PHYS2012 EMP10_2

ELECTRICAL ENERGY STORAGE

CAPACITORS

Reference: Young & Freedman Chapter 24 Capacitance & Dielectrics

CAPACITANCE

A system of two conductive plates carrying equal but opposite charges separated by a dielectric is called a capacitor. Dielectrics are insulators – charges tend not to move easily through them. A capacitor is usually charged by transferring electrons to one plate (-Q) and removing them from the other (+Q). In this way, charge and hence energy can be stored by the capacitor. The capacitance C of the system is defined to be

\[ C = \frac{Q}{V} = \frac{q_{\text{free}}}{V} = \frac{q_f}{V} \] [farad = coulomb / volt 1 F = 1 C.V\(^{-1}\)]

where \( Q \) is the smallest magnitude of the charge on either conductive plate and \( \Delta V \equiv V \) is the magnitude of the potential difference between the plates. Capacitance is often referred to as the “ability” to store charge. Free charges are on conductive plates \( Q = q_{\text{free}} = q_f \)

Capacitors are a basic component of most electronic circuits. They have a multitude of uses, including, timing, filtering, smoothing fluctuating voltages, transmission of ac signals, resonance circuits, flash lights in cameras, pulsed lasers, air bag sensors, ac circuits, etc

The simplest type of capacitor is the parallel plate capacitor. The plates may be thin metallic foils that are separated from one another by a thin dielectric. This “sandwich” is then rolled up, which allows for a larger surface area in a relatively small space. Let \( A \) be the area of each plate and \( d \) the separation distance which is small compared with the dimensions of the plates. We place a charge +Q on one plate and -Q on the other. These charges attract each other and become uniformly distributed on the inside surfaces of each plate so that the electric field inside the conductive plates is zero. The electrical properties of the dielectric separating the plates is given by the dielectric constant (relative permittivity) \( \epsilon_r \), or the permittivity of the dielectric

\[ \epsilon = \epsilon_r \epsilon_0 \quad \epsilon_r \geq 1 \quad \text{For a vacuum } \epsilon_r = 1 \quad [\epsilon \quad \text{C}^2\text{N}^{-1}\text{m}^{-2} \text{ or } \text{F.m}^{-1}] \]

where \( \epsilon_0 \) is the permittivity of free space, \( \epsilon_0 = 8.85 \times 10^{-12} \quad \text{C}^2\text{N}^{-1}\text{m}^{-2} \)

If the dielectric completely fills the space between the plates and plates are close together \( (d^2 \ll A) \) then we can take electric field to be uniform and confined to the region between the plates (i.e. we can ignore any edge effects). Using Gauss’s Law (total electric flux through any closed surface is equal to the net charge enclosed within the surface) and taking the Gaussian surface through the +Q plate we have

\[ \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0} = \frac{q_f - q_b}{\epsilon_0} = \frac{q_f}{\epsilon_r \epsilon_0} \]

Where \( q_{\text{bound}} = q_b \) is the bound charge on the surface of the dielectric

\[ q_b = q_f \left( 1 - \frac{1}{\epsilon_r} \right) \Rightarrow q_b < q_f \]
Since the electric field $E$ is uniform between the plates, the potential difference $V$ between the plates is and using Gauss’s Law

$$V = \int_{-d}^{d} E \cdot dl = E \int_{-d}^{d} dl = E d$$

$$E = \frac{q_f}{\varepsilon_r \varepsilon_0 A} = \frac{V}{d} \quad \Rightarrow \quad V = \frac{q_f d}{\varepsilon_r \varepsilon_0 A}$$

Charge on plates $\rightarrow$ electric field between plates

Hence, for a **parallel plate capacitor**

$$C = \frac{q_f}{V} = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{\varepsilon A}{d}$$

The capacitance only depends upon the geometrical arrangement of the conductors and the dielectric and does **not** depend on either $Q$ or $V$.

When a capacitor is connected to a battery, charge is transferred from one conductor to the other until the potential difference equals the potential difference across the battery terminals. The amount of charge transferred is

$$Q = CV$$

**E906**
STORAGE OF ELECTRICAL ENERGY

When a capacitor is being charged, electrons are transferred from the negative plate to the positive plate. Work must therefore be done to charge the capacitor. Some of this work is stored as electrical potential energy.

