 3.0 MBq.

c. How long do you have before finding a volunteer subject?

d. What mass of $^{133}_{54}Xe$ did you initially acquire?

Workshop Tutorials for Biological and Environmental Physics
Solutions to QR14B: **Radioactivity**

A. Qualitative Questions:

1. Radioactive dirt and detergent testing.
a. The advantage of using radioactively labeled dirt is that it is very easily detected with a Geiger counter or a scintillation detector. Hence one could first measure the radioactivity from the test floor without the radioactively labeled dirt. This will give an estimate of the background radiation in the room. The radioactive dirt is then spread over the floor and the radioactivity measured again. Naturally, one would expect to now detect high levels of radiation. The new enzyme super detergent is used to clean the floor and carefully remove all the contaminated water in a bucket until the floor is dry. At this stage, the radioactivity level from the test floor is measured and compared with the initial measurement without the dirt.
b. This method is very effective because even the smallest particles of dirt (undetectable with normal vision) could easily be detected with a scintillation counter or Geiger counter.

2. Radioisotopes and the replacement reactor.
a. Radioisotopes are used in medicine for imaging and tracing where the patient is given a sample of some sort of radioactive substance, and then the path of the substance is followed to see where it ends up. Radioisotopes are also used therapeutically, in radiotherapy for various cancers. ^{131}I and ^{32}P are used for radiotherapy for thyroid and bone marrow respectively, ^{99m}Tc is the most commonly used isotope for imaging.
b. The radiation levels inside the building are lower than those outside because the building is well shielded. It has two layers of shielding – one around the reactor itself, and one enclosing the entire building. The natural radiation from cosmic rays and other sources outside cannot get into the building, and the radiation from the reactor inside the building is shielded also, so what remains is lower than the natural levels outside the building.
c. Many of the isotopes used for radiotherapy and medical imaging have short half lives, some only a few hours, so if it took a long time to get them to the airport and then to where they are needed, they wouldn't be much use by the time they got there.
d. Heavy water is used rather than the local tap water because it can absorb fewer neutrons and thus allow the reactor to produce more neutrons – which is the purpose of the reactor. Hence heavy water, with some ^{1}H's replaced by ^{2}D's is used.
e. The control rods are used to control the flow of neutrons in the reactor core. They can be inserted to absorb the neutrons and slow down or stop the reaction, or removed to speed up the reaction and bring the reactor up to critical when it is first turned on.

B. Activity Questions:

1. Colleens Cubes
You should have come up with an exponentially decreasing curve for both the number of cubes remaining, and the number of cubes removed at each throw. Many physical processes follow this pattern.

2. Smoke detector
When smoke enters the space between the plates the ions attach themselves to the heavy smoke particles and the flow of current is disrupted, setting off the alarm.

C. Quantitative Questions:

1. Half lives.

a. The half life is the time for the activity to reduce to half its initial value. In this case, determine at what time the activity is 500 counts.s^{-1}. From the graph, this is approximately 6 hours.

b. The decay constant, λ, is given by,

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{6\ \text{hrs}} = 0.116\ \ \text{hr}^{-1}$$

2.

a. The half-life, $T_{1/2}$, is given by, $T_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693}{1.51 \times 10^{-6}\ \text{s}^{-1}} = 458\,940\ \text{s} = \frac{458\,940}{3600 \times 24}\ \text{days} = 5.31\ \text{days}$

b. After 15.93 days, this is 3 half-lives. The activity reduces by two for each half life, after 1sthalf life activity = 2 MBq, after 2nd half life activity = 1 MBq, and after 3rd half life activity = 0.5 MBq.
Hence the activity after 15.93 days (i.e., 3 half lives) is 0.5 MBq
After 18 days, one must use the relationship, $A = A_o\ e^{-\lambda t}$
Now $\lambda = 1.51 \times 10^{-6}\ \text{s}^{-1}$ and t = 18 days = 18 x 24 x 3600 seconds = 1.56×10^{6} s.
Hence $\lambda t = 1.51 \times 10^{-6} \times 1.56 \times 10^{6} = 2.35$
(Note that one must have the time in seconds as the decay constant, λ, is in s^{-1}. Hence the product of λt will then be a dimensionless constant.)
Hence the activity after 18 days is given by,
$A = A_o\ e^{-\lambda t} = 4 \times 10^{6} \times e^{-2.35} = 3.8 \times 10^{5}$ Bq = 380 kBq.

c. Again we use$A = A_o\ e^{-\lambda t}$
Here we are given the activity after a certain time, A, as 3 MBq. The initial activity is 4 MBq and one needs to find the value of the time, t:

$$3 = 4e^{-\lambda t} \quad \text{or} \quad 0.75 = e^{-1.51 \times 10^{-6} t}$$

As t is in the exponent part of the equation, one needs to find the logarithm of both sides; in this case, the natural log of both sides is the more appropriate. Note that the time, t, will be expressed in seconds since λ is given in s^{-1}.
$\ln(0.75) = -1.51 \times 10^{-6}\ t$
$-0.288 = -1.51 \times 10^{-6}\ t$
Hence, t = 190 518 s = 52.9 hours = 2.2 days

d. By definition, 1 g of radium-226 has an activity of 3.7×10^{10} Bq.
That is, it has a specific activity of 3.7×10^{10} Bq/g. If we have a radionuclide of shorter half life and smaller atomic mass, then it will have (in direct proportion) a greater specific activity than Ra-226. (Note for Ra-226, $T_{½}$ = 1600 years and A_R = 226).
Hence, the specific activity of any nuclide, X, of half life T_X and atomic mass A_X is given by,

$$\text{Specific activity} = 3.7 \times 10^{10} \times \frac{226}{A_X} \times \frac{1600\,\text{years}}{T_X\,(\text{in years})}$$

