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SCINTILLATION-INDUCED CIRCULAR POLARIZATION IN PULSARS AND QUASARS

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ABSTRACT

We present a physical interpretation for the generation of circular polarization resulting from the propagation of radiation through a magnetized plasma in terms of a rotation measure gradient, or "Faraday wedges." Criteria for the observability of scintillation-induced circular polarization are identified. Application of the theory to the circular polarization in pulsars and compact extragalactic sources is discussed.

Subject headings: galaxies: magnetic fields — polarization — pulsars: general — turbulence

1. INTRODUCTION

The circular polarization (CP) of radio emission observed from pulsars (Manchester, Taylor, & Huguenin 1975; Radhakrishnan & Rankin 1990; Han et al. 1999) and from some quasars (Roberts et al. 1975; Weiler & de Pater 1983; Saikia & Salter 1988) is not understood. In both cases simple theory suggests that any polarization should be linear, determined by the direction of the projection of the magnetic field in the source region on the plane of the sky. In both cases intrinsic CP is expected as a correction to first order in an expansion in the inverse of the Lorentz factor of the radiating particles, but it does not seem possible to account for the observations in terms of CP intrinsic to the emission process (Radhakrishnan & Rankin 1990; Radhakrishnan 1992; Saikia & Salter 1988). Another possibility is that the CP is imposed as a propagation effect due to the partial conversion of linear polarization into CP resulting from the ellipticity of the natural modes of the medium (Sazonov 1969; Pacholczyk 1973; Jones & O'Dell 1977a, 1977b). However, this also fails to provide a satisfactory explanation for the properties of the observed CP. In this paper we describe a different propagation effect that can lead to CP for a source, independent of its degree of linear polarization. We refer to this as scintillation-induced CP. Although the observed CPs from pulsars and quasars are quite different, we suggest that both might be due to scintillation-induced CP, with the most obvious differences being due to pulsars scintillating in the diffractive regime and quasars scintillating in the refractive regime.

In the formal theory of scattering in a turbulent, magnetized plasma (Melrose 1993a, 1993b) there are differences in the scattering in the two oppositely circularly polarized wave modes of the medium due to their different refractive indices. Most of the terms contributing to the CP are too small to be of interest for scattering in the interstellar medium (ISM). However, we have recently shown (Macquart & Melrose 2000, hereafter Paper I) that there is one much larger contribution to the CP that is a possible candidate for explaining the CP in pulsars and quasars. The CP identified in Paper I is due to a nonzero variance in Stokes V, and the formal theory implies that the dominant contribution is of the form $\langle V^2 \rangle \sim D_{VV}(r_{\rm ref})\langle I \rangle^2$, where $D_{VV}(r)$ is a phase structure function associated with the relative phase, ϕ_V, between the components in the opposite CPs. The distance $r_{\rm ref} = r_{\rm F}^2/r_{\rm diff}$ is the refractive scale, which is defined in terms of the Fresnel scale $r_{\rm F} = (L\lambda/2\pi)^{1/2}$, where L is the distance between the scattering screen and the observer's plane and λ is the wavelength, and the diffractive scale $r_{\rm diff}$, which is defined by writing the phase structure function in the forms $D(r) = (r/r_{\rm diff})^{\beta-2}$ for a power-law spectrum of turbulence. Our purposes in this paper are threefold: first, to provide a physical explanation for the mechanism that leads to this term; second, to use this interpretation to relax some of the restrictive assumptions made in Paper I to obtain a more general semiquantitative expression for the predicted CP; and third, to explore the suggested application to the observed CP in pulsars and extragalactic sources.

