**MAGNETIC PROBLEMS**

**MO14**

You can investigate the behaviour of a toroidal (dough nut shape) electromagnet by changing the core material (magnetic susceptibility $\chi_m$) and the length $d$ of the air gap within the core. The toroid current is $I$ and there are $N$ turns of wire. It has a circular cross section $A$, an average radius $r$ and an average length of $L$. The iron used in the core uses the linear part of the $B$-$H$ curve.

(a) What does this mean – linear part of the $B$-$H$ curve?

(b) Explain why $B = B_{iron} = B_{gap}$.

(c) Show that the $B$ field for the electromagnet is given by the equation

$$B = \frac{\mu_r \mu_0 N I}{(L - d) + \mu_r d}$$

What assumption did you need to make?

(d) Sketch $B$ as a function of $d$ clearly showing the value for maximum value of $B$.

A permanent iron toroid magnet has the same dimensions as the electromagnet ($L = 0.350$ m, $A = 1.65 \times 10^{-4}$ m$^2$, $d = 0.02$ m). For permanent magnets, the relative permeability is a meaningless quantity and the relationship between $B$ and $H$ is given by its hysteresis curve. The magnetization for the permanent magnet is $M = 1.25 \times 10^5$ A.m$^{-1}$.

(e) Draw a diagram of the toroid permanent magnet with an air gap $d$. Indicate on your diagram the N pole, S pole, the directions of $B$, $H$ and $M$ inside the iron and in the air gap. Draw another diagram for the electromagnet and indicate on your diagram the N pole, S pole, the directions of $B$, $H$ and $M$ inside the iron and in the air gap. For a permanent magnet, why should an iron keeper be placed between the poles when the magnet is not being used?

(f) Show that the $B$ field for the permanent magnet is given by the equation

$$B = \frac{\mu_0 M (L - d)}{L}$$

What assumption did you need to make? What does this equation give for the case $L >> d$?
(g) For the permanent magnet, calculate $B$, $H$, $M$, $\phi_m$ in the gap region and in the iron core.

(h) For the electromagnet with $N = 1000$ windings, calculate the current $I$ in the windings for an air core and a soft iron core (magnetic susceptibility iron $\chi_m = 100$) to give the same magnitude of the magnetic field $B$ as for the permanent magnet in part (g). Comment on the result. Why is a soft magnetic core used in an electromagnet?

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**M102**

A silver slab of thickness $t = 1.00$ mm and width $w = 15.0$ mm carries a current of $I = 2.55$ A in a region of magnetic field $B = 1.25$ T which is perpendicular to the slab. The Hall voltage was measured to be $V_H = 0.334$ $\mu$V.

(a) Sketch a diagram of the Hall probe arrangement showing the dimensions of the slab, direction of the current, the magnetic field and the polarity of the Hall voltage.

(b) Derive the expression for the number density $n$ of the charge carriers

$$n = \frac{I B}{t e V_H}$$

(c) Calculate the number density of the charge carriers.

(d) Compare your answer in (c) to the number density of atoms in silver which has a density of $\rho = 10.50 \times 10^3$ kg.m$^{-3}$ and a molar mass of 107.9 g.mol$^{-1}$.

(e) What is the significance of your answers from parts (c) and (d).

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**M169**

A permanent magnet is in the form of circular doughnut with a small gap in it. The magnetic field inside the gap is $B = 1.05$ T. The average radius of the circular doughnut is $r = 0.25$ m and the length of the gap is $a = 18.00$ mm. Calculate the $H$ inside the iron $H_{\text{iron}}$ and in the air gap $H_{\text{air}}$.

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**M174**

A schematic diagram of a copper Hall probe is shown.

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(a) Why is the bottom plate negative?
(b) Derive the expression for the Hall Voltage

\[ V_{\text{Hall}} = \frac{iB}{ne} \]

(c) How can the number density \( n \) of charge carriers be measured using a Hall probe?

