One common paradox students find perplexing in learning about electric current is the apparent contradiction between the tiny drift speed of free electrons in a conductor, say about 1 m/h, and the response of a current “in no time” when the circuit is switched on or off. These phenomena can be understood in terms of the speed of the electrical signal, which travels at or near the speed of light. As soon as the circuit is closed, apart from inductive delay, an electric field is set up almost simultaneously throughout the circuit. It is the electric field that causes electrons to start drifting at all points in the circuit. This paper describes an experiment for measuring the speed of an electromagnetic signal in a coaxial cable.

An experiment to estimate the speed of an electrical pulse in a cable has been explained by T. Duncan. In this paper we describe a setup using more updated measuring instruments. The setup and procedure of our method are given in some detail but the theoretical framework is kept to a minimum. The validity of our method is based on phenomenological reasoning and self-consistence.

Experimental Setup

Our sample is a 15-m long coaxial cable with characteristic resistance $75 \, \Omega$. A function generator with a frequency range of 0 to 10 MHz is used to provide the signal, and a student-grade oscilloscope with a sweep frequency of 20 MHz is used for time measurement. A 10-to-200-Ω noninductive rheostat is connected across the output end of the cable (Fig. 1). The rheostat provides a damping to the outgoing signal and prevents the formation of a standing wave along the cable. In general, the length of the cable required is inversely proportional to the fastest sweeping frequency of the oscilloscope. For a 20-MHz oscilloscope, a length of 15 to 30 m, or the length of a typical physics laboratory, is desirable.

Method

A square wave at 2 V ($\approx 600 \, \text{kHz}$) is applied to Channel 1 and the input end of the cable. The output signal is applied to Channel 2. The rheostat is adjust-
ed until the input trace and the output trace look the same but differ only in phase (Fig. 2). For the sake of easy comparison of waveform and amplitude between the traces, the same gain and zero-level should be used in both channels. The time taken by an EM wave to travel along a single cored coaxial cable is the difference in zero crossings, $\Delta T$, between the traces (Fig. 3).

Theory

The voltage along a coaxial line assumes a general form:

$$v(x,t) = V^+(x-ct) + V^-(x+ct),$$

where $V^+$ represents a wave going in the $+x$ direction (away from the source) and $V^-$ a wave in the $-x$ direction (toward the source, reflected from the far end). Empirically, the functional values of $v(0,t)$ and $v(l,t)$ are represented respectively by the input and output trace displayed on an oscilloscope screen. If the transmission line were infinitely long, there would be no reflected wave and

$$v(x,t) = V^+(x-ct).$$

Although an infinitely long transmission line does not really exist, it can be replaced by connecting a resistive load, $r_0$ (the characteristic resistance of the line), across the far end of a cable of finite length.

Most coaxial cables available in the market have $r_0$ in the range 50 to 200 $\Omega$. By changing the resistance of the rheostat and observing the corresponding changes in waveforms and separation of the traces $v(0,t)$ and $v(l,t)$, one can show that the reflected wave indeed disappears when $r = r_0$. Since the difference in zero crossings can be found without knowing the value of $r_0$, the depth of treatment on characteristic resistance can be adjusted or even skipped at teachers’ own discretion in this measurement exercise. As long as the waveform and amplitude of the traces are identical irrespective of frequency changes, both traces must be represented only by $V^+$ with the same argument ($x-ct =$ constant).

Why Square Wave?

For three reasons, a square wave produces better results than a sine wave or a pulse. First, variations and
distortions in the shape of waveforms are more easily detected. Second, the difference in zero crossings can be measured with higher accuracy using the “vertical part” of these traces. Third, a fairly wide frequency range, say from 50 kHz to 6 MHz, can be used. Below 50 kHz, i.e., when the sweeping frequency of the oscilloscope is much faster than the signal frequency, the traces may become too dim for clear observation.

Experimental Precautions

A. Choice of frequency
In theory, the result of our experiment should be independent of frequency, so the best frequency is the highest frequency with the least distortion in the input square wave. The “highest frequency” is used because it gives rise to the brightest trace.

B. Oscilloscope probes
Since oscilloscope probes are by themselves coaxial cables, identical probes should be used in both channels so that the time lag generated by these leads cancel one another.

C. Oscilloscope adjustment
In the experiment described here, a student-grade oscilloscope was used. With such an instrument, one should increase the time base sensitivity stepwise in order to continually keep the traces on screen. The horizontal magnification ×10 is applied in the last step.

D. Rheostat
If a noninductive rheostat with range 0 to 100 Ω is not available, a more common 500 Ω rotary carbon film B-type (with r ∝ θ) rheostat can be used as a substitute. In the latter case, less than 15% of its full range is used. Adjustment requires practice and some patience.

Result and Uncertainty Analysis

With a suitably long coaxial cable, the uncertainty due to length measurement is less than 1%. Using an oscilloscope with maximum time base sensitivity 0.2 μs cm⁻¹ and ×10 horizontal magnification, the random error generated in time measurement is about 5% [from Fig. 3, trace width/trace separation = (0.2 cm/3.2 cm) × 100% ≈ 6%]. Errors also arise from

(a) distortion produced by the CRO,
(b) nonidentical probes, and
(c) defects in a real and finite transmission line, such as flux leakage at both ends.

These errors render the traces v(0,t) and v(l,t) not exactly identical. By comparing our results here with those obtained from a research-grade oscilloscope, we found that the distortion in the oscilloscope may introduce a percentage error of about 5% in the result. Incorrect judgment of similarity of the input and output trace will introduce an error of approxi-
mately the same magnitude. Results of our experiment are summarized in Table I.

Acknowledgment
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References
3. Radio Frequency Function Generator (Topward Model: 8140, Taiwan) [frequency range 0 to 10 MHz].
4. E.g. Kenwood 20-MHz Oscilloscope Model: CS-4125 (Japan) [max. time base sensitivity = 20 ns cm⁻¹ when “×10 horizontal magnification” is used].
5. A pair of IWATSU Probes SS-082R is used in our experiment.
6. E.g. IWATSU 250-MHz Oscilloscope Model: SS-7825 (Japan) [max. time base sensitivity = 1 ns cm⁻¹ when “×10 horizontal magnification” is used].

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