We start (§ 2) by including birefringence in a simple model for strong scattering in which scattering is attributed to a large number of coherent patches of size $\sim r_{\rm diff}$ within an envelope of size $\sim r_{\rm ref}$ on the scattering screen (e.g., Goodman & Narayan 1989; Narayan 1992; Gwinn et al. 1998). When the birefringence is included, this model reproduces the result $\langle V^2 \rangle \sim D_{VV}(r_{\rm ref})\langle I \rangle^2$ derived from the formal theory of scattering in a magnetized plasma in Paper I. This simple model corresponds to strong diffractive scintillation, suggesting that our expression for $\langle V^2 \rangle$ applies only to a source that exhibits strong diffractive scintillations (which is the case for pulsars but not for quasars). Further physical interpretation of scintillation-induced CP is developed in § 3, where it is argued that it arises from a combination of two processes: a rippling of the wave front, as in the conventional theory for scattering in a turbulent medium, combined with random refractions in the birefringent medium that cause a separation in the rays associated with the opposite CPs. This leads to the following interpretation: the ripples in the wave front for the two CPs become spatially separated due to the random birefringent refractions, so that they do not overlap in the observer's plane. This leads to alternate patches in which one CP and then the other dominates, leading to a nonzero $\langle V^2 \rangle$. This interpretation is the basis for two important generalizations that we propose here. First, an essential requirement in this interpretation is that the images in the two CPs be displaced relative to each other by Δx in the observer's plane. Our interpretation of the formal theory is that this is due to random birefringent scatterings at the putative scattering screen. However, any mechanism that causes such a displacement of the rays corresponding to the two opposite CPs leads to a nonzero Δx and, hence, to a nonzero $\langle V^2 \rangle$. In particular, an alternative to random birefringent refractions is birefringent refraction at a single structure, which

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we refer to as a Faraday wedge (see § 4). Second, although the result derived from the formal theory applies for strong diffractive scintillations, our interpretation implies that a nonzero $\langle V^2 \rangle$ results for any source that exhibits scintillations. Consider a source with a scintillation index m_I on a spatial scale $r_{\rm scint}$ in the observer's plane; it should exhibit fluctuations in the degree of CP, m_V, that is of order $\Delta x/r_{\rm scint}$ times m_I. In particular, a source that exhibits refractive scintillations should exhibit fluctuations in CP on a similar timescale with an amplitude smaller by the factor $\Delta x/r_{\rm scint}$.

For scintillation-induced CP to account for the observed CP in quasars and pulsars, two conditions need to be satisfied. First, the predicted degree of CP, m_V, must be in the observed range, which appears to be ~ 1 for a few pulsars and is $\gtrsim 10^{-3}$ for relevant extragalactic sources. Second, the predicted timescale for the fluctuations in CP must be consistent with the observed variations in the CP. For pulsars this is the diffractive timescale, and for quasars it is the refractive timescale. These requirements are discussed for pulsars in § 5 and for extragalactic sources in § 6.

2. STRONG SCATTERING IN A BIREFRINGENT MEDIUM

In this section we show that the main result found in Paper I is reproduced by a simple model for the scattering. This result is

$$\langle V^2 \rangle \sim D_{VV}(r_{\rm ref}) \langle I \rangle^2 , \qquad (1)$$

which was derived in Paper I for scattering at a single screen for a power-law spectrum of turbulence in a uniform magnetic field. The assumption that the fluctuations are only in the density implies that $D_{VV}(r)$ is proportional to the phase structure function, $D(r)$, for the phase in the absence of birefringence. Assuming a power-law distribution for the turbulence, this implies

$$D_{VV}(r) = \alpha^2 \left(\frac{r}{r_{\rm diff}} \right)^{\beta - 2} , \qquad (2)$$

with $\beta = 11/13$ for a Kolmogorov spectrum of turbulence and with $\alpha = Y \cos \theta$, $Y = \nu_B/\nu$, where ν_B is the electron cyclotron frequency and θ is the angle between the magnetic field and the line of sight.

