ELECTRICAL PROBLEMS

PROBLEM SOLVING GUIDELINES

A NON-LINEAR APPROACH TO PROBLEM SOLVING FOR BOTH QUANTITATIVE AND QUALITATIVE PROBLEMS

In writing an answer to a question it is often necessary to consider many things simultaneously - it is not possible to write out the answer like you would in copying a passage of text from a book. To help you improve your skills in answering questions from examinations, assignments, tutorials, laboratory, etc, you should try what I have called a non-linear approach using

Identify ↔ Setup ↔ Execute ↔ Evaluate

It is better to think "on paper" than in "your head". It is most important that you put information on paper - this makes it possible to access information more easily than organising ideas in your "minds eye".

In hitting a golf ball well, you would have noticed that golfers ("Greg") spend a lot a time setting themselves up. You need to set yourself up before you attempt your answer. You have to visualise what the problem is about, know what information is given and think about what you know. After this setup then you can go into action to execute your answer and evaluate it. However, unlike the golfer, there is a continual back and forward interaction between identify, setup, execute and evaluate.

To be good at golf requires lots of "quality" practice. The same is for you, to become good at physics requires practice, but remember it is only good practice that makes perfect and not time spent.

PRACTICE ONLY MAKES PERMANENT

IDENTIFY

- Identify what the question asking
- Identify the known and unknown physical quantities (units)
- Identify the type of problem

SETUP need a good memory base and understanding

- Visualise the physical situation
- Diagrams - reference frames / coordination system / origin / directions
- Write down key concepts, principles, equations, assumptions that may be needed to answer the question

EXECUTE

- Answer to the question from what you know.
- Numerical questions - solve before calculations - manipulate equations then substitute numbers add comments.

EVALUATE

- CHECK - answer reasonable, assumptions, units, signs, significant figures, look at limiting cases
A basic goal of science is to explain a wide range of phenomena in terms of a small number of powerful, fundamental physical principles.

Learning physics is similar to learning a foreign language such as Chinese. Not only do you have to know the words and meanings but how to put it all together to have successful communication. The language in physics is very specific and the meaning of words can be different to when used in everyday speech. You will have to learn the “restricted” meaning of words and how they relate to describing our physical world, and the basis of describing our physical world is in the use of models.

A major goal for yourself is to strive to achieve is to give good scientific explanations. In answering all questions in examinations, assignments, laboratory, workshops, etc, it is expected that you will use scientific language correctly. The following paragraphs highlight the criteria for good scientific explanations, on which you will be assessed continually.

You should repeatedly refer to the criteria, so that in the examination you will be able to give good scientific explanations in your answers.

A good answer is not just a collection of words, equations and numbers. Often in physics, a good answer to a question combines features of an essay and a mathematical proof - it clearly and coherently communicates your thinking about a question to someone else and it presents a logically valid chain of reasoning based on established principles.

Just as you can tell the difference between a good essay and a poor one, or a good geometry proof and an inadequate one, you will need to learn to distinguish between good and poor scientific explanations.

**Criteria For A Good Scientific Explanations**

- **Explanation based on fundamental physical principles including relevant equations and not just a description**
  
  A description tells what happened; an explanation tells why it happened, in terms of fundamental scientific principles.
  
  **A description:** The charges spread out all over the surface of the metal.
  
  **An explanation:** The charges spread out all over the surface of the metal because the free electrons in metal repel each other (like charges repel) and arrange themselves at the greatest possible distances from each other.

  **Scientific words have very precise meanings**, and they must be used precisely. Unlike everyday speech, where it is permissible to substitute many different words for each other, there are very few synonyms in science. If you use the wrong word, your statement may be meaningless or utterly incorrect. Here are some important words that are frequently misused by novice students: *Work, pressure, force, acceleration, velocity, amplitude, charge, charged, dipole, field, induce, induction, ionize, ionization, neutralize, polarization, polarized, potential*

  For example, a charged object is not the same as a polarized object. Force and charge are utterly different concepts; they are connected conceptually by the fact that a charged object can exert a force on another charged object.

  Here are some examples of **meaningless** statements from students’ answers: "The charge attracts to the positive dipole." and "The metal block is induced by the touching of a positive charge."

- **Explanations of physical phenomena can be given as a number of precise steps (chain argument).**

  **Example** (from *Thinking Physics* by L.C. Epstein & P G Hewitt): Mighty Mouse wants to get the ball bearing up and out of the bowl, but the ball is too heavy and the sides of the bowl too steep
for Mighty Mouse to support the ball's weight. Write a scientific explanation for Mighty Mouse, so that he/she will know how (and why) to get the ball out of the bowl using his/her own strength and without the aid of such things as levers.

- Give the ball a series of little pushes.
- Each push must be at the right time and in the right direction.
- The trick is you must match the rhythm of the pushes to the natural rhythm with which the ball rolls back and forth.
- This natural rhythm is called the natural or resonant frequency of vibration.
- There is usually more than one way a "thing" can vibrate, or resonate. The ball in the bowl can resonate back and forth or it can resonate around a circle. These different ways are called resonant modes.
- Each push then can add a little kinetic energy to the ball.
- Eventually, there will be enough kinetic energy added to the ball to get it to the rim and over the top.

**Diagrams - readable, relevant details, labels**
Scientists draw diagrams all the time. They use diagrams as a tool to support and guide their own thinking, as well as a device for explaining their ideas to others.
Many students are reluctant to draw diagrams. As reasons for their reluctance students say things like "I'm not good at drawing," "It takes too much time," "It's redundant, because I have to explain everything in words and equations anyhow." A common thread in these statements seems to be the perception that a diagram is a decoration, not a tool. Many students have not yet learned to use diagrams in a way that can guide their own reasoning and prevent many errors.

**Readability**
A diagram must be large enough to see and interpret easily. Do not draw little teeny diagrams in the margin of your paper. Make the diagram big enough that all the important information can be included in it, and can be interpreted easily by a reader. A diagram should not be ornate. Use simple, clean lines.

**Labels**
By labelling all distances, charges, and forces in a diagram, you bring together in one place a great deal of information that is scattered throughout the problem. Once it is recorded on your diagram you do not have to search for it again. Labels help to prevent serious errors. Carefully labelled diagrams significantly reduce the number of errors made in problem solutions.

