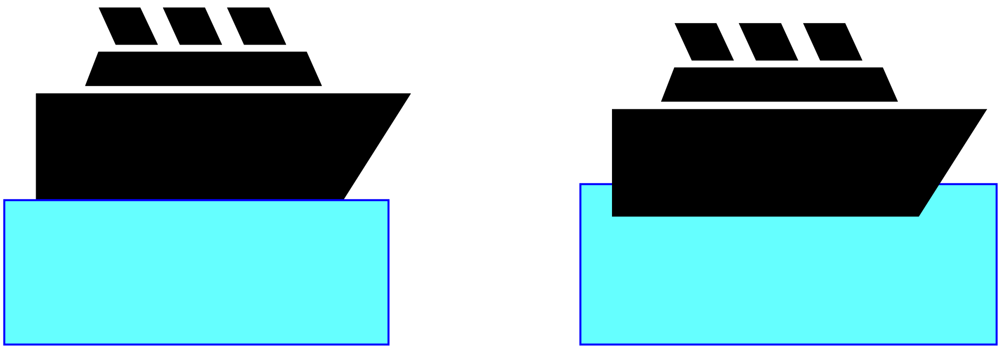
VISUAL PHYSICS
School of Physics
University of Sydney *Australia*

BUOYANCY

FLOATING AND SINKING

? Why do ice cubes float on water?
Why does a hot air balloon rise?

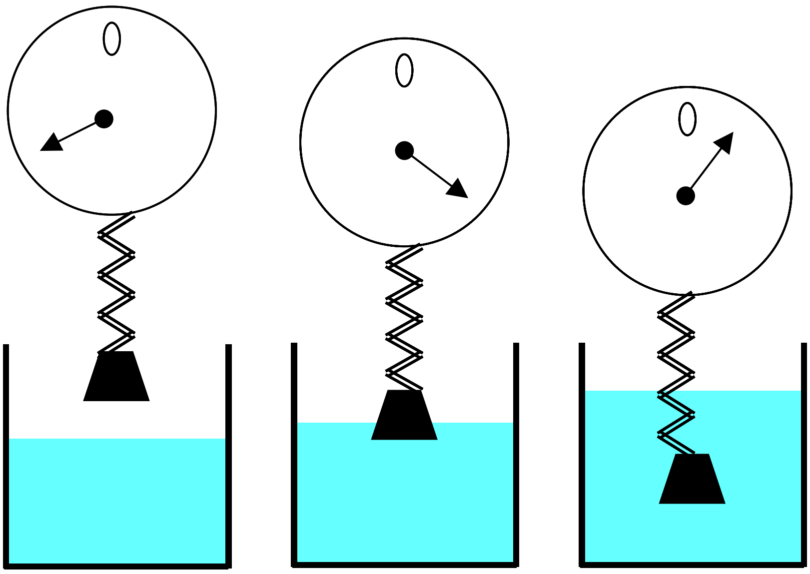
? What is wrong with the picture of the ship on the left?



→ **Archimedes' Principle**

! When an object is completely or partially immersed in a fluid, the fluid exerts an upward force on the object equal to the weight of the fluid displaced by the object.

When a solid object is wholly or partly immersed in a fluid, the fluid molecules are continually striking the submerged surface of the object. The forces due to these impacts can be combined into a single force the **buoyant force**. The immersed object will be "lighter" i.e. It will be buoyed up by an amount equal to the weight of the fluid it displaces.