The model for strong scattering that we introduce to interpret the result (eq. [1]) involves regarding the amplitude of the wave at the observer's screen as a sum of terms that each correspond to a point of stationary phase (e.g., Born & Wolf 1965; Goodman 1985; Gwinn et al. 1998). The model is illustrated in Figure 1. There are two relevant contributions to the phase: a rippling on a scale $r_{\rm diff}$ due to the postulated density fluctuations at the screen, and a geometric effect described by a curvature of the mean wave front on a scale $r_{\rm F}$, with the points of stationary phase corresponding to points where the tangent to the wave front is orthogonal to the line of sight (see Fig. 1). Let there be $N \sim (r_{\rm F}/r_{\rm diff})^4 \gg 1$ such points of stationary phase (e.g., Narayan 1992). The amplitude in this model is of the form

$$u = \sum_{j=1}^{N} A_j e^{i\phi_j} , \qquad (3)$$

where the A_j and ϕ_j are the amplitude and phase, respectively, associated with the jth coherent patch (the jth point of stationary phase). The ϕ_j are assumed to be a random set of phases uniformly distributed in the interval $[0,2\pi]$. For our purposes it suffices to assume that all N amplitudes are identical, writing $A_j = A$ for all j. The mean is defined by averaging over the phases. The intensity is

$$\langle I \rangle = \langle uu^* \rangle = \left\langle \sum_{j,k=1}^{N} A^2 e^{i(\phi_j - \phi_k)} \right\rangle = NA^2 , \qquad (4)$$

and its variance is

$$\langle I^2 \rangle = \langle uuu^*u^* \rangle = \left\langle \sum_{j,k,l,m=1}^{N} A^4 e^{i(\phi_j + \phi_k - \phi_l - \phi_m)} \right\rangle = N(2N-1)A^4 . \qquad (5)$$

For $N \gg 1$ these imply $\langle I^2 \rangle = 2\langle I \rangle^2$, which is the well-known result for diffractive scintillations.

To include the polarization, the wave amplitude, u, is separated into its right-hand, u_+, and left-hand, u_-, circularly polarized components, and the intensities in the two

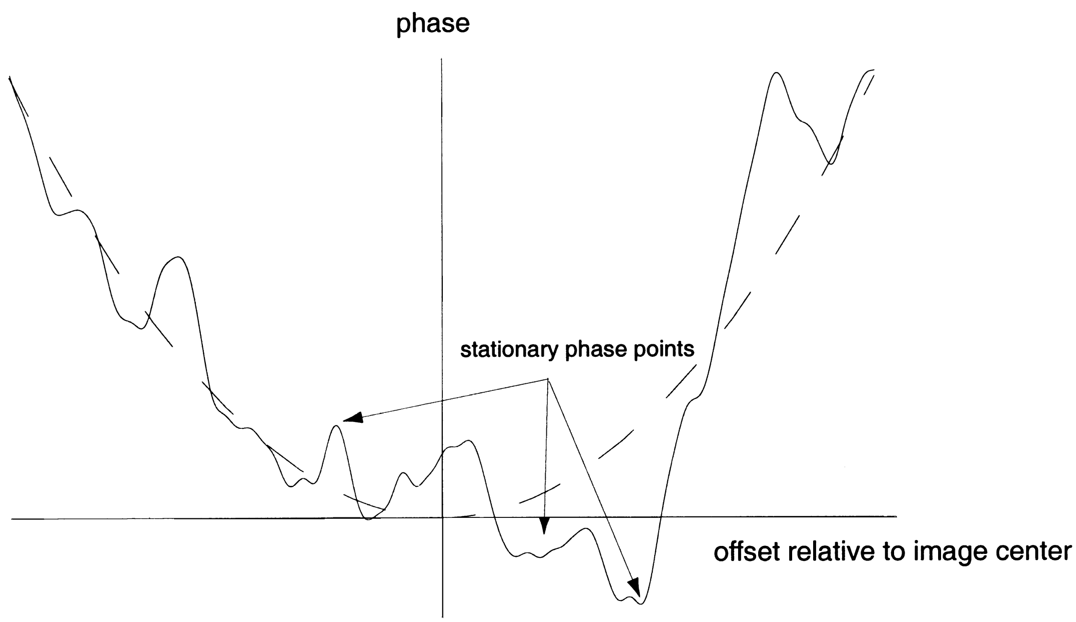

FIG. 1.—Illustration of the scattered wave front, showing the contribution of the geometric phase (*dotted line*), and the combined contribution of the scattering medium and geometric phase (*solid line*). Several points of stationary phase are indicated on the diagram.