The only restriction with units is that both half lives should be in the same units of time. In our case, ${}^{133}_{54}Xe$ has a half life of 5.31 days and has an atomic mass of 133. Hence;

$$\text{Specific activity of Xe-133} = 3.7 \times 10^{10} \times \frac{226}{133} \times \frac{1600 \times 365\,\text{days}}{5.31\,\text{days}} = 6.9 \times 10^{15}\ \text{Bq.g}^{-1}$$

That is, 1 g of Xe-133 has an activity of 6.9×10^{15} Bq, or 1/6.9 x 10^{15}g has an activity of 1 Bq. Hence 4 MBq (initial activity) will have a mass of 4 x 10^{6} /6.9 x 10^{15}g, that is 5.8×10^{-10} g. This very small mass is the amount of pure Xe-133 gas which is mixed with the non radioactive (Xe-131) part.

Workshop Tutorials for Technological and Applied Physics

QR14T: **Radioactivity**

A. Qualitative Questions:

1. Nuclei can decay by emitting particles which can change the energy, mass and charge of the nucleus.
a. How is α decay possible when the α particle must pass an energy barrier which is greater than the energy of the particle? Describe the process involved.
b. If isotope A emits α particles with greater energy than isotope B (of the same element), which will have the longer half life?
c. How can a nucleus change its charge without emitting a charged particle?

2. A chemist at a large detergent manufacturing company is using radioactively labeled dirt to test the effectiveness of their new enzyme powered super strong no scrub floor cleaner.
a. How can you use radioactively labeled dirt to test the effectiveness of a detergent?
b. What is the advantage of using this method to test the effectiveness compared to other methods?

B. Activity Questions:

1. Colleen's Cubes
Shake the bag containing the nuclei (cubes) and pour them into the tray.
Write down the number of cubes with dots showing on top, remove those cubes and replace the rest in the bag.
How many cubes are left of the original 100?
Repeat 10 times. Sketch the number of cubes removed (the activity) as a function of number of throws.
Sketch the number of cubes remaining in the bag as a function of number of throws.

2. Smoke detector
Examine the smoke detector.
It contains a radioactive source, ^{241}Americium, an α emitter.
The α particles ionize air molecules between two charged plates. The positive ions go to the negative plate, the negative ions to the positive plate, which gives a current.
How does smoke disrupt the current?

C. Quantitative Questions:

1. In 1947, in Qumran, near the Dead Sea, young Bedouin shepherds were searching for a lost goat when they found a cave. Inside the cave they found their lost goat, and what became known as the Dead Sea Scrolls. The scrolls were discovered in eleven caves along the northwest shore of the Dead Sea between 1947 and 1956. In all, the remains of about 825 to 870 separate scrolls have been identified. The Dead Sea Scrolls are believed to have been written by the Essenes during the period from about 200 B.C.E to 68 C.E., based on the handwriting styles, materials, and formatting of the manuscripts. The scrolls are mostly made of animal skins or papyrus, but one is made of copper.

Carbon-14 dating of samples taken from ragged edges of manuscript margins was done using a tandem accelerator mass spectrometer, dedicated exclusively to radiocarbon dating. The accelerator sorts and counts carbon isotopes by mass, enabling researchers to directly count ^{14}C atoms using only milligrams of the sample to be dated. The scrolls were dated at 1950 years old.

a. How can the decay of ^{14}C be used to tell how old things are?
b. Why is it not possible to date the copper scroll in this way?
Carbon-14 has a half-life of 5,730 years.
c. What proportion of the original ^{14}C was left in the scrolls?
d. What proportion of ^{14}C would be left after 57, 300 years?
e. Why is carbon dating not used to date things much over 50,000 years old?

2. Nuclear power is used in many places in the world. There are over 400 nuclear power plants currently in operation, over 100 of which are in the USA, providing 20% of the electricity consumed in that country. These plants use uranium fuel ($^{238}_{92}U$ enriched with 3% $^{235}_{92}U$) to produce electricity. It is the fission of the $^{235}_{92}U$ nuclei which provides the majority of the thermal energy that is used to generate the power.

a. Complete the following decay equation: $^{235}_{92}U + n \rightarrow ^{93}_{37}Rb + ^{141}_{55}Cs +$ ____
b. Use the data in the table below to find the energy released in this reaction.
c. How many decays per second would it take to run a 60W light globe?
d. If a power plant only converts 10% of the excess mass into useful energy, how many decays per second would you need?

Useful masses:

Particle	$^{235}_{92}U$	$^{93}_{37}Rb$	$^{141}_{55}Cs$	$^{4}_{2}\alpha$	β	γ
Mass (amu)	235.04392	92.92172	140.91949	4.002603	0.000545	0.0000000

1 amu = 1.660566×10^{-27} kg.

Workshop Tutorials for Technological and Applied Physics
Solutions to QR14T: **Radioactivity**

A Qualitative Questions:

1. Nuclei can decay by emitting particles which can change the energy, mass and charge of the nucleus.

a. The process of α decay can be modelled as a particle in a potential well (the nucleus) escaping from the well. α decay is possible because the potential energy well is not infinitely deep. When the potential energy well is not infinite the wave function of the trapped particle extends beyond the walls, and hence the probability of it existing outside the well, which is equal to the square of the wave function, is non-zero. This process of a particle escaping the well is called tunneling.

b. The higher the energy of the emitted α particles for a given element the shorter the half life. The probability of tunneling increases with increasing energy of the trapped particle (the α particle), hence the higher the energy, the greater the probability of decay, the more decays per unit time and the shorter the half life. (The tunneling probability also depends on the barrier height.)

c. A nucleus can change its charge by capturing an electron from one of the inner orbitals, turning a proton into a neutron. In the process a tiny uncharged particle called a neutrino is emitted.