**Include only relevant details**
A cluttered diagram is hard to interpret. For clarity, include only relevant information. For example, show only excess charges, but do distinguish between charge on a surface and charge inside an object. Do distinguish between free and bound charges - do not make your drawings of polarized molecules in an insulator look the same as a drawing of a polarized metal. These distinctions are important physical distinctions, so diagrams must reflect them unambiguously.

*Often a good diagram can bear the major burden of explanation, with little or no accompanying prose required to make the point. A useful diagram is the centrepiece of a good explanation.*
On a macroscopic scale, when the polarization $P$ is proportional to the electric field $E$ within the dielectric (dielectric constant $\varepsilon_r$)

$$ P = \chi \varepsilon_o E = (\varepsilon_r - 1) \varepsilon_o E $$

where $n$ is the number density (molecules/volume), $p$ is the dipole moment, $q$ is the charge separated and $d$ is the charge separation.

On an microscopic scale an induced electric dipole moment $p$ is proportional to an external electric field $E$ and the constant of proportionality $\alpha$ is called the polarisability of the molecule

$$ p = \alpha E $$

For an atom (assume atom spherical volume of radius $a$), $\alpha = 4\pi \varepsilon_o a^3$

For non-polar molecules (no permanent electric dipole moments)

- For a dilute medium ($\varepsilon_r \sim 1$) $\alpha = \frac{\varepsilon_o}{n} (\varepsilon_r - 1)$
- A better model that can be applied to more dense media, is the Clausius-Mossotti equation

$$ \alpha = \frac{3\varepsilon_o}{n} \frac{\varepsilon_r - 1}{\varepsilon_r + 2} $$

(a) What is a simple way to measure $\varepsilon_r$?

(b) Plot $\frac{n\alpha}{\varepsilon_o}$ vs. $\varepsilon_r$ ($\varepsilon_r$ from 1 to 1.5) for a dilute and dense media.

(c) What is the approximate value of $\varepsilon_r$ when the equation for the polarisability of non-polar molecules for a dilute medium fails?

(d) The dielectric constant for oxygen gas is $\varepsilon_{rg} = 1.000523$ and its molar mass is 32.0 g.

One mole of oxygen gas at STP (273 K, 1 atm) has a volume of 22.4 L. In the liquid state, the density of oxygen is $1.190 \times 10^3$ kg.m$^{-3}$. ($N_A = 6.02 \times 10^{23}$)

(i) Calculate the number densities for oxygen in the gas $n_g$ and liquid $n_l$ states.

(ii) Estimate the radius of an oxygen atom. Is your answer reasonable?

(iii) Assume that the atomic polarizability $\alpha$ is the same in the gaseous and liquid states.

Estimate its dielectric constant for liquid oxygen $\varepsilon_r^l$. Comment on your answer

(e) The oxygen gas was placed into an external electric field $E = 12.5$ kV.m$^{-1}$.

(i) Calculate the induced electric dipole moment $p$.

(ii) Calculate the separation $d$ between the centre of positive and negative charge for the oxygen atoms.

(f)(i) What is the potential in region far from the oxygen electric dipole?

(ii) Why is it difficult to calculate and plot the potential near the dipole?

(iii) Plot in [3D] the potential surrounding the induced electric dipole for an oxygen molecule.

(iv) Does the [3D] plot confirm your answers to parts (fi) and (fii)?
**E040**
A circular parallel-plate capacitor of radius $a$ and plate separation $d$ is connected in series with a resistor $R$ and a switch, initially open, to a constant voltage source $V_o$. The switch is closed at time $t = 0$. Assume that the charging time of the capacitor, $\tau = RC$ is very long compared with $a/c$ and that $d << a$ where $C$ is the capacitance and $c$ is the velocity of light.

Show that the displacement current density $J_d$ as a function of time $t$ is given by

$$J_d = \left(\frac{V_o}{\pi a^2 R}\right) e^{-t/\tau}$$

**E041**
(1) Sketch three separate diagrams to explain how dipoles can be created in a dielectric.

Consider the gas xenon ($\alpha = 3.54 \times 10^{-40} \text{ F.m}^2$, $Z = 54$, $p = 5 \text{ atm}$, $T = 300 \text{ K}$) in an electric field of strength $1.45 \times 10^5 \text{ V.m}^{-1}$.

(2) What is meant by the symbol $\alpha$? Does it depend on the state of matter of the dielectric (gas, liquid, solid)?

(3) How does the polarization of the xenon gas occur in an electric field?

(4) Calculate the dielectric constant of xenon.

(5) Calculate the radius of a xenon atom.

(6) Calculate the charge separation in the induced dipole of a xenon atom.

(7) Calculate the electric dipole moment of the xenon atom.

(8) Calculate the polarization of the xenon gas.

(9) Are the above values sensible?

(10) How would the values be different if the gas was helium?

**E073**
A point charge $Q$ is situated a large distance $r$ from a neutral atom of atomic polarizability $\alpha$.

Draw a labeled diagram of the situation. Find the electric field $E_Q$ at the location of the atom due to the point charge $Q$. Find the electric field $E_p$ produced by the induced dipole of the atom at the location of the point charge $Q$. Hence, find the force of attraction $F$ between the point charge and the induced dipole.

Answers can be expressed in terms of some or all of the quantities: $\pi \varepsilon r Q$ and $\alpha$. 

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**E095**
A parallel plate capacitor is placed in a glass container and then connected to a 12 V battery. The area of the capacitor plates is 0.0100 m$^2$ and the plate separation is 1.00 mm. Find the charge on the plates of the capacitor in the following two cases.

(a) The capacitor is disconnected from the battery and then the container is filled to the top with oil whose dielectric constant is 2.2.
(b) The capacitor is connected to the battery as the container is filled to the top with oil and then disconnected from the battery.

Indicate in cases (a) and (b) how the electric field intensity changes in the capacitor when the oil is poured in.

**E105**
Consider a parallel plate capacitor with two dielectric slabs completely filling the space between the capacitor plates.

Show that the capacitance is

$$C = \frac{\varepsilon_0 A}{d_1 + \frac{d_2}{\varepsilon_{r1} + \varepsilon_{r2}}}$$

**E127**
Capacitors rarely consist of only two plates but are usually made up of multiple plates. For example, the odd plates are connected to one terminal of a battery and the even plates to the other terminal. If there are $M$ plates ($M$ even: 2, 4, ...), each with area $A$ and separation $d$, derive an expression for the capacitance $C$ of such a capacitor. You may ignore edge effects.

