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THE CRAB NEBULA

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I WOULD LIKE to turn to a number of unsolved problems relating to continuing activity in the Crab Nebula. In particular, (1) the Crab emits about 10^{38} erg sec^{-1} in synchrotron radiation and this requires an energy source operating at the present time, (2) the wisps or light ripples which Scargle has just discussed require an energetic exciting mechanism, and (3) the point source of radio emission at 30 to 80 Mc/s requires a coherent emission process which in turn requires an exciting mechanism. My chief concern is with the source of the continued energetic activity evident from these observations.

The evidence for continuing acceleration of the electrons radiating synchrotron radiation in the Crab rests on the fact that the lifetimes of the electrons is less than the age of the nebula (900 years). Most of the power radiated is in the range from near infrared to X-ray frequencies (assuming the latter to bear synchrotron). The synchrotron half-life of the optically-emitting electrons is 100 to 300 years, and for the X-ray emitters is only a few years. For the spectrum to be in a steady state, energy must be being supplied to electrons with energies from 10^{11} to 10^{14} eV at a rate of 10^{38} erg sec^{-1} to balance the synchrotron losses.

The wisps are consistent with being local compressions in the magnetic field. Such a compression from B to bB increases the volume luminosity at fixed frequency by $b^{\gamma+1}$, where γ is the index of the power law of the electron energy spectrum $N(E) \propto E^{-\gamma}$. The wisps themselves have $b \sim 5$, the actual value depending on the geometry of the compression (only the increase in surface brightness can be observed). The wisps appear at around one-tenth the radius of the nebula, i.e. a few times 10^{17} cm from the center of the nebula, propagate at about 0.1 c, appear a few times per year, and have a total energy of the order of 10^{44} erg, again depending on assumptions about their geometry. It would not be inconsistent to assume an average power input of 10^{38} erg $\sec^{-1}$ into the wisps. Throughout the remainder of the nebula less intense compressions are observed, consistent with compressions with $b - 1 \simeq 7\%$, travelling at 10^{-2} c.

Such hydromagnetic disturbances imply a central source of energy in the Crab with outward motions at velocities of at least 10^{-2} c to excite them.

The point source of radio emission is detected by interplanetary scintillation techniques (Bell and Hewish, 1967). Its angular diameter indicates a radius of a few times 10^{15} cm, situated, within observational error, at the center of the Crab. It radiates 10^{33} erg $\sec^{-1}$, has a high effective temperature (10^{13} °K) and shows no significant circular polarization (Andrew et *al.*, 1967). This implies that the radiation mechanism is a coherent one, and indicates that magnetic field effects are probably unimportant. The most plausible emission processes then would involve plasma waves generated through streaming instabilities.

These observations indicate that the continuing activity in the Crab originates from the center of the nebula, and one can infer that it is probably associated with the supernova remnant. One would expect an energy source generating 10^{38} erg $\sec^{-1}$ in the center of the Crab to have associated with it a thermal source radiating at least this power. However, no such thermal source is observed, the largest thermal source in the center of the Crab being a star radiating no more than 10^{34} erg $\sec^{-1}$ in the optical. Thus if there is a thermal source radiating 10^{38} erg $\sec^{-1}$ it must be at unobserved frequencies, the only plausible frequency range being the far ultraviolet or soft X-rays. I shall return to this problem later.

The transport of energy outward from a central energy source would be both radiative and through mass motions. Clearly the latter is required if the energy is to be used to excite the wisps and the point source. Such an outward transport of energy occurs following a solar flare with a large fraction of the energy in the flare going into an interplanetary blast wave.

The primary problem is to understand the nature of an energy source supplying 10^{38} erg $\sec^{-1}$. The possible sources of energy are nuclear, gravitational, rotational, pulsational and magnetic energies. Cameron has shown us that pulsational energy cannot be stored in a neutron star for long periods of time due to the radiation of gravitational waves. Rotational energy can only be an effective energy source if there is some coupling to the surroundings or to a neighboring object, say via a magnetic field. Quantitatively, rotational energy and magnetic energy would be sufficient to account for the 10^{38} ergsec^{-1} over 3×10^{10} sec, but I ignore these here.