(d) Calculate the magnitude of the magnetic field \( B \) for a copper Hall probe of thickness \( t = 125 \, \mu m \) and width \( w = 20.0 \, mm \), current in the strip of \( I = 25.0 \, A \) and Hall voltage across the width of the strip of \( V_{\text{Hall}} = 11.0 \, \mu V \).

Density of copper \( \rho = 8.93 \times 10^3 \, \text{kg.m}^{-3} \) \( N_A = 6.02 \times 10^{23} \, \text{mol}^{-1} \)
Molar mass copper \( M = 63.5 \, \text{g.mol}^{-1} \)

M245
For a bar magnet, make careful sketches of \( M, B \) and \( H \) assuming \( L \) is about \( 2R \).

M269
A 5000 turn toroidal electromagnet is made of iron with a relative permeability of 800. The total path length around the ring is 2.00 m and the width of the gap is 20 mm. The magnetic field in the gap is 0.50 T. Assume there is no bulging of the field in the gap region.

(a) Determine values for \( H \) in the gap and \( B, H, M \) in the iron core.

(b) What value of the current is required to produce the 0.50 T field in the gap?
M282

A hysteresis curve for a sample of iron is shown in the diagram below.

(a) What are the important features of the hysteresis curve?

![Hysteresis Curve for an Iron sample](http://www.whitefang.net/Academics/Physics/I3-Hysteresis/hysteresis.htm)

A Rowland ring is a donut shaped ring or torus of a given ferromagnetic material with two coils around it. The first long coil is used to set up the $H$-field inside the ring by a current $i$. As the current $i$ in this coil changes, an induced emf will be set up in the second coil to give a value for the $B$-field.

Consider a ring that is evenly wound with $N = 200$ turns of wire and has an average circumference, $L = 0.50$ m and cross sectional area, $A = 4.00 \times 10^{-4}$ m$^2$.

It is required that the magnetic flux through the ring to be, $\phi_m = 4 \times 10^{-4}$ T.m$^2$.

(b) What is the required value for the $B$-field?

(c) If the torus has an air core, what current $I$ is required to produce the above magnetic flux $\phi_m$?

(d) If the torus has an iron core whose hysteresis curve is given above (for the calculation, only use the magnetization part of the curve $M \propto H$), what current is needed to give the required magnetic flux $\phi_m$?
(e) What are the permeability $\mu$ and the relative permeability $\mu_r$ of the iron used in your calculation for part (d)?

(f) If an air gap, $a = 1$ mm in length is cut in the ring, what current $I$ is required to maintain the same magnetic flux?

(g) Compare the current through the coil windings for the three cases.

**M295**
As blood is electrically conducting, it is possible to measure the blood velocity in an artery electromagnetically by measuring the voltage produced across the artery when a magnetic field is set up across it.

(a) Draw a diagram showing the artery, the magnetic field, the voltage across the artery and its polarity and the electric field set up in the artery. Show the connections of a voltmeter placed across the artery and the polarity of the meter to give a positive reading.

(b) Carefully explain how the voltage is produced across the artery and how it is related to the magnetic field $B$, the diameter of the artery $d$ and the Hall voltage $V_H$.

(c) An instrument generates a transverse magnetic field of $1.50 \times 10^{-3}$ T. When used on an artery of internal diameter of 5.20 mm, the voltage produced across the artery is 2.52 $\mu$V. Assuming a uniform velocity profile across the artery, calculate the velocity and volume flow rate $\frac{dV}{dt}$ of the blood.

**M301**
Consider a magnetic dipole $\vec{p}_m$ in a uniform magnetic field as indicated in figure (a) below, in which the direction of the $\vec{B}$ field is in the plane of the current loop.

(a) Show that the forces on the magnetic dipole $\vec{p}_m$ lead to a torque that tries to align the magnetic dipole moment $\vec{m}$ with the $\vec{B}$ field.

(b) Now consider the magnetic field $\vec{B}$ to be non-uniform as in figure (b) below, in which the $\vec{B}$ field points in a direction orthogonal to the plane of the current loop. Show that there is a net force that pulls the magnetic dipole $\vec{p}_m$ towards the region of high magnetic field.

