SCATTERING OF HIGHER ENERGY COSMIC RAYS

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Pitch angle scattering of cosmic rays by hydromagnetic waves generated by the cosmic rays themselves is ineffective for $E \gtrsim 10^{11}$ eV. Scattering and the associated parallel diffusion due to waves already present is discussed with particular reference to the model of Lingenfelter et al.(1971). The required spectrum is derived and the presence of the required waves is discussed.

1. Introduction. Cosmic rays can be very effectively scattered by hydromagnetic waves with wavelengths comparable to the cosmic-ray gyroradii. An anisotropic distribution of cosmic rays with degree of anisotropy $\delta \gtrsim v_A/c$, v_A = Alfven velocity, is subject to an instability in which the appropriate hydromagnetic waves grow due to a maser action. The theory of this instability and the associated scattering is discussed by Wentzel (1969a), Kulsrud and Pearce (1969), Tademaru (1969), Skilling (1970) and Melrose and Wentzel (1970). As applied to cosmic rays this theory fails in that higher energy cosmic rays are inefficient in causing the instability to develop. Kulsrud and Cezarski (1971) show that the maser does not turn on at all for the waves generated by cosmic rays with $E \gtrsim 10^{11}$ eV. These authors emphasize that either the relevant waves need to be generated by some source other than the cosmic rays themselves or, if this can be shown not to be the case, some other theory is required to account for the low anisotropy and long confinement time of cosmic rays.

In this paper we deduce the minimum spectrum of hydromagnetic waves required to account for the confinement of cosmic rays with $E \gtrsim 10^{11}$ eV to the galactic disc. The numerical values quoted are based on a model for cosmic ray diffusion developed by Lingenfelter et al. (1971) following Ramaty et al,(1970). These authors consider a model in which cosmic rays are injected at random events (supernova explosions) and diffuse independently both along and perpendicular to the mean magnetic field direction. They find that their model is consistent with the observed abundances of cosmic rays (which change due to spallation) and limits on the degree of anisotropy for scattering lengths $\ell_2 \sim 30$ p.c. perpendicular to the field lines and $\ell_1 \sim 30$ p.c. along the field lines.

The perpendicular diffusion invoked is the random walk of field lines (Jokipii 1966, Jokipii and Paker 1969). This diffusion is associated with hydromagnetic waves with wavelengths $k^{-1} - 30$ – 300 p.c., see Jokipii and Lerche (1969) for example. We are concerned with the parallel diffusion. The wavelengths are comparable to the gyroradii of the cosmic rays with which the waves interact ($k^{-1} \sim 10$A.U. at $E \sim 10^{11}$eV, $k^{-1} - 0.3$ p.c. at $E \sim 10^{15}$eV).

2. <u>Minimum Wave Spectrum</u>. Pitch angle scattering of ultrarelativistic particles (charge q, momentum p, $\mu = \cos(\text{pitch angle})$, distribution function $f(p,\mu)$) by hydromagnetic waves (wave number k along the field line, energy density $W(k)$ in the range dk at k) is described by

$$\frac{\partial f(p,\mu)}{\partial t} = \frac{\partial}{\partial \mu}\left[D(p,\mu)\frac{\partial f(p,\mu)}{\partial \mu} \right] \tag{1}$$

$$D(\mu) = \frac{\pi^2 q^2 (1-\mu^2)}{p^2 c|\mu|} W(k_R) \quad , \tag{2}$$

$$k_R = \frac{|q|B}{pc|\mu|} \quad .$$

Equation (1) is valid provided that the waves propagate at moderate angles ($\lesssim 45^{\circ}$) to the direction of the magnetic field, see Melrose and Wentzel (1970). Terms of the order v_A/c are neglected in (1). The most important of these neglected terms is that leading to the neutral streaming velocity discussed by Wentzel (1969a). This term is in fact absent if, as seems plausible, one has $W(-k) = W(k)$. The neutral streaming velocity $v_{N\emptyset}$ given by Wentzel is modified to

$$v_N = v_{N\emptyset}\frac{W(k) - W(-k)}{W(k) + W(-k)}$$

Pitch angle scattering leads to diffusion along the field lines provided that the degree of anisotropy is small and provided that the scattering length ℓ_2 is much greater than the gyroradii,

$$R_g = \frac{pc(1-\mu^2)}{|q|B} \sim k_R^{-1} \quad . \tag{3}$$

Apart from a numerical factor of order unity the scattering length is given by (Jokipii 1966)

$$\ell_2 \approx \frac{3p^2c^2}{\pi^2 q^2}\left[W\left(k = \frac{|q|B}{pc} \right) \right]^{-1} \quad . \tag{4}$$

If we impose the condition $\ell_2 \leq 30$ p.c., following Lingenfelter et al (1971), and assume that this condition is independent of cosmic-ray energy, equation (4) gives the minimum required spectrum of waves. This spectrum has the functiog at form

$$W(k) \propto k^{-2} \tag{5}$$

The energy $W(k)\Delta k$ in a band width $\Delta k \approx k \sim R_g^{-1}$ required is

$$W(k)\Delta k \sim \left(\frac{R_g}{\ell_2} \right) W_m \quad , \tag{6}$$

where

$$W_m = \frac{B^2}{8\pi}$$

is the energy density in the background magnetic field.