Nuclear and gravitational energy can act as an energy source when matter is accreted onto an object. Nuclear energy can provide 3×10^{18} erggm^{-1} and gravitational energy up to 3×10^{20} erggm^{-1}, corresponding to mass infall at the speed of sound in a relativistic gas. For a neutron star the surface gravitational potential energy is $c^2/30 = 3 \times 10^{19}$ erggm^{-1}. As this seems the largest possible energy available, I assume that the energy source is due to mass infall onto a neutron star. Then to obtain 10^{38} ergsec^{-1}, mass must infall at 3×10^{18} gmsec^{-1}. Over the lifetime of the Crab this requires a mass of 5×10^{-5} $M_\odot$ accreted onto the remnant. The origin of this infalling mass could be the debris from the supernova explosion or could be from a companion object.

If the accreted matter is the debris from the supernova, then 10^{-4} $M_\odot$ must have remained with less than the escape velocity after the explosion. Colgate (1968), from velocity curves for different mass fractions ejected, estimates that 10^{-5} of the ejected mass has less than the escape velocity when the expanding gas becomes optically thin. This is inadequate, as no more than about 1 $M_\odot$ of ejected matter is observed in the Crab. Even if the remnant is a collapsed object so that mass infall supplies 3×10^{20} erggm^{-1}, the mass required is 10^{-5} $M_\odot$. Thus only if a larger fraction than 10^{-5} is left with less than the escape velocity could this source of mass be adequate. As the Crab is anomalous in any event, and notably in having a very slow (for supernovae) expansion velocity, it may well be that an anomalously large fraction of the ejected matter is left with less than the escape velocity.

If the possibility is ignored, then the accreted material must come from a companion object. I consider two possible types of companion object. Firstly if the companion is a Jovian type planet several astronomical units distant from the primary, it is just conceivable that it would supply the necessary mass. The incident flux of radiation is sufficient to boil matter off the planet at the required rate of 3×10^{18} gmsec^{-1} provided that the radius of the planet is blown up to one third the radius of the sun, i.e. to three times the radius of Jupiter. As this model requires some stretching of the parameters, a more reasonable assumption is that the companion is a highly evolved star losing mass by evolutionary processes.

Prendergast and Burbidge (1968) have considered the transfer of mass in such close binary systems. The rate of transfer they consider is 3×10^{19} gm sec^{-1}. The model proposed by these authors has a white dwarf as the primary, their object being to account for the properties of some X-ray sources. The calculations should be little affected if the primary is a neutron star, except that the energy available is much larger, provided that this energy is not too close to the radiation stress limit (when the outward radiation pressure balances the inward force of gravity). For a primary of one solar mass the maximum luminosity allowed by the radiation stress limit is a few times 10^{38} erg sec^{-1}.

It seems desirable to avoid having to appeal to a companion object to supply the mass required. A model involving only the debris from the supernova suffers only from the difficulty of the total mass required. With such a model for the Crab, the energy supplied could be at the radiation stress limit if the remnant is somewhat less massive than the sun. I am aware of no detailed studies of models of this kind. Qualitatively one might expect the thermal velocity and the escape velocity to be comparable below unit optical depth. There would be some outward loss of mass due to free expansion and an inward motion of gas to maintain the energy supply.

Assuming that some adequate source of mass is present, the continual mass infall onto the primary will not only provide an energy source but will also lead to a freely expanding hot gas surrounding the primary. It is plausible to assume that the density of this gas falls off as an inverse square aw, i.e. if n is the number density and r the radius

$$n = n_0 r^{-2} \tag{1}$$

Presumably the energy is transported outward through this region partly in the form of mass motions. To fix the various parameters involved I assume that at $r = 3 \times 10^{17}$ cm these motions become hydromagnetic and then are identified as the wisps.

To begin with, an estimate of the thermal particle number density in the Crab as a whole is required. Assuming that the less intense variations in the Crab travel with the hydromagnetic velocity (the wisps are large amplitude phenomena and so have Mach numbers greater than unity) this velocity is 10^{-2} c. The hydromagnetic velocity is either the Alfvén velocity or the velocity of the suprathermal mode of Parker (1965). In either case this identification leads to a number density of order 3×10^{-2} cm^{-3} or less (Scargle, 1968).

Supposing that the density (Eq. 1) drops to the value $3 \times 10^{-2}\,\mathrm{cm}^{-3}$ at $r = 3 \times 10^{17}$ cm gives

$$n_0 = 3 \times 10^{33} \tag{1'}$$

A large amplitude shock reaching $r = 3 \times 10^{17}$ cm with a number density $0.1\,\mathrm{cm}^{-3}$ (the density compression is necessarily less than a factor 4) requires a velocity of $10^9\,\mathrm{cm\,sec}^{-1}$ if it is to transport $10^{38}\,\mathrm{erg\,sec}^{-1}$. With the surrounding magnetic field of 3×10^{-4} gauss (the field in the freely expanding region falls off as r^{-2} and so is necessarily negligible with respect to this value) these motions would indeed become hydromagnetic at 3×10^{17} cm, and show up as compressions in the field.