(c) Does it matter that the current loop was given a square shape in the figures?
M314
A compass needle of pure iron ($\rho_{Fe} = 7900 \text{ kg.m}^{-3}$) has a length of 30.0 mm, a width of 1.00 mm and a thickness of 0.500 mm. The magnitude of the magnetic dipole moment associated with an iron atom is $p_{mFe} = 2.1 \times 10^{-23} \text{ J.T}^{-1}$

(a) If the magnetization of the needle is equivalent to the alignment of 10% of the atoms in the needle, what is the magnitude of the needle’s dipole moment $p_m$?

(b) With an inexpensive compass needle, we can measure a local magnetic field by disturbing the needle and timing its oscillations. If the compass needle is displaced slightly from its horizontal N-S equilibrium position and the period of oscillation was 2.20 s, what is the horizontal component of the Earth’s magnetic field?

The horizontal component of the Earth’s magnetic field exerts a torque about the needle’s pivot point when the needle is displaced by a small angle $\theta$ to return it to its equilibrium position

$$\tau = -\kappa \theta \quad I = \frac{mL^2}{12} \quad \Rightarrow \quad T = 2\pi \frac{I}{\sqrt{\kappa}}$$

$$\bar{\tau} = \bar{\mu} \times \vec{B} \quad \Rightarrow \quad \tau = -\mu B_h \sin \theta = -\mu B_h \theta$$

$$N_A = 6.02 \times 10^{23}$$

molar mass Fe $M = 55.849 \text{ g}$

M342
A metallic ring of cross-sectional area 2.5 cm$^2$ and mean radius 4.00 m and relative permeability 1500 is wound uniformly with 3000 turns of wire. If a current of 1.6 A passes through the wire, find the mean $B$-field and the magnetization of the ring.

M378
Two long concentric cylindrical conductors of radii $a$ and $b$ ($b > a$) are maintained at a potential difference $V$ and carry equal and opposite currents $i$. Show that an electron with a particular velocity $v$ parallel to the axis may travel un-deviated in the evacuated region between the conductors, and calculate $v$ when $a = 50 \text{ mm}$, $b = 2.0 \text{ mm}$, $i = 100 \text{ A}$ and $V = V_a - V_b = 50 \text{ V}$

It is also possible for the electron to travel in a helical path. By regarding such a path as the combination of a circular motion perpendicular to the axis with a steady velocity parallel to the axis, indicate without detailed mathematics how this comes about.
Consider a long straight and cylindrical wire (radius $a$) that is made a magnetic material with a relative permeability $\mu_r$. The wire carries a current $I$ that is uniformly distributed over its cross-section. Show and justify the following:

$$r \leq R \quad H_{\text{inside}} = \frac{r I}{2 \pi R^2} \quad r \geq R \quad H_{\text{outside}} = \frac{I}{2 \pi r} \quad \text{azimuthal direction}$$

$$r \leq R \quad B_{\text{inside}} = \frac{\mu_r \mu_0 r I}{2 \pi R^2} \quad r \geq R \quad B_{\text{outside}} = \frac{\mu_0 I}{2 \pi r} \quad \text{azimuthal direction}$$

For all $r \quad \phi_n = 0$

(a) Sketch the magnetic field surrounding a long solenoid carrying a DC current.

(b) Find the $H$ and $B$ fields inside a solenoid of length 864 mm, radius 14 mm and 666 turns that carry a current of 4.56 A. Assume that the solenoid is long and that the effects at the ends can be ignored.

(c) A cylindrical piece of iron of radius 2.5 mm and relative permeability 123 was placed along the axis of the solenoid. Calculate the $H$ and $B$ fields inside the iron and in the air gap between the iron core and solenoid windings.
A circular parallel-plate capacitor of radius \( a \) and plate separation \( d \) is connected in series with a resistor \( R \) and a switch, initially open, to a constant voltage source \( V_o \). The switch is closed at time \( t = 0 \). Assume that the charging time of the capacitor, \( \tau = RC \) is very long compared with \( a/c \) and that \( d \ll a \) where \( C \) is the capacitance and \( c \) is the velocity of light.