CPs are identified as $I_\sigma = u_\sigma u_\sigma^*$ with $\sigma = \pm$. The relevant Stokes parameters are given by

$$I = \tfrac{1}{2}(I_+ + I_-)\ , \quad V = \tfrac{1}{2}(I_+ - I_-)\ . \tag{6}$$

The difference in refractive index between wave in the two CPs leads to a phase difference between them, denoted $\sigma\phi_V$. Let $\sigma\phi_{Vj}$ be the additional phase associated with the jth coherent patch. This assumption corresponds to

$$u = \sum_{\sigma=\pm} u_\sigma, \quad u_\sigma = \sum_{j=1}^{N} A_j e^{i(\phi_j + \sigma\phi_{Vj})}\ , \tag{7}$$

The mean CP is then

$$\langle V \rangle = A^2 \left\langle \frac{1}{2} \sum_{i,j=1}^{N} (e^{i(\phi_i + \phi_{Vi} - \phi_j - \phi_{Vj})} - e^{i(\phi_i - \phi_{Vi} - \phi_j + \phi_{Vj})}) \right\rangle\ . \tag{8}$$

We assume that the change in phase due to the birefringence at the screen is sufficiently small that the approximation

$$e^{i(\phi_j + \sigma\phi_{Vj})} = (1 + i\sigma\phi_{Vj} - \tfrac{1}{2}\,\phi_{Vj}^2)e^{i\phi_j} \tag{9}$$

applies. On substituting equation (9) into equation (8), an average over the random phases is nonzero only for $i = j$ and the two terms in equation (8) cancel for $i = j$, giving $\langle V \rangle = 0$.

The variance in V is given by

$$\langle V^2 \rangle = \langle I_+^2 + I_-^2 - 2I_+ I_- \rangle\ , \tag{10}$$

which becomes

$$\langle V^2 \rangle = \frac{A^4}{4} \sum_{i,j,k,l=1}^{N} \left\langle e^{i(\phi_i + \phi_j - \phi_k - \phi_l)} \times \left(\sum_{\sigma=\pm} e^{i\sigma(\phi_{Vi} + \phi_{Vj} - \phi_{Vk} - \phi_{Vl})} - 2e^{i(\phi_{Vi} - \phi_{Vj} - \phi_{Vk} + \phi_{Vl})} \right) \right\rangle\ . \tag{11}$$

On expanding as in equation (9), the only nonvanishing terms are for $i = l$ and $j = k$. There are N^2 such terms, which give

$$\langle V^2 \rangle = N^2 A^4 D_{VV}(r_{ij})\ , \quad D_{VV}(r_{ij}) = \langle (\phi_{Vi} - \phi_{Vj})^2 \rangle\ , \tag{12}$$

where $D_{VV}(r_{ij})$ is the phase structure function for ϕ_V. The distance r_{ij} is the spatial separation of the ith and jth coherent patches, which is typically of order $r_{\rm ref}$. With $NA^2 = \langle I \rangle$ according to equation (4), the result in equation (12) with $r_{ij} \to r_{\rm ref}$ reproduces the functional form of the relation in equation (1).

The numerical coefficient in relation (1) can be evaluated for a power-law spectrum of the turbulence, as given by equation (2). A calculation given in Appendix A leads to a coefficient, $g(\beta)$, of order unity. Thus equation (12) with equation (4) implies

$$\langle V^2 \rangle = g(\beta) D_{VV}(r_{\rm ref}) \langle I \rangle^2\ , \tag{13}$$

with $D_{VV}(r)$ given by equation (2).

The expressions of equation (12) or equation (13) are valid only for $\langle V^2 \rangle \ll \langle I \rangle^2$; this follows from the expansion made in equation (9) and a related expansion for $D_{VV}(r) \ll 1$ made in Paper I. However, there is no reason in principle why this condition must be satisfied. If it is invalid, then the fluctuations in V can approach the maximum possible value, $\langle V^2 \rangle \sim \langle I \rangle^2$.