2. Radioactive dirt and detergent testing.

a. The advantage of using radioactively labeled dirt is that it is very easily detected with a Geiger counter or a scintillation detector. Hence one could first measure the radioactivity from the test floor without the radioactively labeled dirt. This will give an estimate of the background radiation in the room. The radioactive dirt is then spread over the floor and the radioactivity measured again. Naturally, one would expect to now detect high levels of radiation. The new enzyme super detergent is used to clean the floor and carefully remove all the contaminated water in a bucket until the floor is dry. At this stage, the radioactivity level from the test floor is measured and compared with the initial measurement without the dirt.

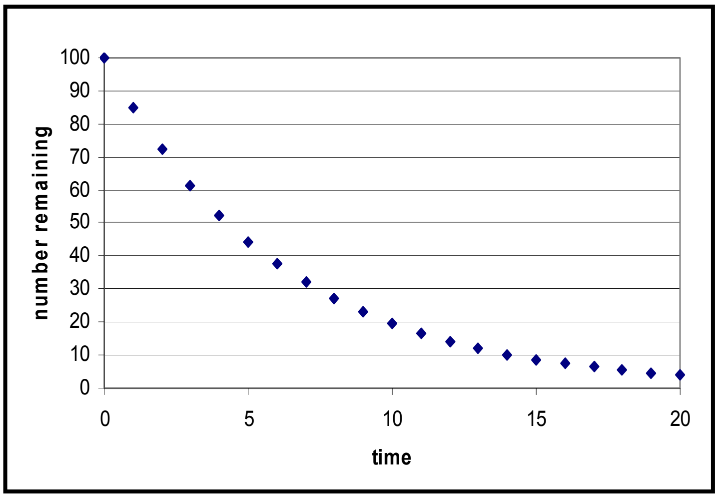
b. This method is very effective because even the smallest particles of dirt (undetectable with normal vision) could easily be detected with a scintillation counter or Geiger counter.

B. Activity Questions:

1. Colleens Cubes

You should have come up with an exponentially decreasing curve for both the number of cubes remaining, and the number of cubes removed at each throw, such as that shown opposite.

Many physical processes, such as cooling of hot coffee, follow this pattern.



2. Smoke detector

When smoke enters the space between the plates the ions attach themselves to the heavy smoke particles and the flow of current is disrupted, setting off the alarm.

C. Quantitative Questions:

1. Carbon dating.

a. There is a constant exchange of Carbon between the body and the environment for all living things. In this way the proportion of ^{14}C is kept approximately the same in the body as in the atmosphere. When an organism dies this exchange stops, and the proportion of ^{14}C gradually decreases due to decay without being replenished. We know the proportion of ^{14}C in the atmosphere, and how long it takes to decay, hence we can measure how much is left in a sample and work out from this when it stopped exchanging C with the environment, and hence how old it is.

b. Copper is not organic, hence it was never exchanging C with the atmosphere. However the ink used was organic and hence the scroll could be dated that way.

Carbon-14 has a half-life of 5,730 years.

c. Use the decay equation $N = N_o\, e^{-\lambda t}$.

The half life is 5730 years, so the decay constant $\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{5730\,\mathrm{y}} = 1.210\times10^{-4}\mathrm{years}^{-1}$.

Putting this into the equation, with t = 1950 years gives

$N = N_o\, e^{-\lambda t} = N_o \times 0.79$. So around 80% is left after 1950 years.

d. After 57, 300 years ten half lives have elapsed. The original amount will have halved 10 times, so

$N = N_o \times ½ \times ½ \times ½ \times ½ \times ½ \times ½ \times ½ \times ½ \times ½ \times ½ = N_o\times(½)^{10} = N_o \times 9.8\times10^{-4}$,

or nearly one thousandth.

e. After 50,000 years there is not enough ^{14}C left in the sample to date it accurately.

2. Energy from nuclear reactions.

a. $^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{93}_{37}Rb + {}^{141}_{55}Cs + 2\ {}^{1}_{0}n$

b. The energy released is given by

$Q = m_i c^2 - m_f c^2$ = (235.043924 u + 1.00866501 u - 92.92172 u – 2 × 1.00866501 u) c^2

= (235.043924 u - 92.92172 u –1.00866501 u) c^2

= 0.194045 × 1.660566 × 10^{27} kg × (3 × 10^8 m.s^{-1})^2

= 2.9 × 10^{11} J.

c. To run a 60W light globe we need 60 J in one second. The number of decays, n, is:

$n = 60\ \mathrm{J} / 2.9 \times 10^{-11}\ \mathrm{J} = 2.068 \times 10^{12} = 3 \times 10^{12}$.

d. A power plant only converts 10% of the excess mass into useful energy, so we only get the energy from one in every ten decays, hence we need 10 times 3 × 10^{12} = 3 × 10^{13} decays.

QR15B: **Radiation and the Body**

A. Qualitative Questions:

1. We are exposed to radiation all the time, indoors and outdoors. This is called background radiation.
a. Give two examples of sources of this background radiation.
b. Which organ generally receives the most background radiation, and why?
There is some concern at the moment that pilots and flight attendants may have significantly higher exposures to radiation than the normal exposure rates for the general public.
c. Why do pilots have a higher exposure to radiation than most other people?