(a) Show that the expression for the displacement current density, \( j_d \) as a function of time, \( t \) is

\[
j_d = \frac{d\sigma}{dt} = \frac{V_o}{\pi a^2 R} e^{-t/\tau}
\]

(b) Show that the expression for the magnetic flux density \( B \) as a function of time \( t \) and of radial position \( r \) between the capacitor plates is

\[
B = \frac{\mu_o r V_o}{2\pi a^2 R} e^{-t/\tau}
\]

(c) Show that the ratio of stored energy in the electric field to the magnetic field is

\[
\frac{U_{M}}{U_e} = \frac{\mu_o d}{8\pi R^2 C} \ll \frac{\mu_o d C c^2}{8\pi a^2} \approx \frac{1}{8}
\]
The figure shows a figure from the 1934 patent application of Ernest Lawrence for cyclotron. Ions are injected in the centre of the device and are accelerated until they are ejected once they make it to outer regions. The two semicircular halves of the devices are connected to a high-speed radio frequency (rf) voltage generator with constant frequency.

(a) Sketch the path of positive ion (H$_2^+$) traversing the magnetic field. Compare this to the path of an electron travelling through the same magnetic field.

(b) Explain how this device can accelerate charged particles.

(c) Show that the radius of the orbit is given by

$$ R = \frac{m v}{q B} $$

(d) Show that the cyclotron frequency is given by

$$ f_c = \frac{q B}{2 \pi m} $$

(e) Show that the kinetic energy of the ion is given by

$$ K = \frac{1}{2} \frac{q^2 B^2 R^2}{m} $$

(f) Show that the magnetic flux $\phi_m$ enclosed by the orbit with radius $R$ is related to the angular momentum $L$ of the ion by the equation

$$ \phi_m = \frac{\pi L}{q} $$

(g) A modern medical cyclotron has a diameter of $d = 1.00$ m and a magnetic field of $B = 2.00$ T. It is used to accelerate H$_2^+$ ions.

(i) Calculate the frequency of the rf source.

(ii) Calculate the maximum speed of the ions.

(iii) Calculate the maximum kinetic energy of the ions in MeV.

(iv) Are relativistic effects important for this cyclotron?
Consider a very long cylindrical solenoid of radius $R$ with $n$ windings per metre and carrying a current $I$. In the following, we will assume that we stay well away from the ends of the solenoid so that end effects can be ignored. We want to determine the magnetic field inside and outside the solenoid in a region near its centre.

(a) Justify the statement: The magnetic field $B$ can’t depend on the azimuthal angle $\phi$ or the on $z$ (direction along the axis of the solenoid).

(b) Consider a closed Gaussian surface as a cylinder of length $L$ and radius $a$ where $a$ is either smaller or larger than $R$. The axis of the Gaussian surface and the solenoid coincide. Use Gauss’s Law $\oint B \cdot d\vec{A} = 0$ to show that $B_r = 0$ (radial component of the magnetic field is zero).

(c) Use Ampere’s Law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$ and integration path (A) to show that $B_z = 0$ outside the cylinder (outside the solenoid, the axial component of the magnetic field is zero).

(d) Use Ampere’s Law and integration path (B) to show that the azimuthal component of the magnetic field outside the solenoid is $B_\phi = \frac{\mu_0 I}{2\pi r}$.

(e) Use Ampere’s Law and integration path (B) and making the radius of the path smaller than $R$, show that the azimuthal component of the magnetic field inside the solenoid is $B_\phi = 0$.

(f) Use Ampere’s Law and integration path (C) to show that the axial component of the magnetic field inside the solenoid is $B_z = \mu_0 n I$.

M587
Discuss the physical interpretation of magnetic dipole moment $\vec{u}$ or $\vec{m}$, and how this relates to the magnetization $\vec{M}$. Material can be classified as diamagnetic, paramagnetic, and ferromagnetic. Discuss these three types of materials and the relationships between $\vec{B}$, $\vec{H}$, and $\vec{M}$ for each type of material.

