Oscillations

Time variations that repeat themselves at regular intervals - periodic or cyclic behaviour

Examples: Pendulum (simple); heart (more complicated)

Terminology:

Period: time for one cycle of motion [s]

Frequency: number of cycles per second [s⁻¹ = hertz (Hz)]

How can you determine the mass of a single E-coli bacterium or a DNA molecule?
**Signal from ECG**

**Period:** time for one cycle of motion [s]

**Frequency:** number of cycles per second [s\(^{-1} = \text{Hz hertz}\)]

\[
1 \text{ kHz} = 10^3 \text{ Hz} \quad 10^6 \text{ Hz} = 1 \text{ MHz} \quad 1 \text{ GHz} = 10^9 \text{ Hz}
\]
Example: oscillating stars
Simple harmonic motion  SHM

- object displaced, then released
- objects oscillates about equilibrium position
- motion is periodic,
- displacement is a sinusoidal function of time (harmonic)

\[ F_e = -kx \]

\[ T = \text{period} = \text{duration of one cycle of motion} \]
\[ f = \text{frequency} = \# \text{ cycles per second} \]
- restoring force always acts towards equilibrium position
Vertical hung spring: gravity determines the equilibrium position – does not affect restoring force for displacements from equilibrium position – mass oscillates vertically with SHM

\[ F_e = -k y \]
Connection SHM – uniform circular motion

\[ \omega = \frac{d\theta}{dt} \]

One cycle: period \( T [s] \)
Cycles in 1 s: frequency \( f [Hz] \)
\[ \omega = \frac{2 \pi}{T} \]
Angular frequency \( \omega \) [rad.s\(^{-1}\)]
Displacement is sinusoidal function of time

\[ x = x_{\text{max}} \cos \left( 2\pi \frac{t}{T} \right) = x_{\text{max}} \cos \left( 2\pi f t \right) = x_{\text{max}} \cos \left( \omega t \right) \]

uniform circular motion
radius \( A \), angular frequency \( \omega \)

\[ \omega = 2\pi f = \frac{2\pi}{T} \]

\( x \) component is SHM:

\[ x = A \cos \left( \omega t \right) \]

\( x_{\text{max}} = A \)
Simple harmonic motion

Displacement is a sinusoidal function of time

\[ x = x_{\text{max}} \cos\left(2\pi \frac{t}{T}\right) = x_{\text{max}} \cos(2\pi ft) = x_{\text{max}} \cos(\omega t) \]

By how much does phase change over one period?
Simple harmonic motion

$$x = x_{\text{max}} \cos(\omega t)$$

$$v = -x_{\text{max}} \omega \sin(\omega t)$$

$$a = -x_{\text{max}} \omega^2 \cos(\omega t)$$

$$a = -\omega^2 x$$

$x_{\text{max}} \equiv A$

equation of motion  \( m \, a = -k \, x \) \hspace{1cm} \text{(restoring force)}

substitute

$$\omega = \sqrt{\frac{k}{m}}$$  \( f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \)  \( T = 2\pi \sqrt{\frac{m}{k}} \)

oscillation frequency and period
$a = -\omega^2 x$

acceleration is $\pi$ rad ($180^\circ$) out of phase with displacement

At extremes of oscillations, $v = 0$

When passing through equilibrium, $v$ is a maximum

\[ v^2 = x_{\text{max}}^2 \omega^2 \sin^2(\omega t) \]
\[ = x_{\text{max}}^2 \omega^2 (1 - \cos^2(\omega t)) \]
\[ = \omega^2 (x_{\text{max}}^2 - x^2) \]

\[ v = \pm \omega \sqrt{(x_{\text{max}}^2 - x^2)} \]
Simple harmonic motion

Hecht: figure 10.25
How do you describe the phase relationships between displacement, velocity, and acceleration?
Problem 2.1

If a body oscillates in SHM according to the equation

\[ x = 5.0 \cos(0.40 t + 0.10) \text{ m} \]

where each term is in SI units. What are
(a) the amplitude
(b) the frequency and period
(c) the initial phase at \( t = 0 \)?
(d) the displacement at \( t = 2.0 \text{ s} \)?

[Ans: 5.0 m, 0.064 Hz, 16 s, 0.10 rad, 3.1 m]
Solution 2.1

Identify / Setup

\[ x = 5.0 \cos(0.40 t + 0.10) \text{ m} \]

\[ x = x_{\text{max}} \cos(\omega t + \phi) \]

\[ \omega = 2 \pi f = \frac{2 \pi}{T} \]

\[ x_{\text{max}} = 5.0 \text{ m} \quad \omega = 0.40 \text{ rad.s}^{-1} \quad \phi = 0.10 \text{ rad} \]

Execute

(a) amplitude \[ A = x_{\text{max}} = 5.0 \text{ m} \]

(b) frequency \[ f = \omega / 2 \pi = (0.40 / (2 \pi)) \text{ Hz} = 0.064 \text{ Hz} \]

\[ T = 1 / f = (1 / 0.064) \text{ s} = 16 \text{ s} \]

(c) initial phase angle \[ \phi = 0.10 \text{ rad} \]

(d) \[ t = 2.0 \text{ s} \]

\[ x = 5 \cos{(0.4)(2) + 0.1} \text{ m} = 3.1 \text{ m} \]

Execute \[ \text{OK} \]
Problem 2.2

An object is hung from a light vertical helical spring that subsequently stretches 20 mm. The body is then displaced and set into SHM. Determine the frequency at which it oscillates.

[Ans: \( \omega = 22 \text{ rad.s}^{-1}; f = 3.5 \text{ Hz} \)]
Solution 2.2

Identify / Setup

**SHM**

\[ \omega = 2\pi f = \sqrt{\frac{k}{m}} \quad \Rightarrow \]

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

\[ F = mg = kx \quad x = 20 \text{ mm} = 20 \times 10^{-3} \text{ m} \]

Execute

\[ mg = kx \quad k = \frac{mg}{x} \]

\[ f = \frac{1}{2\pi} \sqrt{\frac{mg}{mx}} = \frac{1}{2\pi} \sqrt{\frac{g}{x}} = \frac{1}{2\pi} \sqrt{\frac{9.8}{20 \times 10^{-3}}} \text{ Hz} = 3.5 \text{ Hz} \]

Execute  OK