**E147**
A dielectric sphere of radius $R$ carries a polarization $\vec{P} = k \vec{r}$ where $k$ is a constant and $\vec{r}$ is the vector from the centre of the sphere.

(a) Calculate the bound surface charge $\sigma_b$.
(b) Calculate the bound charge density $\rho_b$.
(c) Calculate the electric field inside the dielectric sphere $E_i$.
(d) Calculate the surface charge $Q_S$.
(e) Calculate the interior charge $Q_i$.
(f) Calculate the total charge of the dielectric sphere $Q$.
(g) Calculate the electric field outside the dielectric sphere $E_O$. 
The parallel plate capacitor shown has a plate area $A$ and separation distance $d$ and has a charge of $Q$. The free charge surface density is $\sigma_f = \frac{Q}{A}$. The space between the slabs is filled by two dielectric slabs are equal thickness ($d/2$) but difference dielectric constants ($\varepsilon_{r1}$ and $\varepsilon_{r2}$).

Show and justify the following:

(a) Electric displacement $D = \sigma_f = \frac{Q}{A}$

(b) Electric field $E_1 = \frac{\sigma_f}{\varepsilon_{r1} \varepsilon_0}$, $E_2 = \frac{\sigma_f}{\varepsilon_{r2} \varepsilon_0}$

(c) Polarization $P_1 = \sigma_f \left( \frac{\varepsilon_{r1} - 1}{\varepsilon_{r1}} \right)$, $P_2 = \sigma_f \left( \frac{\varepsilon_{r2} - 1}{\varepsilon_{r2}} \right)$

(d) Potential difference between the plates $V = \left( \frac{Q d}{2 \varepsilon_0 A} \right) \left( \frac{\varepsilon_{r1} + \varepsilon_{r2}}{\varepsilon_{r1} \varepsilon_{r2}} \right)$

(e) Capacitance $C = \left( \frac{2 \varepsilon_0 A}{d} \right) \left( \frac{\varepsilon_{r1} \varepsilon_{r2}}{\varepsilon_{r1} + \varepsilon_{r2}} \right)$

(f) Net bound charge at the interface between the two dielectric slabs $\sigma_{\text{bnet}} = \left( \frac{\sigma_f}{\varepsilon_{r1} \varepsilon_{r2}} \right) (\varepsilon_{r1} - \varepsilon_{r2})$
Consider a copper cable connected from a battery in an electric car. The battery emf is 12.0 V and its internal resistance is 5.00 mΩ. The cable current is 80.0 A, its length is 1.00 m and the maximum permissible power dissipated in the cable is 10.0 W.

Show and justify the following:

(a) Potential difference across the ends of the cable, \( V = 0.125 \) V
(b) Resistance of cable, \( R = 1.56 \times 10^{-3} \) Ω
(c) Cross-sectional area of cable, \( A = 1.08 \times 10^{-5} \) m²
(d) Radius of cable, \( r = 1.85 \times 10^{-3} \) m
(e) The battery voltage, \( V_{\text{battery}} = 11.6 \) V
(f) The current density, \( J = 7.44 \times 10^{6} \) A.m⁻²
(g) Number density, \( n = 8.43 \times 10^{28} \) particles.m⁻³
(h) The drift velocity, \( v_{\text{drift}} = 5.52 \times 10^{-4} \) m.s⁻¹
(i) The time, \( t = 1.81 \times 10^{3} \) s
(j) The relaxation time \( \tau = 2.51 \times 10^{-14} \) s
(k) The mean free path, \( \lambda = 1.39 \times 10^{-17} \) m
(l) Conductivity, \( \sigma = 5.95 \times 10^{7} \) (Ω.m)⁻¹
(m) The electric field in the cable (\( E_1, E_2 \) using \( E = -\frac{dV}{dx} \) and \( J = \sigma E \)).

\[
E_1 = 0.125 \ \text{V.m}^{-1} \quad E_2 = 0.125 \ \text{V.m}^{-1}
\]

Comment on the significance of the results.
A parallel plate capacitor has a square plate area of 0.200 m$^2$ and a plate separation of 0.0100 m. The potential difference between the plates is 3.00 kV. A dielectric slab with a dielectric constant of 3 completely fills the space between the plates of the capacitor. While the capacitor is still connected to the battery, the dielectric slab is partially removed. The slab is withdrawn in a direction parallel to the plates through a distance $x$ (slab fully inserted, $x = 0$ and half space filled, $x = L/2$, slab fully removed, $x = L$). Assume the dielectric slab is removed very slowly so that the kinetic energy of the slab is zero and that edge effects at the ends of the capacitor can be ignored.

(a) Show the following relationship as functions of $x$

**Capacitance**

$$C = \left(\frac{\varepsilon_0 L}{d}\right)(x + \varepsilon_r (L-x))$$

**Charge**

$$Q = \left(\frac{\varepsilon_0 LV}{d}\right)(x + \varepsilon_r (L-x))$$

**Energy stored by capacitor**

$$U_{cap} = \left(\frac{\varepsilon_0 LV^2}{2d}\right)(x + \varepsilon_r (L-x))$$

$$\Delta U_{cap} = \left(\frac{\varepsilon_0 LV^2}{2d}\right)(x + \varepsilon_r (L-x) - \varepsilon_r L)$$

**Change in energy stored by battery**

$$\Delta U_{battery} = -\left(\frac{\varepsilon_0 LV^2}{d}\right)(x + \varepsilon_r (L-x) - \varepsilon_r L)$$

**Work done by external force**

$$W_{me} = -\left(\frac{\varepsilon_0 LV^2}{2d}\right)(x + \varepsilon_r (L-x) - \varepsilon_r L)$$

**External force in removing dielectric**

$$F_{me} = \left(\frac{\varepsilon_0 LV^2}{2d}\right)(\varepsilon_r - 1)$$ independent of $x$

(b) Calculate the above quantities when $x = 0, L/2, L$. 
E293
A hydrogen atom has a radius of $5.29 \times 10^{-11}$ m and is situated between two metal plates 1 mm apart. The potential difference across the plates is 500 V. The measured value for the atomic polarizability is $\alpha / (4\pi \varepsilon_0) = 6.67 \times 10^{-31}$ m$^3$. The hydrogen gas is at a temperature of 300 K and the pressure is atmospheric pressure. $\varepsilon_0 = 8.854 \times 10^{-12}$ F.m$^{-1}$