Let $q$ be the charge transferred at some time interval during the charging process. The potential difference is then $V = q / C$. If a small amount of additional charge $dq$ is now transferred through this potential difference $V$ the potential energy $U$ of the system is increased by

$$dU = V dq = \frac{q}{C} dq$$

Therefore, the total increase in potential energy $U$ as $q$ increases from 0 to the final value $Q$ is

$$U = \int dU = \int_0^Q \frac{q}{C} dq = \frac{1}{2} CV^2 = \frac{1}{2} Q^2$$

$$C = \frac{Q}{V}$$

In an electric car, a bank of capacitors can be charged in the braking process, however, not all the energy can be recovered to charge the capacitors. For example, a battery can be used to charge a battery but only 50% of the energy delivered from the battery can be used to charge the capacitor, the other 50% is dissipated as thermal energy or radiated. This can be seen as follows

Energy supplied by battery ($V$ constant) $U = QV$

Energy transferred to capacitor $U = \frac{1}{2} QV$

In the process of charging the capacitor, an electric field is produced between the plates, and this requires energy. We can then think of the energy stored in the capacitor as energy stored in the electric field

$$U = \frac{1}{2} C V^2 = \frac{1}{2} \left( \epsilon_r \epsilon_0 A \right) \left( \frac{E}{d} \right)^2 = \frac{1}{2} \epsilon_r \epsilon_0 E^2 (Ad)$$

The quantity $Ad$ is the volume of the space between the plates of the capacitor containing the electric field. The energy/volume is called the energy density $u$

$$u = \frac{1}{2} \epsilon_r \epsilon_0 E^2 \quad [u \text{ J.m}^{-3}]$$

This is a general result and is not just true for parallel plate capacitors.

The operation of assembling upon a conductor a group of charges that mutually repel one another requires work and therefore results in the production of potential energy - this potential energy is possessed by the charged conductor itself but it may be more correct to picture the energy stored in the field surrounding the conductor.

For the energy storage in our electric car, we want to maximize the energy stored by the capacitors. How do we do this?

Energy Stored by capacitor $U = \frac{1}{2} CV^2$

Parallel plate capacitor $C = \frac{\epsilon_r \epsilon_0 A}{d}$
We will assume that the potential difference $V$ between the plates is fixed. Therefore, the larger the capacitance, the greater the energy stored. The larger the area $A$ of the plates the larger the capacitance and hence energy stored. We will study ultracapacitors later in which large surface areas are created to give enormous capacitance values in the order of thousands of farads.

The greater the dielectric constant $\varepsilon_r$, the larger the capacitance and energy stored. We will study the electrical properties of insulating materials to account for the dielectric constant of different materials.

The smaller the distance between the plates greater the capacitance value. Is there a limit to how thin the dielectric can be? The electric field between the plates is given by $E = \frac{V}{d}$. As $d$ decreases, $E$ increases. If the electric field exceeds a value known as the dielectric strength, electrons are stripped from the molecules of the dielectric and it no longer behaves as an insulator. This is called electrical breakdown of the dielectric. Therefore, we can’t have an arbitrary small thickness for the dielectric.

Some values for the dielectric constant and dielectric strength are

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric constant $\varepsilon_r$</th>
<th>Dielectric strength V.m$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1.00000</td>
<td></td>
</tr>
<tr>
<td>Air</td>
<td>1.00059</td>
<td>$\sim 3\times10^6$</td>
</tr>
<tr>
<td>paper</td>
<td>3.7</td>
<td>$16\times10^6$</td>
</tr>
<tr>
<td>polystyrene</td>
<td>2.5</td>
<td>$24\times10^6$</td>
</tr>
<tr>
<td>Barium titanate</td>
<td>500 - 6000</td>
<td>$\sim 2\times10^9$</td>
</tr>
<tr>
<td>Titanium dioxide ceramics</td>
<td>15 - 500</td>
<td>$\sim 2\times10^7$</td>
</tr>
</tbody>
</table>
We can increase the capacitance for electrical energy storage by having many capacitors connected in parallel to one another. Generally, capacitors are connected in either series or in parallel.

Capacitors in **series** (charge on each plate is the same)

\[
C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \ldots}
\]

same \( Q \), voltages add

Capacitors in **parallel** (voltage across each capacitor is the same)

\[
C_{eq} = C_1 + C_2 + \ldots
\]

same voltage, charges add

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E127  E551  E815
CAPACITORS AS AN ENERGY STORAGE DEVICES FOR ELECTRIC CARS

For the 250 km journey in an electric car, the energy needed as useful work in moving the car was $2.59 \times 10^8$ J and the rate of energy conversion was 18 kW. **How big a capacitor would be required?**

The calculations can be done easily using Matlab. Below are the Matlab scripts and outputs for the modelling of an air and a ceramic parallel plate capacitor.