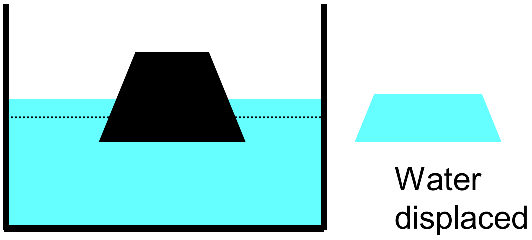
Partially submerged   floating

Floating: partially submerged

Weight of object < weight of fluid that can be displaced by object

Volume of displaced water < volume of object

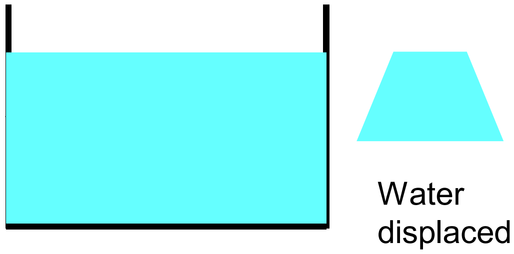
Weight of liquid displaced by partially submerged object = weight of object



Floating: fully submerged

Weight of object = weight of fluid displaced by object

Volume of displaced water = volume of object



Static equilibrium

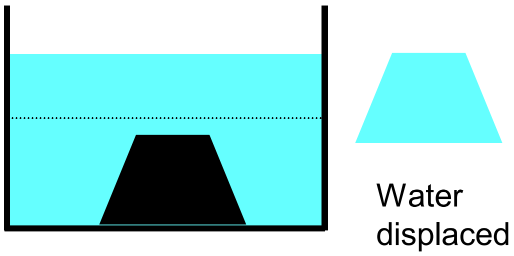
Some fish can remain at a fixed depth without moving by storing gas in their bladder.

Submarines take on or discharge water into their ballast tanks to rise or dive

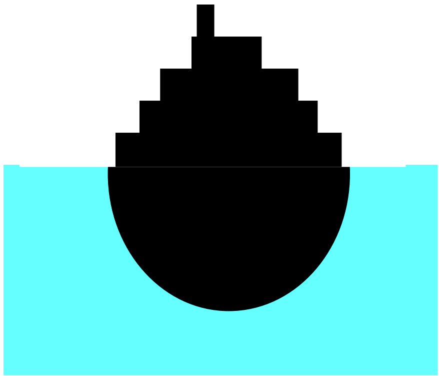
Sinks

Weight of object > weight of fluid displaced by object

Volume of displaced water = volume of object



A steel ship can encompass a great deal of empty space and so have a large volume and a relatively small density.

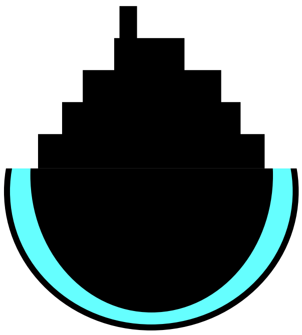


Volume of water displaced

Weight of ship = weight of water displaced

? A 200 tonne ship enters a lock of a canal. The fit between the sides of the lock is so tight that the weight of the water left in the lock after it closes is much less than the ship's weight. Can the ship float?

The buoyant force is equal to the weight of the water displaced, not the water actually present. The *missing water* that would have filled the volume of the ship below the waterline is the displaced fluid.

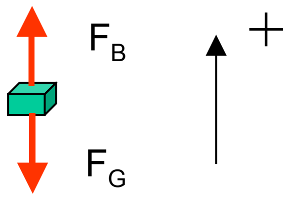


Volume of water displaced. This volume is not necessarily the volume present.

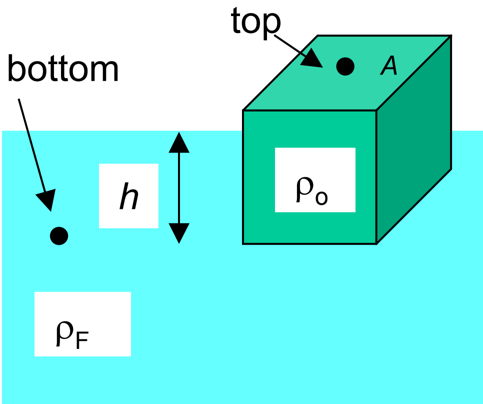
Weight of ship = weight of water displaced

? Why does an object float?

