THERMAL EXPANSION

- Over small temperature ranges, the linear nature of thermal expansion leads to expansion relationships for length, area, and volume in terms of the coefficient of linear expansion $\alpha$.

\[
\Delta L = \alpha L_0 \Delta T \\
\Delta A = 2\alpha A_0 \Delta T \\
\Delta V = 3\alpha V_0 \Delta T = \beta V_0 \Delta T
\]

- **coefficient of volume expansion** $\beta = 3\alpha$

- Bimetallic strips Two flat strips of different metals welded together at one temperature become curved at other temperatures because the metals have different values for $\alpha$. They are often used as thermometers and thermostats.

- Most substances expand on heating. The addition of energy results in molecules effectively pushing their neighbours away from each other.

- Water has an anomalous coefficient of volume expansion, $\beta$ is negative between 0 °C and 4 °C.

- Examples: removing lid from jar, railway tracks, gas pipes, concrete roads, dental filings, bimetallic strips.

- $\alpha_{\text{aluminium}} = 2.4 \times 10^{-5} \text{ K}^{-1}$, $\alpha_{\text{quartz}} \sim 0.04 \times 10^{-5} \text{ K}^{-1}$

  Typical values for $\alpha$ are very small ⇒ only small changes in length for considerable increases in temperature.

  $\alpha$ assume independent of temperature (ok for most practical purposes)

  $\alpha$ depends only upon material object is made from.
• Areal expansion - holes in solids
  As metal expands, the distance between any two points increases. A hole expands just as if it’s made of the same material as the hole.

\[ T_1 < T_2 \]

\[ Q \]

• Volume expansion
  cube \[ V_0 = L_o^3 \]
  \[ V = L^3 = (L_o + \alpha L_o \Delta T)^3 \]
  \[ V = L_o^3 + 3 \alpha L_o^3 \Delta T + 3 \alpha^2 L_o^3 \Delta T^2 + \alpha^3 L_o^3 \Delta T^3 \]
  \[ V - V_0 = \Delta V = 3 \alpha L_o^3 \Delta T = \beta V_0 \Delta T \]

WATER
Liquid water is one of the few substances with a negative coefficient of volume expansion at some temperatures (glass bottles filled with water explode in a freezer) – it does not behave like other liquids

\[ T > 4 \, ^\circ C \quad \text{water expands as temperature increases} \]
\[ 0 < T < 4 \, ^\circ C \quad \text{water expands as temperature drops from 4 \, ^\circ C to 0 \, ^\circ C} \]
\[ T = 3.98 \, ^\circ C \quad \text{water has its maximum density} \]
Lakes freeze from top down rather from bottom up

Water on surface cools towards 0 °C due to surrounding environment. Water as it cools and becomes more dense, it sinks carrying oxygen with it (it is most dense at about 4 °C). Warmer water moves up from below. This mixing continues until the temperature reaches 4 °C. Water then freezes first at the surface and the ice remains on the surface since ice is less dense than water (0.917 g/mL). The water at the bottom remains at 4 °C until almost the whole body of water is frozen. Without this peculiar but wonderful property of water, life on this planet may not have been possible because the body of water would have frozen from bottom up destroying all animal and plant life.
Problem

As a result of a temperature rise of 32 °C a bar with a crack at its centre buckles upward. If the fixed distance between the ends of the bar is 3.77 m and the coefficient of linear expansion of the bar is $2.5 \times 10^{-5}$ K$^{-1}$, find the rise at the centre.

Solution

Setup

\[ \Delta T = 32 \degree C \quad \alpha = 2.5 \times 10^{-5} \text{ K}^{-1} \quad L_o = 3.77/2 \text{ m} = 1.885 \text{ m} \]

\[ h = ? \text{ m} \quad L = ? \text{ m} \]

linear expansion \[ L = L_o + \Delta L = L_o + \alpha L_o \Delta T \]

Action

From Pythagoras’ theorem \[ L^2 = L_o^2 + h^2 \]

\[ h^2 = L^2 - L_o^2 = (L_o + \alpha L_o \Delta T)^2 - L_o^2 = 2 \alpha L_o^2 \Delta T + \alpha^2 L_o^2 \Delta T^2 \]

\[ h = (2 \alpha \Delta T)^{\frac{1}{2}} L_o \quad \text{neglecting very small terms} \]

\[ h = (2(2.5 \times 10^{-5})(32))^{\frac{1}{2}} (1.885) \]

\[ h = 0.075 \text{ m} \]