LECTURE 3       HEAT TRANSFER 1

Text Sections          19.11
Sample Problems        19.9, 19.10
Suggested Questions    12
Suggested Problems     95P
Summary                 Conduction
                        Thermal resistance
                        Convection
                        Radiation

Specific objectives

- Describe the mechanisms of heat transfer (conduction, convection, radiation)
- Solve heat transfer problems
Heat transfer mechanisms

Heat can be transferred from one object to another at a different temperature by conduction, by convection and by radiation.

Unless there is a phase change, the temperature of a system will rise or fall if heat is transferred to or from it respectively. If the system is sufficiently large this temperature change may be negligible. The system is then called a thermal reservoir or heat reservoir; a thermal reservoir has a constant temperature.

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Conduction

In this process heat energy is transferred through a material, or across the boundary of two materials in contact, by vibrations and collisions between nearby atoms. Conduction is the principal heat transfer mechanism for solids.

A metal, for example is made up of a positive ions arranged in a lattice structure with the outer electrons of the atom moving freely throughout the lattice. The ions vibrate more as the temperature is increased.

Consider a uniform flat sheet of a material placed between two thermal reservoirs at different temperatures, $T_C$ and $T_H > T_C$ (See Fig 19-17). Let $A$ be the cross sectional area of the sheet and $L$ its thickness. Assume that there is no significant transfer of heat through the edges of the sheet.

When heat is being transferred at a steady rate, the heat transferred through the boundary with the hot reservoir is equal to that through the boundary with the cold reservoir. The common rate $H$ is given by

$$H = \frac{dQ}{dt} = kA \frac{T_H - T_C}{L}$$

where $k$ is the thermal conductivity of the material. It has SI unit W . m$^{-1}$ . K$^{-1}$. Note that $k$ is largest for materials which conduct heat easily, e.g. metals and smallest for insulating materials such as fibreglass and rockwool (see Table 19-6).

A note of caution: The low thermal conductivities for gases and liquids, although correct, are misleading because convection is usually a more effective method of heat transfer for these materials.
A good heat conductor will feel hotter (colder) to a person touching it than a poor heat conductor at the same higher (lower) temperature because of the more efficient heat transfer.

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**Thermal resistance**

It is often important in the building industry to minimise heat transfer by conduction, e.g. by placing insulation in walls and roofs. Such materials are often labelled with an “R value”; the higher the R value the better the insulation. This is the *thermal resistance* given by

\[ R = \frac{L}{k} \]

and hence

\[ H = A \frac{T_H - T_C}{R} \]

The units for the R value on packages are usually not specified since they are obsolete. Note that R values depend on the thickness of the pieces of material and are not simply attributes of the material itself (like the corresponding values of k).

The name thermal resistance comes from the analogy with electrical conduction \((V = IR)\):

\[ T_H - T_C \leftrightarrow V \quad H \leftrightarrow I \quad R / A \leftrightarrow R \]

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**Heat conduction through composite sheets**

Consider two uniform flat sheets of a material with thermal conductivities \(k_1\) and \(k_2\) respectively placed between two thermal reservoirs at different temperatures, \(T_C\) and \(T_H\) > \(T_C\) (See Fig 20-18). Let \(A\) be the cross sectional area of each of the sheets and \(L_1\) and \(L_2\) respectively their thicknesses. Assume that there is no significant transfer of heat through the edges of the sheets.

When heat is being transferred at a steady rate, the heat transferred through the boundary with the hot reservoir is equal to each of those through the boundary between the two sheets and through the boundary with the cold reservoir. The common rate \(H\) is given by

\[ H = k_1 A \frac{T_H - T_x}{L_1} = k_2 A \frac{T_x - T_C}{L_2} \]
where $T_X$ is the temperature at the boundary between the two sheets.

The second equation gives

$$\left( \frac{k_1}{L_1} + \frac{k_2}{L_2} \right) T_X = \frac{k_1 T_H}{L_1} + \frac{k_2 T_C}{L_2}$$

and hence from the first equation

$$H = A \left( \frac{T_H - T_C}{L_1 / k_1 + L_2 / k_2} \right) = A \left( \frac{T_H - T_C}{R_1 + R_2} \right)$$

These last two equations can easily be generalised to the case where there are more than two sheets.

Note that in the special case where the two sheets are of the same material, ie. $k_1 = k_2$, then

$$T_X = \frac{L_1 T_H + L_2 T_C}{L_1 + L_2} = T_H - \frac{L_1}{L_1 + L_2} (T_H - T_C)$$

This means that the temperature decreases uniformly through the combined sheets.