Calculate the following
(a) Number density, $n$.
(b) Atomic polarizability, $\alpha$ [F.m$^{-2}$]
(c) Dielectric constant using $\varepsilon_r = 1 + \frac{n\alpha}{\varepsilon_0}$
(d) Dielectric constant using $\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{n\alpha}{3 \varepsilon_0}$
(e) Dielectric constant using $\varepsilon_r = 1 + 4\pi n a^3$ Comment on the three values for $\varepsilon_r$.
(f) Electric field between the plates, $E$.
(g) The separation distance between the two charges in the induced electric dipole, $d$.
(h) The ratio of the Bohr radius, $a$ to the charge separation $d$ for the induced electric dipole moment, $a/d$. Comment of the value for this ratio.
(i) The plate voltage you would need with this apparatus to ionize the hydrogen atom, $V_i$.
(j) The work done on the electron to remove it from the hydrogen atom, $W_i$. Compare this value with the result of quantum theory: ionization energy $E_i = 13.6$ eV.

E288
A parallel plate capacitor has an area $A = 20.0$ cm$^2$ and a plate separation of $d = 4.00$ mm. (a) Find the capacitance in air and the maximum voltage and charge the capacitor can hold. (b) A Teflon sheet is slid between the plates, filling the entire volume, Find the new capacitance and maximum charge. (c) Before the insertion of the Teflon, the plates are connected to a 24.0 V battery and then disconnected. What are the energies in the capacitor before and after the Teflon is inserted? (d) Was work done in inserting the Teflon?

Suppose now, that the Teflon sheet that is inserted between the plates is only 2 mm thick and fills only half the volume. Before the Teflon is inserted, the disconnected capacitor carries a charge of 1.00 nC. (e) Find the electric field everywhere. (f) Find the new capacitance.

Dielectric strength (air) = $3.00 \times 10^6$ V.m$^{-1}$ Dielectric strength (Teflon) = $6.00 \times 10^7$ V.m$^{-1}$

Dielectric constant (Teflon) = 2.1

E305
The dipole moment of the water molecule has a value of $6.2 \times 10^{-30}$ C.m. Find the maximum electric polarization of water vapour at 100 °C and atmospheric pressure.

Use the Langevin equation (when $p E / k T \ll 1$) and the dielectric constant of water at 20 °C is 81 to determine the electric dipole moment of the water molecule.

Comment on any discrepancy with the two values.
E333
Consider a daily journey to and from work, a distance of 60 km. Assume a petrol driven car with a fuel consumption of 10 L/100 km. The efficiency of the petrol engine in delivering useful work is 20%. For an electric car, using lithium-ion batteries that have an efficiency of 90%, what is the required mass of the lithium-ion batteries? If the batteries are charged from a single 240 V, 15 A powerpoint, how long would it take to recharge the batteries.

Energy density of petrol = 46 MJ.kg\(^{-1}\)
Density of petrol = 750 kg.m\(^{-3}\)
Lithium-ion battery, energy density = 160 W.h.kg\(^{-1}\)

Show and justify the following:
- petrol - volume used = 6.00\(\times\)10\(^{-3}\) m\(^3\)
- petrol - mass used = 4.50 kg
- petrol - energy supplied = 2.07\(\times\)10\(^8\) J
- car - energy as useful work = 4.14\(\times\)10\(^7\) J
- batteries - energy need to be supplied = 4.60\(\times\)10\(^7\) J
- batteries - mass required = 79.9 kg
- powerpoint - power = 3.60\(\times\)10\(^3\) W
- batteries - charging time = 3.55 h

What conclusions can be drawn from the numerical results for the simple mode used in modeling the performance of the petrol and electric cars?

E361
A 12 V car battery is used to charge a 100 \(\mu\)F capacitor. (a) How much energy is stored in the capacitor? (b) Compare this with the energy stored in the battery itself given that battery is rated at 100 A.h. (c) What potential difference would be needed for the capacitor to provide the same energy as the battery? (d) If the potential difference across the capacitor was 12 V what would be the required capacitance value to store the same energy as the battery?

Batteries are a practical way to store large amounts of energy. A battery energy's is stored in chemical bonds rather than the macroscopic separation of charge in capacitor. Batteries are a practical way to store large amounts of energy for long periods, but not a practical way to deliver the energy quickly. Capacitors can deliver energy very quickly and can also be used to deliver energy slowly if required.

The 100 \(\mu\)F capacitor was fully charged by the 12 V car battery and then disconnected from the battery. (e) How long would it take to fully discharge the capacitor (t \(\sim\) 5\(\times\) time constant) through a 1.5 \(\Omega\) resistor and what is the final charge on the capacitor plates? (f) How much energy would be delivered by the battery in this time if it was connected to the through 1.5 \(\Omega\) resistor? Would this be possible?
E382

Two coaxial cylindrical metal tubes (inner tube, radius \( a \) and outer tube radius \( b \)) stand vertically in a tank of dielectric oil (dielectric constant \( \varepsilon_r \) and mass density \( \rho \)). The inner tube is maintained at a potential \( V \) and the outer one is grounded. The length of the vertical tubes is \( L \). To what height \( h \) does the oil rise in the space between the tubes?