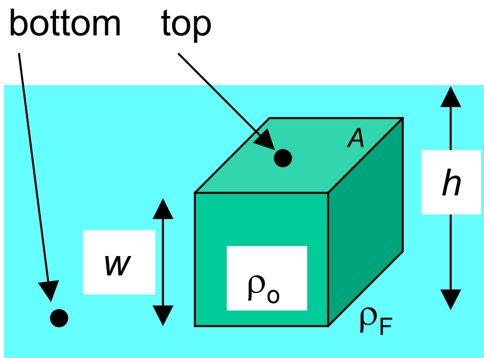
FLOATING: weight of object = buoyant force



Object partially submerged



Object fully submerged



An object floats because of the pressure difference between the top and bottom of the object.

For object partially submerged

$V_{\text{fluid displaced}} = V_F$

m_F is the mass of fluid displaced

$\Delta p = p_{\text{bottom}} - p_{\text{top}} = \rho_F\, g\, h + p_{\text{atm}} - p_{\text{atm}} = \rho_F\, g\, h$

$F_B = \Delta p\, A = \rho_F\, g\, h\, A$

$V_{\text{submerged}} = h\, A = V_F$

$F_B = \rho_F\, g\, V_F = m_F\, g =$ weight of fluid displaced

For object fully submerged

$V_{\text{fluid displaced}} = V_F$

$\Delta p = p_{\text{bottom}} - p_{\text{top}} = + p_{\text{atm}} + \rho_F\, g\, h - \{p_{\text{atm}} + \rho_F\, g(h - w)\}$
$= \rho_F\, g\, w$

$F_B = \Delta p\, A = \rho_F\, g\, w\, A$

$V_{submerged} = V_{object} = w\, A = V_F$

$F_B = \rho_F\, g\, V_F = m_F\, g$ $\Rightarrow$ weight of fluid displaced

For static equilibrium

$F_B = F_G \Rightarrow$

Archimedes's principle weight of object (F_B)
= weight of fluid displaced (F_G)

Hence, mass of object (m_O) = mass of fluid displaced (m_F)

Floating and sinking
Consider an object fully submerged ($V_F = V_o = V$)

! $\boldsymbol{F_B - F_G = \rho_F\, g\, V_F - \rho_o\, g\, V_o = (\rho_F - \rho_o) g\, V}$

Net force acting on object $\Rightarrow$

If $\rho_F > \rho_o \Rightarrow F_B - F_G > 0 \Rightarrow$ object will accelerate upward and rise to the surface and then **float** on the surface partially submerged.

If $\rho_F = \rho_o \Rightarrow F_B - F_G = 0 \Rightarrow$ static equilibrium $\Rightarrow$ object will **float** fully submerged

If $\rho_F < \rho_o \Rightarrow F_B - F_G < 0 \Rightarrow$ object will accelerate downward and **sink** to the bottom.

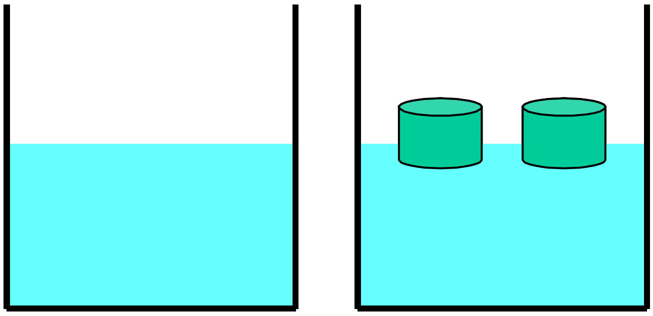
?

A

Two cups are filled to the same level. One cup has ice cubes floating on it. Which weight more?