%% Parallel Plate Capacitor - air dielectric
close all; clear all; clc;
% Data     SI units unless stated otherwise
eps0 = 8.85e-12;  % permittivity of free space
V = 12;           % battery voltage
E_ds = 3e6;       % dielectric strength
U = 2.59e8;       % energy stored by capacitor

% Calculations
C = 2 * U / V^2;  % capacitance
d = V / E_ds;     % min thickness of dielectric
A = d * C / eps0; % area of capacitor plates
L = sqrt(A);      % length of square plate

% Print answers
fprintf('capacitance, C = %6.2e  F 
',C)
fprintf('min thickness, d = %6.2e  m 
',d)
fprintf('plate area, A = %6.2e  m^2 
',A)
fprintf('length square plate, L = %6.2e  km 
',L/1000)

%%

**capacitance, C = 3.60e+006  F**

**min thickness, d = 4.00e-006  m**

**plate area, A = 1.63e+012  m^2**

**length square plate, L = 1.28e+003  km**

It is impossible to use a bank of air filled capacitors to provide the necessary energy for the car, the total capacitance value and plate area are too enormous. ($C \sim 10^6$ F & $A \sim 1000$ km $\times$ 1000 km).

%% Parallel Plate Capacitor - ceramic dielectric
close all; clear all; clc;
% Data     SI units unless stated otherwise
eps0 = 8.85e-12;  % permittivity of free space
epsR = 1000;      % relative permittivity
V = 12;           % voltage across capacitor
E_ds = 3e7;       % dielectric strength
U = 2.59e8;       % energy stored by capacitor

% Calculations
C = 2 * U / V^2;  % capacitance
\[ d = \frac{V}{E_{ds}}; \quad \text{min thickness of dielectric} \]
\[ A = d \times C / (\varepsilon R \varepsilon_0); \quad \text{area of capacitor plates} \]
\[ L = \sqrt{A}; \quad \text{length of square plate} \]

% Print answers
fprintf('capacitance, C = %6.2e  F
n',C)
fprintf('min thickness, d = %6.2e  m
n',d)
fprintf('plate area, A = %6.2e  m^2
n',A)
fprintf('length square plate, L = %6.2e  km
n',L/1000)

\begin{align*}
\text{capacitance, } C &= 3.60e+006  \text{ F} \\
\text{min thickness, } d &= 4.00e-007  \text{ m} \\
\text{plate area, } A &= 1.63e+008  \text{ m}^2 \\
\text{length square plate, } L &= 1.28e+001  \text{ km}
\end{align*}

Again, it is impossible to use a bank of ceramic capacitor which have large values for their dielectric constant and breakdown voltage to provide the necessary energy for the car, the capacitance value and plate area are too enormous. \( C \sim 10^6 \text{ F} \) & \( A \sim 10 \text{ km} \times 10 \text{ km} \).

\textbf{E361}

\textbf{ULTRACAPACITORS}

Ultracapacitors, also known as supercapacitors, electric double layer capacitors or electrochemical double layer capacitors (EDLCs) are electrochemical capacitors that have unusually high energy densities when compared common capacitors, typically on the order of thousands of times greater than a high capacity electrolytic capacitor. For instance, a typical D-battery sized electrolytic capacitor will have a capacitance in the range of tens of millifarads. The same size electric ultracapacitor capacitor would have a capacitance of several farads, an improvement of about two or three orders of magnitude in capacitance, but usually at a lower working voltage. Typical double layer construction consists of two carbon electrodes immersed in an organic electrolyte (in essence we have a charge separation of a few atomic layers). During the charging process the electrically charged ions in the electrolyte migrate towards the electrodes of opposite polarity due to the electric field created by the applied voltage creating the two separated charged layers. Since no chemical reactions are involved as in a battery, the effect is easily reversed. However, ultracapacitor capacitors have a low working voltage of only a few volts to avoid electrolysis of the electrolyte with the consequence of gas emission.

An ultracapacitor can be viewed as two non-reactive porous plates suspended within an electrolyte, with a voltage applied across the plates. The applied potential on the positive plate attracts the negative ions, while the potential on the negative plate attracts the positive ions. This effectively creates two layers of capacitive storage, one where charges are separated at the positive plate and another at the negative plate.
Using an ultracapacitor for energy storage for our 250 km journey

%% (3) Ultra capacitors

close all; clear all; clc;

% Data  SI units unless stated otherwise

C = 5000;  % capacitance
V = 3.0;  % voltage across capacitor
u_cap = 30;  % energy density of capacitor [W.h/kg]
W_car = 2.59e8;  % work need to be done on car for 250 km journey
e = 0.80;  % efficiency of electric motor
U_car = W_car / e;  % energy required by electric car for 250 km journey