**Hints - Show and justify the following:**

Let \( \lambda \) represent the charge per unit length

In the air part

\[
E_{\text{air}} = \frac{\lambda_{\text{air}}}{2\pi \varepsilon_0 r}, \quad V = \left( \frac{\lambda_{\text{air}}}{2\pi \varepsilon_0} \right) \ln(b/a)
\]

In the oil part

\[
E_{\text{oil}} = \frac{\lambda_{\text{oil}}}{2\pi \varepsilon_r \varepsilon_0 r}, \quad V = \left( \frac{\lambda_{\text{oil}}}{2\pi \varepsilon_r \varepsilon_0} \right) \ln(b/a) \quad \lambda_{\text{oil}} = \varepsilon_r \lambda_{\text{air}}
\]

Charge on inner conductor

\[
Q = \lambda_{\text{air}} \left( L + (\varepsilon_r - 1)h \right)
\]

Capacitance

\[
C = \left( \frac{2\pi \varepsilon_0 \left( L + (\varepsilon_r - 1)h \right)}{\ln(b/a)} \right)
\]

Change in energy of system of capacitor and battery

\[
dU_{\text{system}} = -\frac{1}{2} CV^2
\]

The force attracting the oil into the tubes

\[
|F_{\text{oil}}| = \frac{\pi \varepsilon_0 (\varepsilon_r - 1) V^2}{\ln(b/a)}
\]

The gravitation force

\[
|F_G| = \rho \pi \left( b^2 - a^2 \right) h g
\]

The height of the oil

\[
h = \frac{\varepsilon_0 (\varepsilon_r - 1) V^2}{\rho \left( b^2 - a^2 \right) g \ln(b/a)}
\]
A metal sphere of radius $a$ and charge $+Q$ is surrounded by a thick spherical layer of dielectric material with a dielectric constant $\varepsilon_r$ and it has an outer radius $b$ as measured from the centre of the metal sphere. Starting with Gauss’s Law in the form $\oiint \vec{D} \cdot d\vec{A} = q_f$ show and/or justify the following relationships inside the dielectric material ($a < r < b$):

(a) electric displacement $D = \frac{Q}{4\pi r^2}$ radial outward

(b) electric field $E = \frac{Q}{4\pi \varepsilon_r \varepsilon_0 r^2} = \frac{Q - Q_b}{4\pi \varepsilon_0 r^2}$ radial outward

(c) bound charge $|Q_b| = Q \left( \frac{\varepsilon_r - 1}{\varepsilon_r} \right)$

(d) polarization $P = \frac{Q}{4\pi r^2} \left( \frac{\varepsilon_r - 1}{\varepsilon_r} \right)$ radial outward

(e) bound charge density $\rho_b = -Q \left( \frac{\varepsilon_r - 1}{\varepsilon_r} \right) \delta^3(\vec{r})$ use $\nabla \cdot \left( \frac{\hat{r}}{r^2} \right) = 4\pi \delta^3(\vec{r})$

Bound charge at centre of dielectric material

$$\iiint \rho_b \, d\tau = -\iiint Q \left( \frac{\varepsilon_r - 1}{\varepsilon_r} \right) \delta^3(\vec{r}) \, d\tau = -Q \left( \frac{\varepsilon_r - 1}{\varepsilon_r} \right) \iiint \delta^3(\vec{r}) \, d\tau = -Q \left( \frac{\varepsilon_r - 1}{\varepsilon_r} \right)$$

(f) surface charge density

$$\sigma_b(r = a) = -Q \left( \frac{\varepsilon_r - 1}{\varepsilon_r} \right)$$

$$\sigma_b(r = b) = \frac{+Q}{4\pi b^2 \varepsilon_r}$$

(g) total charge of dielectric material $Q_{\text{dielectric}} = 0.$

Sketch the electric field lines inside and outside the dielectric material. Explain why the number of electric field line change at the boundaries of the dielectric material.

Sketch the distribution of the free and bound charges for the metal sphere and dielectric material.

How does the electric field surrounding the metal sphere (charge $+Q$) and dielectric layer compare with the electric field surrounding a point charge $Q$? Explain.
**E551**
A fuel gauge uses a capacitor to determine the height $h$ of the fuel in the tank. When the tank is empty, air fills the space between the capacitor plates and when the tank is full, fuel fills the space. The effective dielectric constant $\varepsilon_{\text{eff}}$ of the capacitor changes as the level of fuel changes. Appropriate electronics can determine the value of the effective dielectric constant. Determine an expression for the effective dielectric constant $\varepsilon_{\text{eff}}$ as a function of the height of the fuel $h$, height $l$ and dielectric constant $\varepsilon_r$. Consider the fuel to be either petrol ($\varepsilon_r = 1.95$) or methanol ($\varepsilon_r = 33.0$).

Sketch graphs of this function for both the petrol and methanol. What are the values for $\varepsilon_{\text{eff}}$ for both petrol and methanol when the tank is empty, $\frac{1}{4}$ full, $\frac{1}{2}$ full, $\frac{3}{4}$ full and full? For which fuel is the fuel gauge more practical?

**E567**
Consider a parallel plate capacitor with a thin dielectric material ($\varepsilon_r = 3.00$) inserted between the plates. The surface charge density on the plates is 3.00 $\mu$C. Find electric displacement, electric field and polarization outside and inside the dielectric for the region between the plates of the capacitor. Ignore any edge effects. Find the bound charge. If the electric displacement is represented by 12 field lines in the region between the plates of the capacitor, how many fields are there for the electric field and polarization inside and outside the dielectric. Sketch a diagram of the capacitor with the dielectric showing the free and bound charges and the field lines for the electric displacement, electric field and polarization.

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{N}^{-1} \text{m}^{-2}$$
**E582**
Sulfur has a density of $2.1 \times 10^3$ kg.m$^{-3}$, is number 16 in the periodic table and has an atomic mass of 32. Sulfur has a dielectric constant of 4. The internal electric field in a piece of sulfur is $10^7$ V.m$^{-1}$. Assuming all electrons to be displaced by the same distance $d$ with respect to the nucleus, find $d$. How does $d$ compare to the size of an atom? Comment on this result.

$$e = 1.60 \times 10^{-19} \text{ C} \quad N_A = 6.02 \times 10^{23} \text{ particles.mol}^{-1}$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2\text{N.m}^{-2}$$

1. Consider a single sulfur atom. Draw a diagram showing the electron distribution.
2. The internal electric field in a piece of sulfur is $10^7$ V.m$^{-1}$. Draw a diagram showing the electron distribution and the electric field.
3. On the diagram show the electric dipole moment of a sulfur atom (vector), the charge that has been separated and the separation distance $d$ and the equation for the electric dipole moment of a single sulfur atom.
4. What does the internal electric field do to the piece of sulfur?
5. Determine the electric susceptibility of the sulfur.
6. Determine the number density of the sulfur.
7. Determine the electric dipole moment of a sulfur atom.
8. Determine the charge separation distance $d$.
9. Comment on the magnitude of $d$.
10. You can also approach the problem from an algebraic point of view. Express the separation distance $d$ in terms of the following quantities $\varepsilon_r$, $\varepsilon_0$, $E$, $M$ (molar mass), $Z$ (atomic number), $\rho$, $N_A$

**E588**
The table gives the values for the dielectric constant of a few materials at DC and an optical frequency.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\varepsilon_r$ (DC)</th>
<th>$\varepsilon_r$ (optical)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diamond</td>
<td>5.68</td>
<td>5.66</td>
</tr>
<tr>
<td>NaCl</td>
<td>5.90</td>
<td>2.34</td>
</tr>
<tr>
<td>LiCl</td>
<td>11.59</td>
<td>2.78</td>
</tr>
<tr>
<td>water</td>
<td>80</td>
<td>1.33</td>
</tr>
</tbody>
</table>

(a) What is meant by the terms DC and optical?
(b) Explain the changes in the dielectric constant with frequency for each material.