B

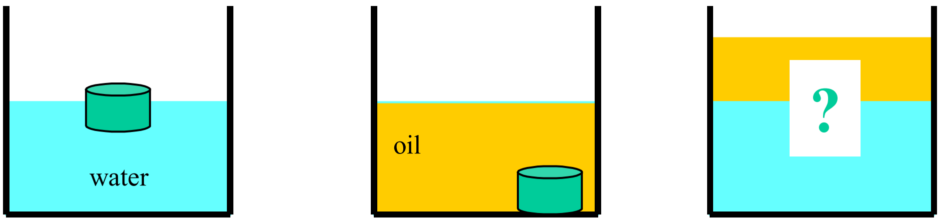
Two cups are filled to the same level. One of the cups has ice cubes floating in it. When the ice melts, in which cup is the level higher?



Answer A   The cups weight the same. Assume static equilibrium, then weight of the ice cubes is equal to the buoyant force. The buoyant force is equal to the weight of the water displaced by the ice cubes. This means that the weight that the ice cubes add to the cup is exactly what an amount of water that is equal to that submerged volume of ice cubes would add.

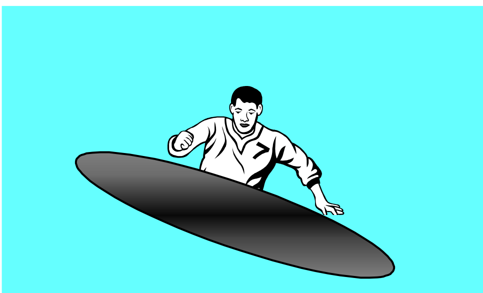
Answer B   The level is the same. The weight of the ice cubes is equal to the weight of the water that would fill the submerged volume of the cubes. When the cubes melt into the water the volume of melted water is exactly equal to the volume of water that the cubes were displacing.

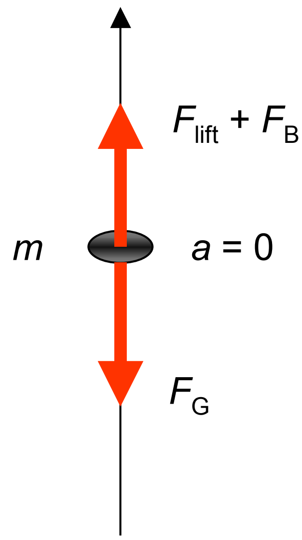
? Consider an object that floats in water but sinks in oil. When the object floats in water half of it is submerged, then oil is slowly poured on the top of the water so it completely covers the object. Will the object move up? stay in the same place? or move down?



When the oil is poured over the object it displaces some oil. This means it feels a buoyant force from the oil in addition to the buoyant force from the water. Therefore it rises higher.

? A giant clam has a mass of 470 kg and a volume of 0.350 m^3 lies at the bottom of a freshwater lake. How much force is needed to lift it at constant velocity?





$$F_{lift} + F_B = F_G$$

m = 470 kg

V_{clam} = 0.350 m^3

g = 9.8 $m.s^{-2}$ $\rho_{water} = 10^3$ $kg.m^{-3}$

$V_{\text{displaced}} = V_{\text{clam}}$

$\Sigma F = 0 \;\Rightarrow\; F_{\text{lift}} + F_B = F_G$

$F_{\text{lift}} = ?$ N

$F_B = \rho_{\text{water}}\, g\, V_{\text{displaced}} = \rho_{\text{water}}\, g\, V_{\text{clam}}$

$F_G = m\, g$

$F_{\text{lift}} = F_G - F_B = m\, g - \rho_{\text{water}}\, g\, V_{\text{clam}}$

$F_{\text{lift}} = (470)(9.8) - (10^3)(9.8)(0.350)$ N $= 1.18\times10^3$ N

? A ring weighs 6.327×10^{-3} N in air and 6.033×10^{-3} N when submerged in water.