A crude estimate of the temperature of the freely expanding gas can be made by assuming that at $r = 3 \times 10^{17}$ cm (= 1/10th the radius of the Crab) the expansion velocity is 1/10th the expansion velocity of the outermost regions of the Crab, i.e. is equal to $10^7\,\mathrm{cm\,sec}^{-1}$. Identifying this with the speed of sound gives a temperature of $10^6\,°\mathrm{K}$.

This model is obviously a crude one. Built into it are the requirements for an energy transport of $10^{38}\,\mathrm{erg\,sec}^{-1}$ and the requirement for the excitation of the wisps. As a check one can estimate the amount of mass lost by a free expansion at $10^7\,\mathrm{cm\,sec}^{-1}$ over a spherical surface of 3×10^{17} cm with a number density of $3 \times 10^{-2}\,\mathrm{cm}^{-3}$. This mass is $3 \times 10^{17}\,\mathrm{gm\,sec}^{-1}$, in reasonable agreement with the $3 \times 10^{18}\,\mathrm{gm\,sec}^{-1}$ injected.

It seems plausible that such an energy source would have a thermal source associated with it. One can estimate the temperature of such a source by using Eqs (1) and (1') to find unit optical depth for the Compton cross section. This gives $r \sim 10^9$ cm. A black body radiating nearly $10^{38}\,\mathrm{erg\,sec}^{-1}$ with this radius has a temperature of $5 \times 10^5\,°\mathrm{K}$. So a thermal source would be expected in the soft X-ray region (100–200 eV). In this range the Compton cross section underestimates the true absorption so that the temperature of the source may be lower. Even with interstellar absorption of soft X-rays (which is poorly known) one might expect to observe such a thermal source.

Another feature of the continuing activity in the Crab is the point source of radio emission. Two models for this have been presented in the literature, by Ginzburg and Ozernoi (1966) and by Zhelezniakov (1967). Neither seems satisfactory.

The model of Ginzburg and Ozernoi assumes that the emission process is Rayleigh scattering of electron plasma waves. The model requires a high plasma density, $n \simeq 10^7\,\mathrm{cm}^{-3}$, for the observed frequency to equal the plasma frequency. The temperature is around $10^6\,°\mathrm{K}$ and so the free-free absorption length is 10^{11} cm compared to 3×10^{15} cm for the dimensions of the source. The authors assume that the plasma is magnetically confined, presumably so that the large density drop required to allow the radiation to escape can occur. This requires a field of 0.1 gauss over 3×10^{15} cm.

However, such a field must be fixed to some centrally condensed object if it is to confine the plasma. As such a field cannot fall off less slowly than r^{-2}, this requires fields in excess of 10^8 gauss for an ordinary star ($r = 10^{11}$ cm) and 10^{18} gauss for a neutron star ($r = 10^6$ cm). These fields are impossibly large. Consequently, magnetic confinement does not seem possible; then the short absorption length appears to rule out a plasma density of 10^7 cm^{-3} and the adopted radiation mechanism must also be ruled out.

Zhelezniakov assumes the radiation mechanism to be coherent synchrotron radiation. This overcomes the difficulty with the free-free absorption length, requiring a lower plasma density. The plasma plays only a passive role in this process, relativistic electrons with a peaked energy distribution being the active radiators. This model suffers from the drawback that there is no energy source for the relativistic electrons, and the way that a peaked spectrum might be set up is ignored.

If the basic energy source for the radio emission is the plasma waves, as in the model of Ginzburg and Ozernoi, then to account for the observations the frequency radiated must be greater than the plasma frequency. One way of accomplishing this is to use the plasma waves to accelerate the electrons. This acceleration process effectively accelerates mildly relativistic electrons and will produce the peaked energy spectrum required for the coherent synchrotron process (Tsytovich, 1966). However, a further mechanism is then possible, namely bremsstrahlung generated by the relativistic electrons off the plasma waves themselves, a process closely analogous to inverse Compton radiation.