**E600**
The diagram shows a parallel plate capacitor with a plate area of $1.00 \times 10^{-2}$ m$^2$ and a plate separation of $1.00 \times 10^{-3}$ m containing a dielectric sheet ($\varepsilon_r = 2.00$) of unknown thickness $t$ inserted. (a) Calculate the capacitance without the dielectric sheet inserted. (b) If the capacitance with the sheet inserted is found to be $1.01 \times 10^{-10}$ F then calculate the thickness of the dielectric sheet.
**E617**
Two capacitors of equal capacitance $C_1$ are connected in parallel and charged to a voltage $V_1$. The two capacitors still connected in parallel are isolated from the voltage source. Then a dielectric slab of dielectric constant $\varepsilon_r$ is inserted into one capacitor to completely fill the space between the plates. Express the final voltage $V_2$ across the two capacitors connected in parallel and the charges $q_A$ and $q_B$ on the plates of each capacitor in terms of $C_1$, $V_1$ and $\varepsilon_r$.

**E664**
Under which condition must more work be done in moving apart the plates of a capacitor:
(1) when the capacitor is connected to the voltage source all the time or
(2) when it is disconnected after the initial charging.
Explain your answer.
A parallel plate capacitor has a square plate area of 0.200 m\(^2\) and a plate separation of 0.0100 m. The potential difference between the plates is 3.00 kV. Then a dielectric sheet of thickness 5.00\(\times\)10\(^{-3}\) m and dielectric constant of 3 is inserted midway between the plates of the charged capacitor that is still connected to the battery.

1. Sketch the capacitor without the dielectric and label it with the values of \(A\), \(d\) and \(V\).
2. Sketch the capacitor with the dielectric showing the distribution of the free and bound charges for the plates and dielectric. On another sketch, show the field lines for \(\vec{E}\), \(\vec{D}\), \(\vec{P}\).
3. Show that the values in the following table are correct. Explain why the quantities increase, decrease or stay the same.

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<thead>
<tr>
<th></th>
<th>No dielectric</th>
<th>dielectric</th>
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</thead>
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<tr>
<td>(\varepsilon_0)</td>
<td>8.85(\times)10(^{-12})</td>
<td>8.85(\times)10(^{-12})</td>
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<tr>
<td>(\varepsilon_r)</td>
<td>1</td>
<td>3</td>
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<tr>
<td>(A)</td>
<td>0.200</td>
<td>0.200</td>
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<tr>
<td>(d)</td>
<td>10.0(\times)10(^{-3})</td>
<td>10.0(\times)10(^{-3})</td>
</tr>
<tr>
<td>(V)</td>
<td>3.00(\times)10(^3)</td>
<td>3.00(\times)10(^3)</td>
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<tr>
<td>(t)</td>
<td>5.00(\times)10(^{-3})</td>
<td>5.00(\times)10(^{-3})</td>
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<tr>
<td>(E)</td>
<td>3.00(\times)10(^5)</td>
<td>9.00(\times)10(^5)</td>
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<tr>
<td></td>
<td>(E_{\text{free space}} = 3.00\times10^5)</td>
<td>(E_{\text{inside}} = 3.00\times10^5)</td>
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<tr>
<td>(C)</td>
<td>1.77(\times)10(^{10})</td>
<td>5.31(\times)10(^{10})</td>
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<tr>
<td>(Q)</td>
<td>5.31(\times)10(^{7})</td>
<td>1.59(\times)10(^{6})</td>
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<tr>
<td>(\sigma_f)</td>
<td>2.66(\times)10(^6)</td>
<td>7.97(\times)10(^6)</td>
</tr>
<tr>
<td>(\sigma_b)</td>
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<td>5.31(\times)10(^6)</td>
</tr>
<tr>
<td>(D)</td>
<td>2.66(\times)10(^6)</td>
<td>7.97(\times)10(^6)</td>
</tr>
<tr>
<td>(P)</td>
<td>0</td>
<td>5.31(\times)10(^6)</td>
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<td>(\chi_e)</td>
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<td>2</td>
</tr>
<tr>
<td>(U)</td>
<td>7.97(\times)10(^{-4})</td>
<td>2.39(\times)10(^{-3})</td>
</tr>
<tr>
<td>(u)</td>
<td>0.398</td>
<td>1.20</td>
</tr>
<tr>
<td>(\Delta U_{\text{cap}})</td>
<td>(\times\times\times)</td>
<td>1.59(\times)10(^3)</td>
</tr>
<tr>
<td>(\Delta U_{\text{battery}})</td>
<td>(\times\times\times)</td>
<td>-3.19(\times)10(^3)</td>
</tr>
</tbody>
</table>
| \(|\Delta U_{\text{battery}}/\Delta U_{\text{cap}}|\) | 2             | Battery supplies twice the energy that was given to capacitor
Consider two concentric conducting spherical shells with radii $R_1$ and $R_2$. The space between the shells is taken up by a material with dielectric constant $\varepsilon_r$. What is the capacitance $C$? Show as $R_2 \rightarrow R_1$ ($d = R_2 - R_1$) and for large radii of curvature that you obtain the result for a parallel plate capacitor. Determine the capacitance of Earth, assuming it to a conductor.

$$k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \quad 4\pi\varepsilon_0 = \frac{1}{9\times10^9} \quad R_E = 6.38\times10^6 \text{ m}$$

(a) Assume vacuum filled capacitor to start.
- Radius of inner conducting sphere (charge $+Q$), $R_1$
- Radius of outer conducting sphere (charge $-Q$), $R_2$

Draw a labeled diagram of the physical situation.