2. A radiology nurse was found in the surgery late at night with her boyfriend in front of the x-ray machine. She explained that she regularly gave him doses of x-rays as a contraceptive measure.
a. Discuss the value of x-rays as a contraceptive, and the possible side effects.
b. What is the difference between ionizing and non-ionizing radiation? Why does one cause genetic effects but the other mainly somatic effects?

B. Activity Questions:

1. Measuring Radiation
Several different means of measuring radiation are shown.
Explain how they work.
Which ones would be suitable monitoring devices for persons working in a radiation area?

2. Exposure levels
Look at the chart showing the recommended exposure limits.
How do these compare with the dosages shown in the table?
What sort of professions do you think have the highest exposure?

3. Common sources of radiation
Use the counter to measure the radiation coming from the various sources.
How do they compare to background radiation?
How do they compare to the recommended maximum dosages?

C. Quantitative Questions:

1. Gold-198* is used to trace factory waste and sewage causing ocean pollution, and to trace sand movement in river beds and on ocean floors. It decays as follows

$^{79}Au \rightarrow {}^{80}Hg + \beta^- + \gamma$.

You are measuring the attenuation of the radiation from a sample of ^{198}Au through a new type of shielding material which you intend to use when working at the sewage plant.

The proportion of γ radiation penetrating a material decreases exponentially with the thickness of the material. A shielding material is rated according to its attenuation coefficient, $\mu = \ln2/HVL$. The HVL is the half-value layer, which is the thickness which stops one half of the incident radiation. This thickness depends on the material, and also on the radiation. It will be greater for more penetrating radiation.

a. Write an equation which gives the γ radiation intensity at a distance *d* through some material.

b. Sketch the intensity of the γ radiation as a function of distance.

c. Name another process which follows this form.

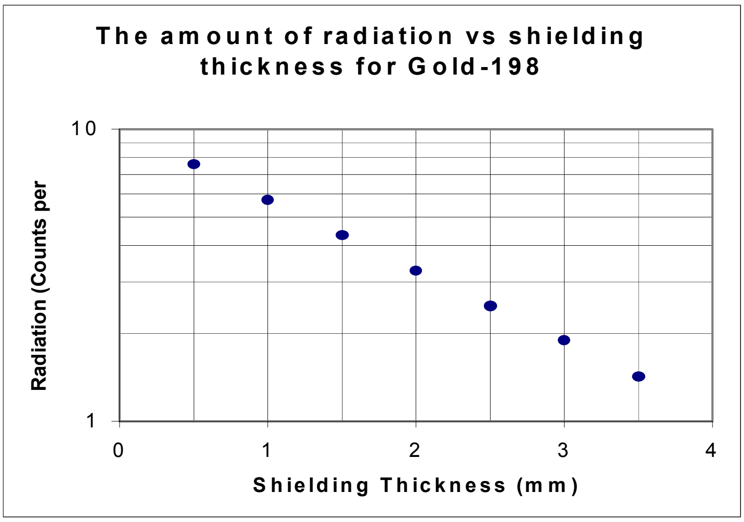
Use the graph to answer the following questions:

d. What is the half thickness of this shielding?

e. What is the attenuation coefficient of the lead shielding?

f. Using the graph, estimate the radioactivity directly in front of the source with no shielding.

g. You directly measure this activity with a Geiger counter and find that it is much higher than that predicted by the graph. Why is this?



2. Brent is changing the battery in the smoke detector when the phone rings. While he is answering the phone Barry the dog chews up the detector and swallows the Americium source which it contains! Brent is somewhat worried by this.

a. Complete the following reaction equation:

$^{241}_{95}Am \rightarrow$ _______ $+ ^{4}_{2}\alpha$.

It takes 9 hours for the source to pass through Barry, who weighs 25 kg. The half life for $^{241}_{95}Am$ is 433 years. The $^{241}_{95}Am$ source in a standard smoke alarm has an activity of 1μCi (37 kBq), and each emitted α particle has an energy of 5.4 MeV. The RBE factor for this radiation is around 15.

b. What is the resulting physical dose in Grays which Barry receives?

c. What is the dose equivalent in Sieverts?

d. Why are α sources only considered dangerous when inhaled or ingested?

Workshop Tutorials for Biological and Environmental Physics

Solutions to QR15B: **Radiation and the Body**

A. Qualitative Questions:

1. Background radiation.

a. Background radiation comes from many sources; from outer space, from the earth and from inside our bodies. Radiation from outer space, called cosmic radiation, comes from exploding stars and from the Sun and includes penetrating gamma rays and fast moving nuclear particles. The rocks and soils on earth contain small quantities of radioactive uranium, thorium and potassium-40 with their daughter products. The radioactivity levels in granite areas are higher due to a greater concentration of these elements. Indeed, the natural background activity is 2 to 3 times higher than the average background radiation in these areas. Small traces of naturally occurring isotopes (potassium-40 and carbon-14) are present inside all of our bodies!

b. The lungs generally receive the highest dose of radioactivity, from airborne radioisotopes such as radon-222 gas. The lungs exchange more material with the environment than any other organ. Smokers may have a much higher dosage rate than non-smokers as many cigarettes use tobacco which is dried in the open air and accumulates radon dust which settles on it.

c. The background radiation at higher altitudes is greater because of a higher cosmic radiation. The atmosphere acts as a shield and attenuates (reduces) the cosmic radiation reaching the earth considerably. Thus as one goes to higher altitudes, the 'filtering' action of the atmosphere is reduced. For example, the mean dose rate at sea level is 0.2 mSv/year whereas the dose rate at 4,000 m is 1 mSv/year. This is the reason that pilots and flight attendants have a higher exposure to radiation than most other people as they spend a lot of time at high altitudes.