% Calculations
U_cap = 0.5 * C * V^2;  % energy stored by the ultra capacitor
N_cap = U_car / U_cap;  % number of capacitors required to supply the energy for 250 km journey
u_cap = 30*3600;  % energy density of capacitor [J/kg]
M_cap = U_car / u_cap;  % total mass of ultra capacitors

% Print answers
fprintf('Capacitor - energy stored, U_cap = %6.2e  J\n',U_cap);
fprintf('No. of capacitors, N_cap = %6.2e  \n',N_cap);
fprintf('Energy density - capacitor, u_cap = %6.2e  J/kg\n',u_cap);
fprintf('Total mass of ultra capacitors, M_cap = %6.2e  kg\n',M_cap);

Capacitor - energy stored, U_cap = 2.25e+004  J
No. of capacitors, N_cap = 1.44e+004
Energy density - capacitor, u_cap = 1.08e+005  J/kg
Total mass of ultra capacitors, M_cap = 3.00e+003  kg

It is certainly not feasible to even think about using traditional types of capacitors for the enormous amount of energy required for a car for a journey of only 250 km. However, the results for ultracapacitors are much more promising and the mass of ultra capacitors is similar to that of lead acid batteries. Batteries require long charging times (~ hours) whereas ultra capacitors can be charged rapidly (~ minutes). Batteries have a limited number of discharge / charge cycles but ultracapacitors, the number of cycles is thousands of times greater before they have to be replaced. Remember an electric car connected to a normal power point takes about 20 h for the recharging. Capacitors store energy in an electrostatic field rather than as a chemical state in batteries. Since no chemical actions are involved with capacitors means very long cycle life is possible.

For long trips it is still not feasible to power electric motors by ultracapacitors but they may be suitable for short journeys or replace batteries in hybrid systems.

Since normal capacitors store charge only on the surface of the electrode they have lower energy storage capacity and lower energy densities compared with batteries. The charge / discharge reaction is not limited by ionic conduction into the electrode bulk, so capacitors can be run at high rates and provide very high specific powers but only for short periods of time.
Electrostatic capacitor

\[ C_{\text{max}} \approx 0.01 \text{ F} \quad u_{\text{max}} \approx 0.01 \text{ W.h.kg}^{-1} \]

Electrolytic capacitor

\[ C_{\text{max}} \approx 0.1 \text{ F} \quad u_{\text{max}} \approx 0.1 \text{ W.h.kg}^{-1} \]

Electrochemical double layer capacitor

\[ C_{\text{max}} \approx 5000 \text{ F} \quad u_{\text{max}} \approx 30 \text{ W.h.kg}^{-1} \]

For a normal capacitor, energy is stored by the removal of electrons, from one metal plate and depositing them on another. This charge separation creates a potential between the two conductive plates. The total energy stored is proportional to both the amount of charge stored and the potential between the plates. The amount of charge stored is essentially a function of size and the material properties of the plates and the potential between the plates is limited by dielectric breakdown of the substance separating the plates.

\[
C = \varepsilon_r \varepsilon_0 \frac{A}{d} \quad Q = CV \quad U = \frac{1}{2} QV \quad E = \frac{V}{d}
\]

Ultracapacitors do not have a normal dielectric. Rather than two separate plates separated by an intervening substance, these capacitors use "plates" that are in fact two layers of the same substrate, and their electrical properties, the so-called "electrical double layer", result in the effective separation of charge despite the vanishingly thin (on the order of nanometers) physical separation of the layers. The lack of need for a bulky layer of dielectric permits the packing of "plates" with much larger surface area into a given size, resulting in extraordinarily high capacitances in practical-sized packages. Each layer by itself is quite conductive, but the physics at the interface where the layers are effectively in contact means that no significant current can flow between the layers.

In general, ultracapacitors improve storage density through the use of a nanoporous material, typically activated charcoal, in place of the conventional insulating barrier. Activated charcoal is a powder made up of extremely small and very "rough" particles, which, in bulk, form a low-density volume of particles with holes between them that resembles a sponge. The overall surface area of even a thin layer of such a material is many times greater than a traditional material like aluminum, allowing many more charge carriers (ions or radicals from the electrolyte) to be stored in any given volume. The charcoal, which is not a good insulator, is taking the place of the excellent insulators used in conventional devices, so in general EDLCs can only use low potentials on the order of 2 to 3 V. Activated charcoal is not the "perfect" material for this application. The charge carriers are actually quite large – especially when surrounded by solvent molecules – and are often larger than the holes left in the charcoal, which are too small to accept them, limiting the storage. Most recent research in ultracapacitors has focused on improved materials that offer even higher usable surface areas. Experimental devices developed at MIT replace the charcoal with carbon nanotubes, which can store about the same charge as charcoal (which is almost pure carbon) but are mechanically arranged in a much more regular pattern that exposes a much greater suitable surface area.