(a) What is the volume of the ring?
(b) What is the density of the ring?
(c) What is the ring made of?

weight of ring in air = $F_{\text{GRair}} = 6.327\times10^{-3}$ N

weight of ring in water $F_{\text{GRwater}} = 6.033\times10^{-3}$ N

buoyant force = F_B = ? N

weight of water displaced = F_{GF} = ? N

volume of water displaced = V_F = ? m^3

mass of water displaced = m_F = ? kg

density of water = $\rho_F = 10^3$ kg.m^{-3}

volume of ring = V_R = ? N

density of ring = ρ_R = ? kg.m^{-3}

Decrease in weight due to buoyant force

$F_B = F_{\text{GRair}} - F_{\text{GRwater}} = 0.294\times10^{-3}$ N

Archimedes' Principle: buoyant force equals weight of water displaced.

$F_{GF} = F_B = 0.294\times10^{-3}$ N $= m_F\, g = \rho_F\, V_F\, g$

Since ring fully submerged

$V_R = V_F = F_B / \rho_F\, g = (0.294\times10^{-3}) / \{(10^3)(9.8)\}$ m^3

$V_R = 3.00\times10^{-8}$ m^3

$\rho_R = m_R / V_R = F_{GRair} / (g\, V_R)$

$\rho_R = (6.327\times10^{-3}) / \{(9.8)(3.00\times10^{-8})\}$ kg.m^{-3}

$= 21.5\times10^3$ kg.m^{-3}

Maybe gold

? A wooden raft has a density of 0.500×10^3 kg.m^{-3} and dimensions of 3.05 m $\times$ 6.10 m $\times$ 0.305 m. How deep does it sink into the water when unloaded? What is the maximum number of 70 kg people can the raft carry before it sinks?

density of water = ρ_F = 10^3 kg.m^{-3}

volume of water displaced = V_F = ? m^3

density of raft = $\rho_R = 0.500\times10^3$ kg.m^{-3}

volume of raft = V_R = (3.05)(6.10)(0.305) m^3 = 5.675 m^3

cross-sectional area of raft = A_R = (3.05)(6.10) = 18.61 m^2

height by which raft sinks = h_R = ? m

mass of raft = $m_R = \rho_R\, V_R = (0.500\times10^3)(5.675)$ kg

$= 2.837\times10^3$ kg

mass of raft and people = m = ? kg

mass of 1 person = m_1 = 70 kg

number of people before raft sinks = N = ?

Archimedes Principle $\Rightarrow m_R = m_F$

$m_F = m_R = 2.837\times10^3$ kg = $\rho_F V_F$

$V_F = m_R / \rho_F = (2.837\times10^3 / 10^3)$ m^3

$V_F = A_R h_R = 2.837$ m^3

$h_R = V_F / A_R = 2.837 / 18.61 = 0.152$ m

Maximum volume of water that can be displaced is equal to the volume of the raft

$V_R = 5.675$ m^3 = V_F

Archimedes Principle $\Rightarrow m = m_F$

$m = m_F = \rho_F V_F$

$m = (10^3)(5.675)$ kg

$m = 5.675\times10^3$ kg

mass of load = $N m_1 = m - m_R$

= $(5.675\times10^3 - 2.837\times10^3)$ kg = 2.84×10^3 kg

$N = 2.84\times10^3 / 70 = 40.6$

The raft can hold 40 people safely

Archimedes' Principle

It can be argued that the study of hydrostatics was begun by Archimedes in the third century B.C. The greatest physicist of ancient time, Archimedes was apparently a kinsman of Hieron II, Tyrant of Syracuse. Legend has it that the king ordered a solid gold crown to be made. But when the piece was delivered, although its weight was right, Hieron suspected that his jeweler had substituted silver for gold in the hid-den interior. Archimedes was given the challenge of determining the truth without damaging the royal treasure. After pondering the problem for some time, its solution came to him while he was musing in a warm tub at the public baths. The distracted philosopher leaped from the water and ran home naked, shouting through the streets, *"Heureka! Heureka!"* I have found it! I have found it! What he found, as we shall soon see, was far more valuable than Hieron's crown.