This simple model for $\langle V^2 \rangle$ suggests the following interpretation for the result of equation (1). The assumptions made in the calculation in Paper I apply to a source that, in the absence of birefringence, exhibits strong diffractive scintillations. The inclusion of birefringence leads to random birefringent refractions that cause the two oppositely circularly polarized rays to emerge from the scattering screen propagating in slightly different directions. This angular separation between the rays implies that the peaks and troughs in the wave fronts in the opposite CPs are displaced from each other in the observer's plane. This separation corresponds to alternating patches of opposite CPs, which is described by $\langle V^2 \rangle$.

3. RANDOM BIREFRINGENT REFRACTIONS

In this section we discuss the interpretation of the function $D_{VV}(r_{\rm ref})$ in equation (1) in terms of random birefringent refractions. We then argue that this interpretation allows one to relax two restrictive assumptions made in Paper I to obtain a more general, semiquantitative expression for the predicted scintillation-induced CP.

Random refractions in an isotropic medium lead to an angular ("scatter") broadening of the source. In scattering theory this is described in terms of a bundle of rays diffusing in angle of propagation. For our purposes here a semiquantitative discussion suffices. In refractive scattering, the screen acts like a large lens, of size $\sim r_{\rm ref}$, that causes a phase change of order $\delta\phi \sim [D(r_{\rm ref})]^{1/2}$. The phase change includes a tilt of the wave front that corresponds to a change in the ray direction. The typical change in the ray angle is $\delta\psi \sim (\lambda/2\pi)\delta\phi/r_{\rm ref}$. When the birefringence is included, there is an additional phase change between the two wave modes, and this leads to a separation of the ray angles for the opposite CPs. The relative phase change between the wave fronts for the two CPs is of order $\delta\phi_V \sim [D_{VV}(r_{\rm ref})]^{1/2}$, and the angular separation between the rays in the two CPs is $\Delta\psi_V \sim (\lambda/2\pi)\delta\phi_V/r_{\rm ref}$. This angular separation times the distance, L, to the screen leads to a spatial displacement of the images in the two CPs of $\Delta x \sim \Delta\psi_V L$, which implies

$$D_{VV}(r_{\rm ref}) \sim \frac{(\Delta x)^2}{r_{\rm diff}^2}\ . \tag{14}$$

The expression for $\langle V^2 \rangle$ obtained in Paper I, and rederived in equation (13) above, then implies

$$\langle V^2 \rangle \sim \frac{(\Delta x)^2}{r_{\rm diff}^2} \langle I \rangle^2\ . \tag{15}$$

We interpret this form (eq. [15]) as follows. According to the model in § 2, the source is exhibiting diffractive scintillations that correspond to a rippling of the wave front such that there are patches of constructive and destructive interference with a characteristic size $r_{\rm diff}$. The birefringence causes a relative spatial displacement by Δx of this pattern in the opposite CPs. For $\Delta x \lesssim r_{\rm diff}$, the partial separation of the images in the opposite CPs leads to an image with a degree of CP $\sim \Delta x / r_{\rm diff}$.

The foregoing discussion shows that two ingredients are essential for scintillation-induced CP: scintillations that produce a pattern of variations with a scale size $r_{\rm scint}$ in the observer's plane, and birefringent refraction that causes a displacement, Δx, between the images in the opposite CPs. The resulting degree of CP is then given by

$$m_V \sim \frac{\Delta x}{r_{\rm scint}} m_I \; , \quad m_V = \frac{\langle V^2 \rangle^{1/2}}{\langle I \rangle} \; , \quad m_I = \frac{[\langle (I^2 \rangle - \langle I \rangle^2]^{1/2}}{\langle I \rangle} \; . \tag{16}$$

With the assumptions made in the models discussed above, the scintillations are diffractive, $m_I \sim 1$, $r_{\rm scint} = r_{\rm diff}$, and Δx is due to random birefringent refractions. More generally, equation (15) should apply with m_I, $r_{\rm scint}$ corresponding to the observed scintillations, and Δx could be due to birefringent refraction either by a single structure (Faraday wedge) or by a spectrum of turbulence. For example, for refractive scintillations one has $m_V = (\Delta x / r_{\rm ref}) m_{\rm ref}$, where $m_{\rm ref}$ is the intensity modulation due to refractive scintillation.