Sketch and label $B-H$ curves for each type of material.

Reference
http://www.ndt-ed.org/EDU/Materials/Magnetism/Magnetism.htm
An excellent site to visit.
M670
A Rowland ring made of iron has a mean circumference of 0.500 m and a cross sectional area of $4.00 \times 10^{-4}$ m$^2$. It is found with 450 turns of wire that carry a current of 1.20 A. The relative permeability of iron under these conditions is 550. (a) What is the magnetic flux through the ring? (b) What would be the magnetic flux through the ring if a gap of 20 mm were to be cut in its length, assuming the flux did not spread from the gap.

(Physics Problem Solver 734)

M736
Take the relative permeability of iron to be $\mu / \mu_0 = 1000$, and the atomic magnetic dipole moment to be $2\mu_B$, where $\mu_B$ is the Bohr magneton. The density of iron is 7860 kg.m$^{-3}$, and it has an atomic mass of 56.

(a) Calculate $M$ and make sure it has the dimensions you’d expect.
(b) Find $H$ and $B$.

M780
Two iron bars look alike, but only one is a magnet. How can you determine which is the magnet only by investigating their interactions with each other?

M872
(a) A current $i = 30.0$ A is maintained in a thin, tightly wound coil of $N = 15$ turns, with a radius, $R = 0.200$ m. Using the Biot-Savart Law, show that the magnetic field, $B$ at the centre of the coil is

$$B = \frac{\mu_0 N i}{2 R}$$

and calculate its value and give its direction.
(b) Using Ampere’s Law show that the magnetic field, \( B \) along the axis of a very long solenoid of length 
\( L = 0.25 \text{ m} \) of \( N = 1000 \) turns carrying a current 
\( i = 15 \text{ A} \) is

\[
B = \frac{\mu_0 NI}{L}
\]

and calculate its value and give its direction.

(c) Using Ampere’s Law show that the magnetic field, \( B \) in an air-core toroid of cross 
sectional area \( A = 1.00 \times 10^{-3} \text{ m}^2 \) and a mean circumferential length, 
\( L = 0.500 \text{ m} \) and closely wound with \( N = 100 \) turns of wire carrying a current 
\( i = 15 \text{ A} \) is

\[
B = \frac{\mu_0 NI}{L} = \frac{\mu_0 Ni}{2\pi r}
\]

and calculate its value and give its direction and the 
value of the 
magnetic flux \( \phi_m \).

M919
A toroid with a length \( L = 0.150 \text{ m} \) has inside it a sample of magnetic material of 
susceptibility \( \chi_m = 0.0200 \) and wound 
with 1000 turns of wire carrying a current 
\( i = 2.00 \text{ A} \). Calculate the following:

(a) The relative permeability \( \mu_r \)

(b) The permeability of the magnetic 
material \( \mu_r \).

(c) The magnetic field intensity \( H \) 
produced by the current.

(d) The magnetization \( M \) in the material.

(e). The magnetic field \( B \) resulting from 
the current and the magnetization of the 
material.
**M961**
Assume an electromagnet is very long. The relative permeability of its iron core is 200 and there are 700 turns per meter and the windings carry a current of 3.00 A. Find \( B, H, M \) and \( \chi_m \) in the regions inside the iron core, the gap region and completely outside the electromagnet.

**M978**
In a helium atom, one of its electrons is a d-state. The atom is placed in a strong magnetic field \( B = 2.00 \) T. By how much does the magnetic field change the energy of the electron in its d-state?

**M988**
Under conditions of maximum magnetization, the dipole moment per unit volume of cobalt is \( 1.5 \times 10^5 \) A.m\(^{-1}\). Assuming that this magnetization is due to completely aligned electrons, how many such electrons are there per unit volume? How many aligned electrons are there per atom? The density of cobalt is \( 8.9 \times 10^3 \) kg and the atomic mass is 58.9 g.mol\(^{-1}\).