The relative importance of the synchrotron and plasma wave bremsstrahlung processes depends on the ratio of the energy density in the magnetic field to that in the plasma waves. The major difference in the two processes is the frequency radiated. An electron with Lorentz factor $\gamma = E/mc^2$ radiates synchrotron radiation at the frequency

$$\omega = 2\pi f \simeq \Omega \gamma^2 \tag{2a}$$

where $\Omega = eB/mc$ is the non-relativistic electron gyrofrequency. The plasma wave bremsstrahlung occurs at a frequency (Gailitis and Tsytovich, 1964)

$$\omega = 2\pi f \simeq kcy^{Z} \tag{2b}$$

where k is the wave number of the plasma waves. In fact, for the latter mechanism mildly relativistic electrons can radiate at the frequency observed. For instance with a phase velocity of the plasma waves $v_\varphi \simeq 10^9\,\mathrm{cm\,sec^{-1}}$ and a plasma frequency $\omega_0 \simeq 2\pi \times 10^5\,\mathrm{sec^{-1}}$ (corresponding to $n \simeq 3 \times 10^2\,\mathrm{cm^{-3}}$ for $r \simeq 3 \times 10^{15}$ in Eq. (1)), setting $k = \omega_0/v_\varphi \simeq 2\pi \times 10^{-4}$ in Eq. (2b) requires $\gamma \sim 3$ for $f = 3 \times 10^7$ c/p.

The plasma wave bremsstrahlung process requires further investigation as the radiation process for the point source is coherent (as indicated by its high effective temperature) and the conditions under which this process can be coherent have not been investigated.

A further radiation mechanism involving plasma waves, but resulting in the radiation of frequencies much higher than the plasma frequency, is that suggested by Colgate (1967). In this case a continuous scattering of electromagnetic waves off plasma waves results in an upward diffusion in their frequency. As indicated above, the requirement that the frequency radiated be much larger than the plasma frequency is imposed by the condition that the free-free absorption length exceed 10^{15} cm. The only other escape from this requirement is to ignore the observation of a finite angular size for the point source, so that the radius is much less than 10^{15} cm.

One can understand why plasma waves should be copiously generated by noting that an outwardly propagating shock passing through a decreasing density distribution can only be collisionally dominated if the collision mean free path, l, is much less than the dimensions over which the density changes appreciably. Here this requires $l \ll r$. As

$$l \simeq 3.5 \times 10^4\, T^2 n^{-1}\ \mathrm{cm} \tag{3}$$

with the distribution of Eq. (1), $l \ll r$ requires $r \ll 10^{17}$ cm. At $r = 3 \times 10^{15}$ cm $l \simeq 10^{-1} r$. Thus significant streaming motion occurs, and one expects electrostatic stresses to be set up to compensate for the reduced collisional stresses. Under these conditions plasma waves are generated.

Thus the presence of the point source seems consistent with the overall model presented. The actual radiation mechanism however cannot yet be identified with any degree of confidence.

There are two other problems associated with the Crab Nebula which I would like to mention briefly. The first is the nature of the so-called central star. This star, if it is in the Crab, has a luminosity of about that of the sun, but is the color of an A type star. With the overall model just presented, this star could be the evolving companion, or the low frequency optical bremsstrahlung tail of the thermal source associated with the remnant. In deciding the nature of this object further spectra are required, as there remains some controversy as to whether it shows lines or not.

Secondly there remains the problem of how the highly relativistic electrons are accelerated in the Crab. Since hydromagnetic motions are observed, probably supplied with sufficient energy (10^{38} erg sec^{-1}) to act as an energy source, it is highly plausible that these lead to the acceleration. Scargle (1968) finds that there is a direction of maximum hydromagnetic activity in the Crab, and that in this direction the spectral index of the optical synchrotron radiation (and so presumably of the energy spectrum) is systematically flatter than in the remainder of the nebula.

I consider two acceleration mechanisms which might operate in the Crab. The first is physically similar to the gyrorelaxation effect (see Chandrasekhar, 1960). When a particle distribution is subjected to an impressed periodic magnetic field, pitch angle anisotropies are generated. If isotropy is re-established by some other process in a time shorter than the period of the impressed magnetic field, then the particles systematically gain energy at the expense of the impressed magnetic field perturbations.

In the Crab Nebula the hydromagnetic motions provide the magnetic field variations, and the anisotropies so generated are removed by the emission of higher frequency hydromagnetic waves (Melrose 1968a). When these secondary waves reach a state of quasiequilibrium due to their emission and reabsorption, they provide an intermediate state which allows the gyro-relaxation effect to operate.

An estimation of the acceleration resulting in this case (Melrose 1968b) shows that the mechanism is capable of accounting quantitatively for the acceleration of electrons with energies up to the energy range for the emission of optical synchrotron radiation. The mechanism can also account for the flattening of the energy spectrum observed. However, it cannot account for the acceleration of the highest electrons.