Ultracapacitors are also being made of carbon aerogel (very low mass density). This is a unique material providing extremely high surface area of about 400-1000 m²/g. The electrodes of aerogel ultracapacitors are usually made of non-woven paper made from carbon fibers and coated with organic aerogel, which then undergoes pyrolysis. The paper is a composite material where the carbon fibers provide structural integrity and the aerogel provides the required large surface. Small aerogel ultracapacitors are being used as backup electricity storage in microelectronics, but applications for electric vehicles are expected. Aerogel capacitors can only work at a few volts; higher voltages would ionize the carbon and damage the capacitor. Carbon aerogel capacitors have achieved energy densities of 90 W.h.kg⁻¹ and power densities 20 W.g⁻¹.

Ultracapacitors: (1) high self-discharge rates, much higher than batteries, but the capacitors can be charged-discharged millions of times compared with ~ 1000 cycles for rechargeable
batteries. (2) extremely low internal resistance and consequent high cycle efficiency. (3) contain no corrosive electrolyte as do batteries and are made from low toxicity materials.

<table>
<thead>
<tr>
<th>Approximate values only</th>
<th>max energy density (W.h.kg⁻¹)</th>
<th>max power density (W.kg⁻¹)</th>
<th>charge / discharge times (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Petrol*</td>
<td>12000 (2400)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lead acid battery</td>
<td>40</td>
<td>50</td>
<td>&gt; 1000</td>
</tr>
<tr>
<td>Lithium ion battery</td>
<td>160</td>
<td>100</td>
<td>&gt; 1000</td>
</tr>
<tr>
<td>Normal capacitor</td>
<td></td>
<td>5000</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>Ultra capacitor</td>
<td>30</td>
<td>5000</td>
<td>&lt; 1</td>
</tr>
</tbody>
</table>

* Petrol engines operate at about 20% tank-to-wheel efficiency reducing the effective energy density.

+ Power density: rate at which energy can be delivered to a load. Batteries - movement of charge carriers in a liquid electrolyte, have relatively slow charge and discharge times. Capacitors - can be charged or discharged at a rate that is typically limited by current heating of the electrodes. So while existing ultracapacitors have energy densities that are perhaps 1/10th that of a conventional battery, their power density is generally 10 to 100 times as great.

China is experimenting with a new form of electric bus, known as Capabus, which runs without continuous overhead lines (is an autonomous vehicle) by using power stored in large onboard ultracapacitors which are quickly recharged whenever the vehicle stops at any bus stop (under so-called electric umbrellas), and fully charged in the terminus. A few prototypes were being tested in Shanghai in early 2005. In 2006 two commercial bus routes began to use ultracapacitor buses; one of them is route 11 in Shanghai. Some newer buses with ultracapacitors can supply 10 W.h.kg⁻¹.

The buses have very predictable routes and need to stop regularly every 4.8 km or less, allowing quick recharging at charging stations at bus stops. A collector on the top of the bus rises a few feet and touches an overhead charging line at the stop; within a couple of minutes the ultracapacitor banks stored under the bus seats are charged. The buses can also capture energy from braking, and the company says that recharging stations can be equipped with solar panels. A third generation of the product, which will give ~30 km of range per charge.

Estimates claim that the buses have one-tenth the energy cost of equivalent diesel buses and can achieve lifetime fuel savings of $200,000 per bus. The buses use 40% less electricity even than an electric trolley bus, mainly because they are lighter and have the regenerative braking benefits. The ultracapacitors are made of activated carbon and have an energy density of 6 W.h.kg⁻¹; for comparison, a high-performance lithium-ion battery can achieve 200 W.h.kg⁻¹ but the ultracapacitor bus is about 40% cheaper than a lithium-ion battery bus and far more reliable.

The shortcomings of ultracapacitors make them unsuitable as a primary energy source for cars, however, they are ideal for temporary energy storage for capturing and storing the energy from regenerative braking and to provide a booster in charge response to sudden power demands and thus the primary battery can be downsized. An array of ultracapacitors in series coupled to a load in parallel with a storage battery creates a hybrid energy source with higher energy and power densities than either device in a stand-alone configuration. A battery is quickly degraded when large currents are drawn from it for any length of time. The storage battery used in conjunction with a bank of ultracapacitors prevents these large current spikes resulting in longer battery life.