2. X-rays as a contraceptive measure.

a. Low doses of x-rays (100 mGy) can result in a reduction in the number of spermatozoa. As the dose is increased, the depletion of spermatozoa becomes more significant and lasts longer. For example, a dose of 2 Gy will produce temporary sterility that commences 2 months after the exposure and persists for up to 12 months. A high dose of 5 Gy will produce permanent sterility. Even for low doses of 100 mGy, there is an increased possibility of genetic mutations which can produce undesirable effects in the children produced by this person. Furthermore, the possibility of the man developing radiation-induced leukemia or some other type of cancer is more likely and this increases with increasing dose.

b. Ionising radiation is radiation that is sufficiently energetic to ionise atoms and molecules. In other words, the energy of the radiation is sufficient to eject electrons from atoms and molecules, to produce ion pairs; ie, an electron and a positive ion. Examples of ionising radiation include alpha, beta, gamma, x-ray radiation as well as neutrons and protons. Non-ionising radiation is radiation which has insufficient energy to cause ionisation. Examples include radio waves, microwaves (as emitted by mobile phones), infra red, visible and ultraviolet radiation and ultrasound. The effects of radiation on cells are divided into two classes, namely somatic and genetic. The somatic effects arise from damage to ordinary cells in the body and only affect the person that has been exposed. On the other hand, genetic effects are due to damage to the cells in the reproductive organs, which have an effect on the next or subsequent generations.

B. Demonstration Questions:

1. Measuring Radiation

Two methods of monitoring radiation are the film badge and the Geiger counter.

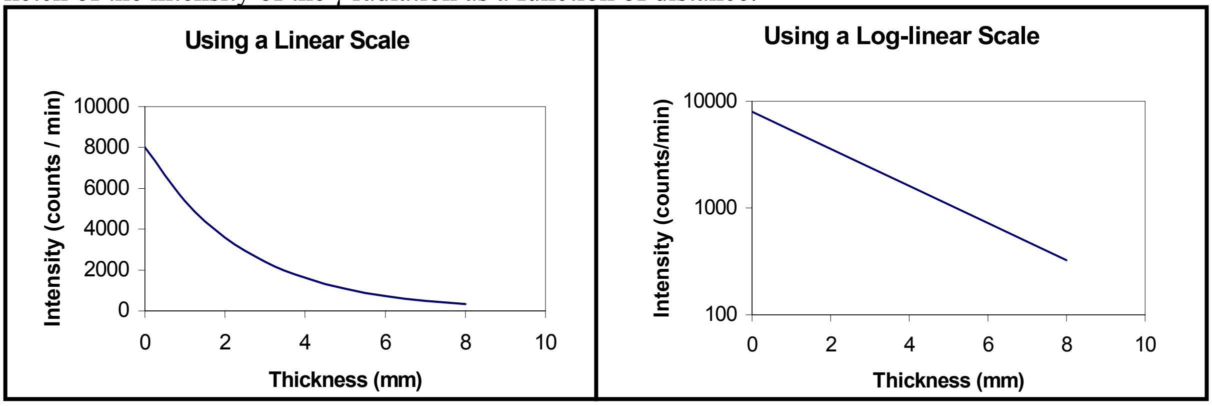
Film badges contain exactly what their name implies -- a piece of photographic film and several types of thin metal strips, which act as absorbers and allow for detection of various energies of radiation.

A Geiger counter is a device used to detect radiation from a radioactive source. It detects and records the number of radioactive particles. The Geiger counter consists of metal tube filled with a gas at low pressure, such as argon, which detects the presence of radiation. A wire runs down the centre of the tube and is maintained at a high positive voltage compared with the outer tube, which is negatively charged. When a particle enters the window at one end of the tube it ionises a few gas atoms. The freed electrons are attracted to the central positive wire and ionise other gas atoms as they accelerate towards the wire. A large number of electrons are quickly produced and these produce a voltage pulse at the wire. This pulse is transferred to an electronic counter. The pulses can also be sent to a loudspeaker to be heard as a clicking sound.

2 and 3. Exposure levels and Common sources of radiation
Radiation comes from many natural sources, as well as man made sources, and the typical exposures are well below recommended limits. People who work with radiation such as x-ray sources, especially radiographers and dentists, and aircraft pilots who are exposed to more cosmic radiation receive higher dosages than other people.

C. Quantitative Questions:

1. Shielding of radiation sources.
a. Intensity at a distance *x* through some material is given by: $I = I_o e^{-\mu x}$, where I = intensity of radiation after travelling through a thickness, *x* mm, of shielding, I_o = initial intensity of the radiation without any shielding, μ = fractional reduction (per mm) of the shielding material and *x* = thickness of shielding in mm
b. Sketch of the intensity of the γ radiation as a function of distance:



c. Another process which follows this exponential form is the way in which radioactive decay varies with time.
d. The half value layer is the thickness of shielding which reduces the intensity to half of the original intensity. If one draws a straight line of best fit through the experimental points and extends it back to zero thickness, the initial intensity is 10 counts/second. One can then use the graph to find the thickness where the intensity is reduced to 5 counts/second; this is approximately 1.25 mm. Hence the half-value layer, HVL = 1.25 mm.
e. The attenuation coefficient can be calculated from the HVL: $\mu = \frac{0.693}{HVL} = \frac{0.693}{1.25\ \text{mm}} = 0.55\ \text{mm}^{-1}$

f. Here we are to use the graph to estimate the radioactivity directly in front of the source with no shielding. We did this earlier, by extending the line of best fit to zero thickness and found it to be 10 counts/second.
g. The gold-198 emits gamma radiation and beta particles. The beta particles are strongly attenuated by a small thickness of shielding whereas the gamma rays are more penetrating. As thicker and thicker shielding is used, the gamma radiation is attenuated in an exponential fashion. So if we base our initial radiation on the line of best fit only, we have actually neglected the effect of the beta particles. Hence when there is no shielding, one will measure the radiation from **both** the beta and gamma radiation.