Show that the electric field between the spheres is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

(b) Derive the following equation for the magnitude of the potential in the region between the two spheres

$$|V| = \frac{Q}{4\pi\varepsilon_0} \int_{R_1}^{R_2} \frac{dr}{r} = \frac{Q}{4\pi\varepsilon_0} \left( \frac{1}{R_2} - \frac{1}{R_1} \right) = \frac{Q}{4\pi\varepsilon_0} \frac{R_2 - R_1}{R_1 R_2}$$

(c) Derive an expression for the capacitance $C$.

Comment on your result – what does $C$ depend upon?

(d) Modify the equation you derived in (3) to account for the presence of a dielectric between the metal spheres. When a dielectric is inserted between the plates of a capacitor, does the capacitance $C$ decrease, increase or stay the same? Explain your answer.

(e) Show as $R_2 \rightarrow R_1$ ($d = R_2 - R_1$) and for large radii of curvature you obtain the result for a parallel plate capacitor

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

(f) Show that the capacitance of an isolated sphere (radius $R$) is given by

$$C = 4\pi\varepsilon R$$

(g) Determine the capacitance of Earth, assuming it to be conducting. $C_{\text{Earth}} = \ ? \ \text{ F}$
Consider a daily journey to and from work, a distance of 60 km in an electric car. Assume the energy required for the journey is \( U = 4.60 \times 10^7 \) J (see problem E333). The electric car is powered by banks of ultracapacitors. The capacitance of each capacitor is \( C_u = 3000 \) F and the maximum voltage across each capacitor is \( V_u = 2.00 \) V. The voltage across the capacitor bank is \( V = 100 \) V.

(a) Draw a diagram of how the capacitors can be arranged in a series and parallel combination to provide the energy for the journey.

(b) What is the total charge \( Q \) that must be stored by the capacitor bank to provide the energy for the journey?

(c) What is the equivalent capacitance of the capacitor bank \( C \)?

(d) What is the charge stored by an individual ultracapacitor \( Q_u \)?

(e) How much energy is stored by each individual capacitor \( U_u \)?

(f) What is the number of capacitors \( N_u \) required to store the energy necessary for the journey?

(g) What number of capacitors \( N_s \) can be added in series to provide the 100 V operating voltage?

(h) What is the charge \( Q_s \) stored in each series branch?

(i) What is the capacitance \( C_s \) of each series branch?

(j) How much energy \( U_s \) is stored in the series connection of \( N_s \) capacitors?

(k) What number of parallel series branches \( N_p \) is required to stored the energy for the journey?

A parallel plate capacitor is connected to a battery and is in equilibrium. Now a dielectric slab of dielectric constant \( \varepsilon_r \) is slowly inserted between the plates and the system allowed to come to equilibrium again. Describe and explain, what happens to the potential difference, capacitance, charge on the capacitor, electric field and stored energy. Is work done on the slab?
E797
The space between the plates of a parallel plate capacitor contains two dielectric slabs. The slabs have a thickness $a$ and the total distance between the plates is $5a$. Slab 1 has a dielectric constant of 2, and slab 2 has a dielectric constant of 1.5. The free charge density on the top plate is $+\sigma_f$ and on the bottom plate $-\sigma_f$.

(a) Find the electric displacement in each slab.
(b) Find the electric field in the free space and in each slab.
(c) Find the polarization in each slab.
(d) Find the potential difference between the plates.
(e) Find the amount of bound charge associated with each slab and the location of all charges.
(f) Now that you know all the charges (free and bound), apply four Gaussian surface to verify the values for the electric field given by part (b).

E815
One conductive plate has a charge $-Q$ and another plate has a charge $+1.2Q$. The plates are separated from each other by a distance $d$ to form an air-filled parallel plate with area $A$.

(a) What is the capacitance $C$ of the capacitor?
(b) What is the electric field between the plates of the capacitor?
(c) What is the potential difference between the plates of the capacitor?
(d) Draw a diagram showing the charge distribution on each plate using 10 minus signs (-) to represent the charge $-Q$. Draw the electric field lines to represent the electric field between the plates. For the conductive plates, indicate the region where the electric field is zero.

Answers can be expressed in terms of all or some of the quantities $Q$, $d$, $A$ and $\varepsilon_0$. 
**E851**
A parallel plate capacitor is filled with a material having a dielectric constant \( \varepsilon_r \). The material is not a perfect insulator but has a resistivity \( \rho \). The plate area is \( A \) and the separation distance is \( d \). The capacitor is initially charged with a charge of magnitude \( Q_o \) each plate, that gradually discharges through the dielectric, the charge at any time \( t \) being \( Q \). Show and justify the following:

(a) Conduction current at any instant \( t \)
\[
i_c = \frac{Q}{\rho \varepsilon_r \varepsilon_0}
\]

(b) Displacement current density at any instant \( t \)
\[
i_d = \frac{Q}{\rho \varepsilon_r \varepsilon_0}
\]

(c) The total current is **zero** at any instant \( t \).

**E888**
(a) What is the Claussius-Mossotti equation and define the terms involved?

(b) Explain how this equation can be simplified for a dilute medium.

(c) Given \( \alpha = 2.3355 \times 10^{-40} \text{ C}^2 \cdot \text{m} \cdot \text{N}^{-1} \), for a helium atom, calculate the following

(1) Radius of the helium atom.

(2) Induced dipole moment for helium atoms in an electric field of 10 kV.m\(^{-1}\).

(3) Distance of separation between the centre of the charges.

(4) Dielectric constant for helium at atmospheric pressure and 20 °C.