The second mechanism is analogous to a Fermi type process. This can occur when the amplitude of the wave is large enough so that the classical momentum (e/c) A (A is the vector potential in the wave) exceeds the particle momentum p. Then the changes in sign of p $-$ (e/c) A imply reflections of the particles from wave crests and troughs. The rate of acceleration is then just the Fermi form

$$\frac{dE}{dt} = \frac{v_0^2}{c\lambda} E \tag{4}$$

with λ the wavelength of the waves and v_0 the velocity of wave propagation. There is the added requirement

$$\left|\frac{e}{c} A\right| > |p|$$

or equivalently, in terms of the wave energy density W_w and the Lorentz factor γ of the electrons,

$$W_w > \frac{\pi mc^2}{r_0\lambda^2}\gamma^2 \qquad (5)$$

where $r_0 = e^2/mc^2$ is the classical radius of the electron.

Applying this mechanism to acceleration by Alfven waves associated with the wisps (see Scargle 1968) one has $\lambda \simeq 3 \times 10^{16}$ cm, $v_0 \simeq 3 \times 10^9$ cm sec^{-1}, and the energy density in the waves $W_w \simeq 10^{-8}$ erg cm^{-3}. These are the most favourable values one could choose, and with these values Eq. (5) allows for acceleration up to $\gamma \simeq 10^9$, which is just the highest energy observed in the Crab. The acceleration time for the highest energy electrons follows from Eq. (4) to be a few years, about the same as the synchrotron half-life-time for $\gamma = 10^9$. Thus, with a favorable choice of the parameters, this mechanism can just account for the acceleration of the highest energy electrons in the Crab.

In conclusion it is clear that many aspects of the Crab Nebula and its continuing activity remain unexplained. The presence of an extremely energetic energy source operating in the Crab seems to require something along the lines discussed above to account for the remarkable features observed. Our further understanding of the Crab, the closest supernova remnant which can be observed in detail, will no doubt further our understanding of post-supernova conditions, even though there remains the complication factor that the Crab is an unusual supernova remnant.

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DISCUSSION

P. Morrison Has anyone considered putting a star at the position of the filaments to account for the point source?

D. Melrose That would certainly do it. An energy output of 10^{33} erg/sec is less than the solar luminosity; one would use a star as the energy source if it occurred in the filaments.

L. Sartori If the x-rays are not synchrotron radiation, and the life-time for the optical synchrotron radiation is 300 years, I don't see why continual replenishment is still needed. The Crab Nebula would then be only three lifetimes old and if the particles have some component of motion along the field lines the lifetime could conceivably be extended to 1000 years. Isn't it then possible to trace all the energy back to the original explosion?

L. Woltjer We must account for both the synchrotron loss and the expansion velocity, so I think that it is almost impossible to take the electrons back to the outburst.

D. Melrose Also, we don't know what the conditions were like in the earlier stages. Presumably the magnetic field was larger, for example. However, I agree that one could explain the energy source in this manner, but it is difficult to do so.

S. Colgate One way in which energy can be released from a magnetic field is by having flux lines which pass through a neutron star released through resistive instability. If the energy stored in a dipole field in a neutron star is released in this manner, the time scale will be of the order of the age of the Crab Nebula. A flux line released from the surface of the star by resistive instability would move out at essentially the local Alfven speed associated with the field value at the surface. As far as the energy requirements are cony cerned, the magnetic field energy is of the order of 1% of the binding energe of the neutron star. Actually, only 10^{-3} of the total energy is needed, sinct the binding energy is roughly 10^{52} ergs, and we must account for an outpu- of 10^{49} ergs released over 1000 years.

A. G. W. Cameron Is it clear, if most of the internal field is wrapped up inside the star with a more-or-less quiet exterior, that field loops are going to escape, or will they decay inside and just heat the star?

S. Colgate The problem is where does the bifurcation of the mass, i.e., that which falls in versus that which is ejected, take place relative to the field distribution? If the Crab Nebula is the result of a supernova in a star with a very large dipole field component, it is conceivable that 10^{-3} of the internal energy of the star would be released, and we would not have to consider the geometry of the field lines.

A. G. W. Cameron Nevertheless, if we consider a region in which at one stage convection has reduced the field to a relatively small size and at other times differential rotation set in so that much of the internal field were wrapped into an internal toroid about the center, then I would think that the field would contract inward as it dissipated.