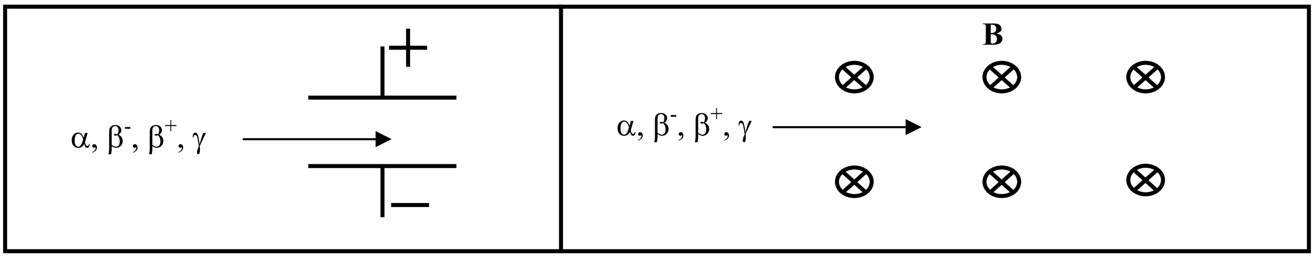
2. Barry the dog chews up the smoke detector and swallows the Americium source which it contains.
e. ${}^{241}_{95}Am \rightarrow {}^{237}_{93}Np + {}^{4}_{2}\alpha$.
The half life for ${}^{241}_{95}Am$ is 433 years, so the activity will be virtually the same, 1μCi (37 kBq), for the 9 hours for the source to pass through Barry. Hence we can simply use the activity given. Each emitted α particle has an energy of 5.4 MeV. The RBE factor for this radiation is around 15.
f. The physical dose in Grays is the energy absorbed per kilogram. α particles have very little penetrating power, hence virtually all the α particles will be absorbed by Barry. The number of emitted α particles is the activity × the time = 37 kBq × 9 h = 37×10^3 decays per second × 32400 s = 1.2×10^8 α particles.
Each α particle has a kinetic energy of 5.4 MeV = 8.6×10^{-13} J. So the total energy absorbed by Barry from the ${}^{241}_{95}Am$ is 8.6×10^{-13} J per α × 1.2×10^8 α particles = 1.0×10^{-4} J.
The resulting dose in Grays is 1.0×10^{-4} J / 25 kg = 4.1×10^{-6} Gy = 4.1 μGy.
g. The dose equivalent in Sieverts is the RBE × dose (in Grays) = 15 × 4.1×10^{-6} Gy = 6.2×10^{-5} Sv = 62 μSv
For comparison, the NHMRC recommended radiation dose limit for the public is 1000 μSv per year.
h. α sources are only considered dangerous when inhaled or ingested because of their poor penetrating power. They are easily stopped by air, and cannot penetrate clothing or skin easily. They are most dangerous when inhaled, as even the lining of the gut is hard for the α particles to penetrate, and this lining is constantly shed and replaced.

Workshop Tutorials for Technological and Applied Physics

QR15T: **Interactions with Radiation**

A. Qualitative Questions:

1. Beams of α, β^- and γ radiation of approximately the same energy pass through electric and magnetic fields as shown below.



a. Show the path taken by each particle in the two fields. Why do they follow these paths?
b. Which particle is the most penetrating? Explain your answer.
c. Which has the highest ionising power?
d. How are β^-, β^+ and electrons different?
e. How are x-rays, γ rays and photons different?

2. People who work with radiation, such as dentists using x-ray machines, usually use shielding of some type, such as lead aprons, to protect themselves from the radiation.
a. Why do materials composed of heavy elements make the best radiation absorbers for x-rays?
The research reactor at Lucas Heights is used to produce medical isotopes and is a source of neutrons for experiments and materials testing, for example aeroplane components. To use neutrons for some experiments, they have to be slowed down, or moderated. This is done by allowing them to repeatedly collide with atoms or molecules at a lower temperature than the neutrons
b. What would the properties of an ideal moderator material be?
c. Neutrons, electrons and x-rays can all be used to investigate the structure of materials. What information do neutrons give that electrons and x-rays don't?

B. Activity Questions:

1. Measuring Radiation
Several different means of measuring radiation are shown.
Explain how they work.
Which ones would be suitable monitoring devices for persons working in a radiation area?

2. Nuclear Power Stations
Use the diagram to explain how the energy from the nuclear decay is used to produce electricity.
What forms does the energy take before becoming electrical energy?
What do the control rods of a nuclear reactor control and how do they do this? How are they operated (i) to reduce the power level and (ii) on a long term basis as fuel is consumed?

C. Quantitative Questions:

1. The proportion of γ radiation penetrating a material decreases exponentially with the thickness of the material. A shielding material is rated according to its attenuation coefficient, μ = ln2/HVL. The HVL is the half-value layer, which is the thickness which stops one half of the incident radiation. This thickness depends on the material, and also on the radiation. It will be greater for more penetrating radiation.

a. Write an equation which gives the γ radiation intensity at a distance *d* through some material.

b. Sketch the intensity of the γ radiation as a function of distance.

c. Name another process which follows this form.