3. Are the Required Waves Present? With $\ell_1 \lesssim 30$ p.c. in equation (6) the required r.m.s. fluctuations ΔB in the magnetic field in the range $\Delta k \sim k$ at $k \sim R_g^{-1}$ is

$$\frac{\Delta B}{B} \gtrsim 10^{-10} (R_g)^{\frac{1}{2}} \tag{7}$$

with

$$R_g \approx 10^{14} \left(\frac{E_{\ell N}}{10^{11}} \right) \text{cm} \tag{8}$$

where we take the background magnetic field to be $3\mu g$.

Scintillation of pulsars, provided information on the r.m.s. value of the fluctuations An in the electron cohcentration in the interstellar medium (Wentzel 1969b). Typical characteristic lengths are around 10^{11} cm (Rickett 1970) which is considerably shorter than the lengths (8) of interest. Downs and Reichley (1971) suggest that fluctuations with correlation lengths $\sim 10^{14}$ cm and $\Delta n \sim 5 \times 10^{-7} \text{cm}^{-3}$ may be present. However we are unaware of any discussion of observational evidence for fluctuations in the range between 10^{14} cm and a few parsecs (which is the range of interest here).

With an interstellar electron density $n \approx 10^{-2} \text{cm}^{-3}$ the fluctuations suggested by Downs and Reichley give

$$\frac{\Delta n}{n} \approx \frac{\Delta B}{B} \approx 10^{-3} \quad .$$

This suggests that (7) may be marginally satisfied for the waves required to scatter cosmic rays with $E \sim 10^{11}$ V .

For the longer wavelengths one can only speculate on the presence of the waves. We offer two arguments in favour of the presence of waves with $k^{-1} \gtrsim 10^{14}$ cm . Firstly, one expects the motion of stars, at around 10 km sec^{-1} , with stellar-wind cavaties similar to that of the sun to generate waves with $k^{-1} \sim 10^{15}$ cm . This is so if the velocity of the star relative to the interstellar gas exceeds the Alfven velocity. Secondly we now argue that damping of the waves is a maximum at $k^{-1} \sim 10^{14}$ cm . It is then plausible that the spectrum of waves has a minimum at $k^{-1} \sim 10^{14}$ cm.

Kulsrud and Pearce (1969) discuss the damping of hydromagnetic waves due to collisions between ions and neutrals. Let n be the number density of ions or electrons, n_H be the number density of neutrals and ν_o the collision frequency.

Write $\epsilon = n/n_H$. For $w \gg \nu_o \epsilon^{\frac{1}{2}}$ the waves propagate only in the ionized gas with a damping time ν_o^{-1} . For $w \ll \nu_o$ the neutrals move with the ions in the wave and the damping time is $\nu_o \omega^{-2}$. In these two cases the Alfven velocity is given by

$$v_{A1} \approx \frac{6 \times 10^5}{n^{\frac{1}{2}}} \text{ cm sec}^{-1} \qquad (\omega \gg \nu_o \epsilon^{\frac{1}{2}}) \quad , \qquad (9a)$$

and

$$v_{A2} \approx \frac{6 \times 10^5}{n_H^{\frac{1}{2}}} \text{ cm sec}^{-1} \qquad (\omega \ll \nu_o) \quad , \qquad (9b)$$

respectively, for a field of 3pg . With (Kulsrud and Pearce 1969, Sche er and Tsytovich 1970)

$$\nu_o^{-1} \approx \frac{5 \times 10^8}{n_H} \text{ sec} \quad , \qquad (10)$$

and $n_H \approx 1 \text{ cm}^{-3}$ the maximum damping occurs at

$$k^{-1} \approx v_{A2} \nu_o^{-1} \approx v_{A1} \nu_o \epsilon^{\frac{1}{2}} \approx 3 \times 10^{14} \text{cm} \quad ,$$

where both limiting cases overlap. For $k^{-1} > 3 \times 10^{14}$ cm the damping time increases as k^{-2} . Furthermore the relevant Alfven velocity (9b) is smaller than typical stellar velocities.

4. Discussion. In models for cosmic-ray confinement and escape, based on parallel diffusion alone one requires that the streaming velocity be equal to (length of path to escape from the disc)/(confinement time). The neutral streaming velocity, first recognized by Wentzel (1969a), is in reasonable agreement with this requirement. Because scattering in which the cosmic rays generate their own waves leads to this neutral streaming velocity as the maser action approaches saturation, models based on parallel diffusion alone are tenable if the waves are generated by the cosmic rays. The inefficiency of the maser action for $E \gtrsim 10^{11}$ V undermines any model based on parallel diffusion alone. It is unreasonable to require that waves generated in other ways be just at the level, no more and no less, to give the required streaming velocity.

In the model developed by Lingenfelter et al. (1971) escape is due to perpendicular diffusion. All that is required of the parallel diffusion is that it be efficient enough to prevent escape at a faster rate than does perpendicular diffusion. This requires that the waves be excited above a minimum level, determined above. The requirement on the waves in this model is less stringent than in models based on parallel diffusion above. Furthermore the waves need not be homogeneously distributed throughout the disc to impede streaming adequately. (One expects these waves to be excited in isolated regions if they are generated by the motion of stellar cavities.)

Although we are unable to draw any positive conclusions regarding the presence of waves at or above the required minimum level, present knowledge does not forbid their presence. If subsequent investigation shows that the minimum levels are not attained models for the confinement and escape of cosmic rays will require further revision.

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