A completely submerged body displaces a volume of liquid equal to its own volume. Experience also tells us that when an object is submerged, it appears lighter in weight; the water buoys it up, pushes upward, partially supporting it somehow. That much would be obvious to anyone who ever tried to submerge an inflated tire tube or a beach ball. Archimedes quantified the phenomenon. His Buoyancy **Principle** asserts that *an object immersed in a fluid will be lighter (that is, it will be buoyed up) by an amount equal to the weight of the fluid it displaces.* The upward force exerted by the fluid is known as the buoyant force. A 10 N body that displaces 2 N of water will "weigh" only 8 N while submerged.

Buoyant force is caused by gravity acting on the fluid. It has its origin in the pressure difference occurring between the top and bottom of the immersed object, a difference that always exists when pressure varies with depth (as it does for a fluid).

When a solid object is wholly or partly immersed in a fluid, the fluid molecules are continually striking the submerged surface of the object. The forces due to these impacts can be combined into a single force the **buoyant force.**

? A hydrometer is a simple instrument used to measure the density of a liquid by measuring how deep it sinks in a liquid.

A particular hydrometer was made of a uniform cylindrical rod that could float vertically in a liquid. The length of the rod was 0.250 m and its cross sectional area was 2.00×10^{-4} m^2. The mass of the rod was 4.50×10^{-2} kg. To calibrate the hydrometer it is placed into water that had a density of 1.000×10^{3} $kg.m^{-3}$. How far from the bottom end of the rod should a mark of 1.000 be placed to indicate the relative density of the water? The hydrometer sinks to a depth of 0.229 m when placed into an alcohol solution. What is the density of the alcohol solution?

Solution

Hydrometer

$L = 0.250$ m

$A = 2.00\times10^{-4}$ m^2

$m = 4.50\times10^{-2}$ kg

$m\,g = (4.50\times10^{-2})(9.8)$ N $= 4.41\times10^{-1}$ N

For the hydrometer to float, the weight of the hydrometer must equal the buoyant force.

Archimedes' Principle: buoyant force equal weight of fluid displaced.

Let h = height to which hydrometer is submerged

$F_B = \rho\,A\,h\,g = m\,g$ where $\rho = 1.000\times10^{-3}$ $kg.m^{-3}$

$h = m\,/\,(\rho\,A) = 4.50\times10^{-2}\,/\,\{(1.000\times10^{-3})(2.00\times10^{-4})\}$ = **0.225 m**

Hydrometer placed into alcohol solution

Again, $F_B = \rho\,A\,h\,g = m\,g$ where ρ = ? $kg.m^{-3}$ and $h = 0.229$ m

$\rho = m\,/\,(A\,h) = = 4.50\times10^{-2}\,/\,\{(2.00\times10^{-4})(0.229)\}$ = **983 $kg.m^{-3}$**

? A Cartesian diver can be made by completely filling an empty soft drink bottle with water. A medicine dropper is then placed in the bottle so that a small air pocket is trapped at the top of the dropper. When the bottle is squeezed, the medicine dropper sinks. When the bottle is release, the diver rises.

Why does the dropper sink when the bottle is squeezed? Your answer should include a list of the important physical quantities involved, the names and descriptions of the physical principles involved.

Solution

Pascals Principle – when pressure is applied to an enclosed fluid, the pressure is transmitted undiminished to every point in the fluid and to every point on the walls of the container.

$\Rightarrow$ the pressure exerted on the air pocket is increased

Boyle's Law $p\,V$ = constant T = constant

$\Rightarrow$ p increased $\Rightarrow$ V decreased

Density $\rho = m / V$

V decreased $\Rightarrow \rho$ increased $\Rightarrow$ average density of the dropper has increased

Archimedes Principle – When an object is floating in a fluid, the fluid exerts an upward force on the object called the buoyancy force. This buoyant force is equal to the weight of the fluid displaced by the body

* average density of object $\leq$ density of surrounding fluid $\Rightarrow$ object floats

* average density of object > density of surrounding fluid $\Rightarrow$ object sinks

Hence by squeezing the bottle, the average density of the dropper is increased and becomes larger than the surrounding fluid and the dropper sinks.