There are several isotopes of potassium which are radioactive. They emit γ and β radiation. The half value layers for the γ radiation from ^{40}K for several materials are shown below.

material	HVL (cm)
Lead	1.2
Iron	1.8
aluminium	5.0
water	12
air	10,000
concrete	5.6

d. By what fraction would the radiation be decreased by 25 cm of aluminium?

e. You are designing a shield for a hospital storage room in which ^{40}K is to be stored. If the radiation must be reduced to 1% of its unshielded level, what thickness of lead do you need to use?

f. What thickness would you need in part **e** if you were using concrete? Why do you think concrete is more commonly used as a shield?

2. Gold-198* is used to trace factory waste and sewage causing ocean pollution, and to trace sand movement in river beds and on ocean floors. It decays as follows

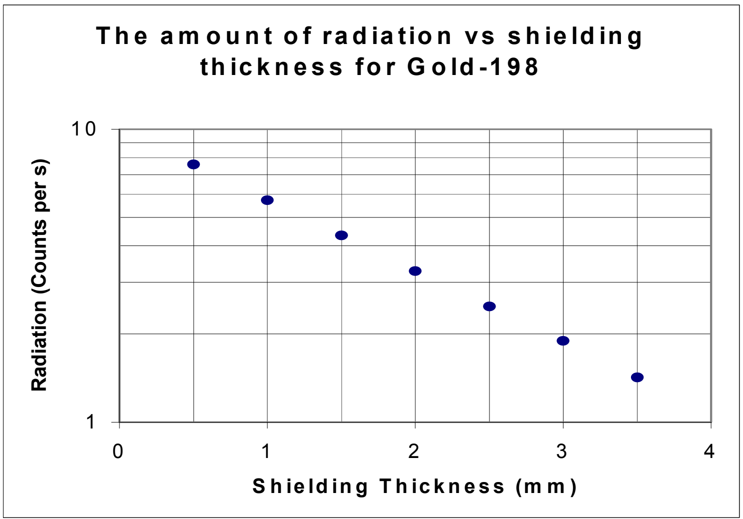
$$^{79}Au \rightarrow {}^{80}Hg + \beta^- + \gamma.$$

You are measuring the attenuation of the γ radiation from a sample of ^{198}Au through a particular type of shielding material. Use the graph below to answer the following questions.

a. What is the half thickness of this shielding?

b. What is the attenuation coefficient of the shielding?

c. Using the graph below, estimate the radioactivity directly in front of the source with no shielding.



d. You directly measure this activity with a Geiger counter and find that it is much higher than that predicted by the graph. Why is this? (Look at the decay equation and consider the penetrating power of different radiation types.)

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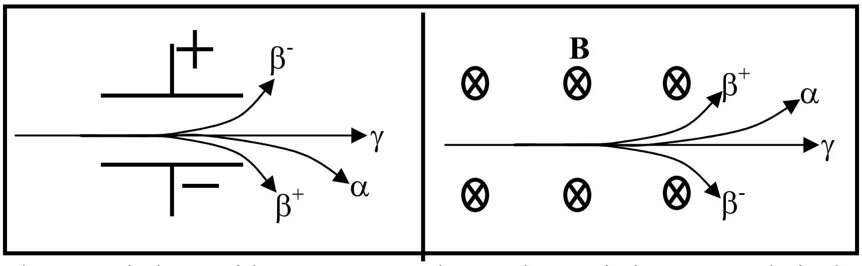
Solutions to QR15T: **Interactions with Radiation**

A. Qualitative Questions:

1.



a. The charged particles are deflected and follow a curved path in an electric or magnetic field. The γ particles are uncharged and hence are not affected. The α's are charged, but are very heavy and are less deflected than the βs.

b. The γ particle is the most penetrating because it has neither mass nor charge, hence it interacts relatively weakly with matter.

c. The α has the highest ionising power, with the largest charge (+2*e*), but are not very penetrating.

d. β^-'s and electrons are exactly the same, they were named β's before it was known that they were electrons. β^+'s are positrons, or positively charged electrons. They have the same mass as an electron, and the same magnitude of charge, but are positive.

e. x-rays and γ rays are both photons, they have no charge and no mass. γ rays have higher energy (and higher frequency) than x-rays, and both are higher energy than photons of visible light.

2. a. X-rays interact with electrons and heavier elements have more electrons for a given volume.
b. You want the moderator to be light so that it recoils a lot (and takes energy off the neutron) and you don't want it to absorb the neutron or you will run out of neutrons before you get to use them. That's why heavy water is a good moderator, and is used at many reactors.
c. Neutrons are uncharged, hence interact weakly with matter, so they penetrate allowing you to look deep inside big samples. (They have even been used to image cracks inside engine blocks.) Because X-rays and electrons scatter off the electrons in a material, they are not very good for looking at elements with few electrons (hydrogen being the obvious one). Neutrons see hydrogen very strongly. Neutrons interact with nuclei, rather than electrons, so they can give information about different isotopes in a sample which electrons and X-rays do not give. (In addition, neutrons have a magnetic moment, so they can be used to investigate magnetic properties of materials.)