**E890**
A charge \( Q = 5.0 \mu \text{C} \) is placed on a parallel plate capacitor of area \( L \times L, \ L = 0.50 \text{ m} \) and plate separation \( d = 1.0 \text{ mm} \). The capacitor is filled with a dielectric material, \( \varepsilon_r = 2.3 \). Show and justify the following

(a) surface charge induced on the surface of the dielectric \( Q_b = 2.8 \times 10^{-6} \text{ C} \)

(b) electric field \( E = 9.8 \times 10^5 \text{ V} \cdot \text{m}^{-1} \)

(c) potential difference \( V = 9.8 \times 10^2 \text{ V} \)

(d) capacitance \( C = 5.1 \times 10^{-9} \text{ F} \)

(e) stored energy \( U = 2.5 \times 10^{-3} \text{ J} \)

**E893**
A short cylinder of radius \( a \) and length \( L \) carries a “frozen-in” polarization \( \vec{P} \) parallel to its axis. Find the bound charge \( \sigma_b \) and sketch the electric field for (i) \( L >> a \), (ii) \( L << a \) and (iii) for \( L \approx a \). This devise is known as a **bar electrets** – it’s the electrical equivalent to a bar magnet. Only special materials e.g., barium titanate.
A conducting sphere of radius \( a \) carrying a charge \( q \) is half-submerged into a non-conducting liquid of dielectric constant \( \varepsilon_r \). Find the electric field outside the sphere (in the liquid and in the air) and the charge density on the surface of the sphere.

**Boundary conditions at a discontinuity**

\[
D_{\text{above}} - D_{\text{below}} = P_{\text{above}} - P_{\text{below}} \quad \quad D_{\perp \text{above}} - D_{\perp \text{below}} = \sigma_f
\]

\[
E_{\text{above}} - E_{\text{below}} = 0 \quad \quad E_{\perp \text{above}} - E_{\perp \text{below}} = \frac{\sigma}{\varepsilon_0}
\]

**ISEE approach**

1. Draw a diagram of the sphere showing the air, non-conducting liquid, the radius \( a \) and a Gaussian surface \( S \) of radius \( r > a \). By symmetry, what can you say about the electric field and electric displacement?

2. Apply Gauss’s Law to the surface \( S \)

\[
\oint_S \vec{D} \cdot d\vec{A} = q_{\text{enclosed}}
\]

3. Re-write Gauss’s Law applied to the surfaces \( S_{\text{air}} (D_{\text{air}}) \) and \( S_{\text{liquid}} (D_{\text{liquid}}) \).

4. Hence, show that \( D_{\text{air}} + D_{\text{liquid}} = \frac{q}{2\pi r^2} \)

5. Explain why the following equations are correct

\[
D_{\text{air}} = \varepsilon_0 E_{\text{air}} \quad \quad D_{\text{liquid}} = \varepsilon_r \varepsilon_0 E_{\text{liquid}}
\]

6. At an interface between two dielectric materials, the tangential components of the electric fields must be equal \( E_{\text{t1}} = E_{\text{t2}} \) (\( E_{\parallel} \equiv E_{\parallel} \)). Show this fact on your diagram.

Hence, explain why \( E = E_{\text{air}} = E_{\text{liquid}} \).

7. Show that the electric field is

\[
E = \frac{q}{2\pi \varepsilon_0 (\varepsilon_r + 1) r^2}
\]

8. Show that

\[
D_{\text{air}} = \frac{q}{2\pi (\varepsilon_r + 1) r^2} \quad \quad D_{\text{liquid}} = \frac{\varepsilon_r q}{2\pi (\varepsilon_r + 1) r^2}
\]

9. Give expressions for the charged density on the surface of the sphere for the exposed (air) and submerged (liquid) parts

\[
\sigma_{\text{air}} = ? \quad \quad \sigma_{\text{liquid}} = ?
\]

10. Verify the answers are correct by substituting \( \varepsilon_r = 1 \).

11. What is the electric field inside the sphere?

12. Sketch a diagram showing the distribution of the charge on the conducting sphere.

13. Sketch the electric field and the electric displacement surrounding the charged sphere.

14. Hence, explain why the electric field surrounding the charged conducting sphere is the same within the air and in the liquid.

This approach to studying more completed problems by considering the electric displacement first then the electric field is a most useful one.
E900
Consider a dielectric sphere of radius \( a \) and dielectric constant \( \varepsilon_r \). The sphere contains a total of \( Q_f \) that have been uniformly distributed with density \( \rho_f \). Show and justify the following:

(a) Electric field inside the sphere is given by
\[
\vec{E} = \frac{\rho_f}{3\varepsilon_0 \varepsilon_r} \hat{r}
\]
Electric field outside the sphere is given by
\[
\vec{E} = \frac{\rho_f a^3}{3\varepsilon_0 r^2} \hat{r}
\]
where \( \hat{r} \) is the unit vector pointing in the radial direction.

(b) Potential \( V \) at the centre of the sphere with respect to infinity
\[
V = \left( \frac{\rho_f a^2}{3\varepsilon_0} \right) \left( 1 + \frac{1}{2\varepsilon_r} \right)
\]

(c) Total bound charge at the surface of the dielectric sphere
\[
q_b = Q_f \left( \frac{\varepsilon_r - 1}{\varepsilon_r} \right)
\]

(d) Total bound charge in the interior of the dielectric sphere
\[
q_b = -Q_f \left( \frac{\varepsilon_r - 1}{\varepsilon_r} \right) \quad \text{use} \quad \rho_b = -\nabla \cdot \vec{P} \quad \text{and} \quad \nabla \cdot \vec{r} = 3 \quad \text{(prove)}
\]

(e) Net bound charge of the dielectric sphere \( q_{b\text{net}} = 0 \)

(f) Total electromagnetic energy of the system
\[
U = \left( \frac{2\pi}{45} \right) \left( \frac{\rho_f^2 a^5}{\varepsilon_0 \varepsilon_r} \right) (1 + 5\varepsilon_r)
\]

E906
One conductive plate has a charge \(-Q\) and another plate has a charge \(+Q\). The plates are separated from each other by a distance \( d \) to form an air-filled parallel plate with area \( A \). A thin conductive plate of thickness \( t \) \((t < d)\) is placed between the two plates.

(a) What is the potential difference between the plates?
(b) What is the electric field between the plates of the capacitor?
(c) What is the capacitance \( C \) of the capacitor?
(d) Draw a diagram showing the charge distribution on each conductive plate using 10 minus signs \((-\)\) to represent the charge \(-Q\). Draw the electric field lines to represent to the electric field between the plates. For the conductive plates, indicate the region where the electric field is zero.

Answers can be expressed in terms of all or some of the quantities \( Q, d, A \) and \( \varepsilon_0 \).
<table>
<thead>
<tr>
<th>1 Cars</th>
<th>2 Energy Storage</th>
<th>3 dielectric (micro)</th>
<th>4 dielectrics (macro)</th>
</tr>
</thead>
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<td>039</td>
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