B. Activity Questions:

1. Measuring Radiation

Two methods of monitoring radiation are the film badge and the Geiger counter.
a. Film Badge: Film badges contain exactly what their name implies -- a piece of photographic film and several types of thin metal strips, which act as absorbers and allow for detection of various energies of radiation. Eg. the attenuation of beta rays depends only on the density of the medium through which it travels and plastic filters of low atomic number can be used to assess beta radiation dose. Cadmium can be used to assess a neutron dose. As the gamma radiation emitted, when neutrons are captured by the cadmium atoms, will blacken the film underneath the cadmium filter. If the radiation beam is only gamma rays then the blackening will be the same under a tin-lead filter and under a cadmium-lead filter. However, the presence of neutrons will produce excess film blackening under the cadmium-lead filter. The amount of exposure to the film determines the amount of radiation exposure that the individual received during that period.
b. A Geiger counter detects and records the number of radioactive particles. The Geiger counter consists of metal tube filled with a gas at low pressure, such as argon, which detects the presence of radiation. A wire runs down the centre of the tube and is maintained at a high positive voltage compared with the outer tube, which is negatively charged. The voltage is not sufficient to ionise the gaseous atoms in the tube. When a particle enters the window at one end of the tube it ionises a few gas atoms. The freed electrons are attracted to the central positive wire and ionise other gas atoms as they accelerate towards the wire. A large number of electrons are quickly produced and these produce a voltage pulse at the wire. This pulse is transferred to an electronic counter. The pulses can also be sent to a loudspeaker to be heard as a clicking sound. Counters enable radioactive tracers to be followed as they make their way through complex organisms such as the human body. They are used also to follow radioactive isotopes in chemical reactions.

2. Nuclear Power Stations

The nuclear interactions take place inside the reactor. Neutrons bombarding $^{235}_{92}U$ cause disintegration of the nucleus into two lighter nuclei and two neutrons. These neutrons are then slowed down by a moderator so that they will cause further interaction. In the interaction mass is lost and this is found as kinetic energy of the products of the disintegration. The kinetic energy is lost to thermal energy as these nuclei slow down in the reactor and the reactor heats up. Water surrounding the reactor is thus heated and turns to steam which is used to turn a turbine which then generates electricity.

The energy changes are:

Nuclear binding energy → kinetic energy → thermal energy → mechanical energy → electrical potential energy.

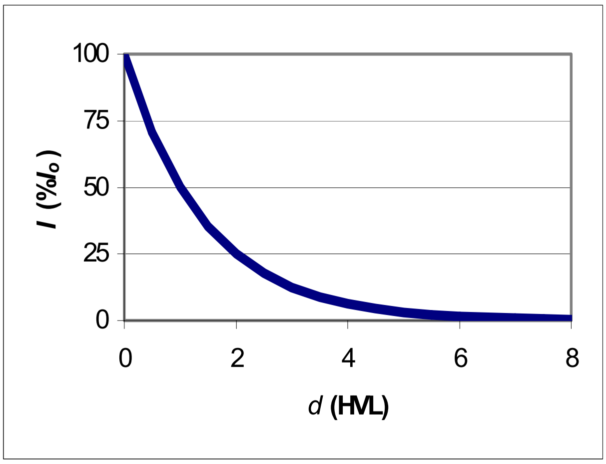
As seen in the description of the reaction above the number of neutrons increases at each nuclear decay and it is possible that the process could get out of control and the reactor overheat. The control rods (often made of cadmium) absorb neutrons and thus can be used to control the process. (i)To reduce the power level the rods are pushed further into the reactor.(ii)As the fuel is consumed and more decay product builds up, they can be moved out slowly.

C. Quantitative Questions:

2. The proportion of γ radiation penetrating a material decreases exponentially with the thickness of the material. A shielding material is rated according to its attenuation coefficient, $\mu = \ln 2/HVL$. The HVL is the half-value layer, which is the thickness which stops one half of the incident radiation. This thickness depends on the material, and also on the radiation. It will be greater for more penetrating radiation.

a. The intensity at distance *d* is $I(d) = I_o\, e^{-\mu d}$.

b. See opposite.



c. Many processes follow this form, such as radioactive decay as a function of time, cooling, and population growth and decay.

d. Each HVL decreases the intensity by 50% or ½. The HVL for Aluminium is 5.0 cm, so 25 cm is 5 HVLs. The reduced intensity is therefore $(½)^5$ = 1/32 or 3%, a reduction of 97%.

You can also do this using

$I(25cm) = I_o e^{-\mu\, 25cm} = I_o e^{-0.028\times 25cm} = 0.03\, I_o$.

e. If the radiation is to reduced to 1% the $I/I_o = 1/100$, so using $I(d) = I_o\, e^{-\mu d}$ we have $1/100 = e^{-\ln 2\, x/1.2}$. Taking natural logarithms of each side gives x = 7.97 cm.

f. For concrete using the same relation, $1/100 = e^{-\ln 2 \times /5.6}$, which gives x = 37.2 cm. Concrete is used as it is much cheaper, and easily incorporated into structures.

2. Radiation shielding.

a. Draw a straight line through the points and take any reading. Take a second reading where the radiation has fallen to a half. You will find the difference in thickness (along the x axis) is always about 1.3mm-the half value layer. (Note that the y-axis is logarithmic)

b. $\mu = \ln 2/HVL = 0.693/1.3mm = 0.533\ mm^{-1}$.

c. By extending the line to the value x = 0, the value on the Y – axis is 10 counts per s.

d. Both γ and β radiation are products of the nuclear reaction. As the β's will be absorbed by a very small thickness of shielding they will not contribute to the readings that gave the graph and extension of the graph to the x = 0 value will be purely for the γ's. 10 counts per s will not be the true reading as both radiations will be present and the reading will be greater.