4. THE FARADAY WEDGE

The random birefringent refractions that contribute to scintillation-induced CP through the factor $D_{VV}(r_{\rm ref})$ in equation (1) are due to gradients in rotation measure (RM) associated with the turbulence at the scattering screen. Scintillation-induced CP requires that the wave front be rippled and that the ripples in the opposite CPs be displaced laterally along the wave front, but it is not necessary that these two features be imposed at a single scattering screen. In this section we introduce the concept of a Faraday wedge in which the lateral displacement along the wave front of the ripples in the opposite CPs is attributed to birefringent refraction at a single structure along the line of sight. We consider two idealized models for the wedge; in the first the wedge is modeled as a birefringent region with a density gradient, and in the second the wedge is modeled as a prism with sharp edges. These two models lead to essentially the same semiquantitative result for the angular separation of the rays corresponding to the two CPs.

4.1. *Refraction in a Birefringent Medium*

It is useful to view the effects of a Faraday wedge in terms of both geometric optics (rays) and physical optics (wave fronts). In terms of geometric optics, as illustrated in Figure 2, a Faraday wedge splits an incident ray into two oppositely circularly polarized emerging rays propagating in slightly different directions, separated by an angle $\Delta\zeta$ say. After propagating the distance L from the wedge to the observer's plane, the left- and right-hand images are separated by $\Delta x = \Delta\zeta L$. For $\Delta\zeta L \gtrsim r_{\rm scint}$ an observer sees well-separated oppositely circularly polarized images of the source for a scintillating source. In terms of physical optics, as illustrated in Figure 3, the Faraday wedge causes an incident wave front to split into two wave fronts whose normals are in slightly different directions, separated by $\Delta\zeta$. Each wave front is rippled, and the effect of the Faraday wedge applies to the average (over the ripples) wave fronts. (One wave front is also slightly delayed relative to the other, but this effect is neglected here.) At the observer's plane, corresponding ripples on the two wave fronts are displaced along the wave front by $\Delta x = \Delta\zeta L$. The observer sees a net CP varying on the timescale associated with the scintillations as the whole pattern sweeps across the observer's plane.

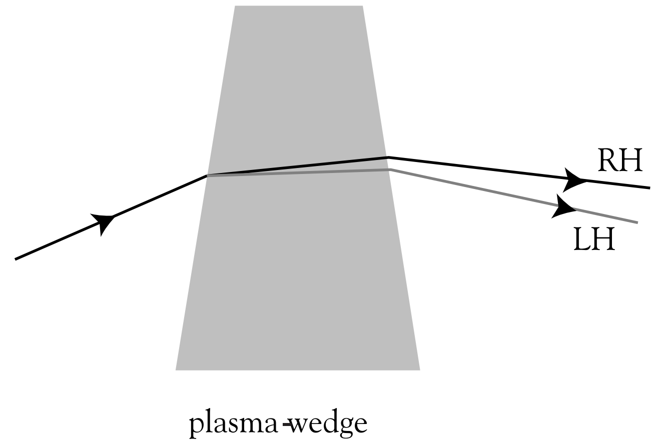

FIG. 2.—Schematic of refraction of rays at a Faraday wedge: an incident ray splits into two circularly polarized rays propagating in slightly different directions.

The following estimate of the angle $\Delta\zeta$ applies when there is a smooth gradient in rotation measure (RM). Consider a planar wave front propagating parallel to the z-axis incident upon a slab of material of thickness Δz, containing spatial variations in the plane transverse to the z-axis. As shown in Appendix A, using the paraxial approximation, the angular deviation of the ray in terms of the refractive index variations on the scattering screen is determined by

$$\frac{d(n_\sigma \; \boldsymbol{\kappa})}{dz} = \frac{\partial n_\sigma}{\partial \boldsymbol{r}} \; , \tag{17}$$

where $\sigma = \pm$ denotes the two wave modes, $\boldsymbol{r} = (x, y)$ is the plane orthogonal to the z-axis, $\boldsymbol{\kappa} = \boldsymbol{k}/k$ is the direction of the wavevector, and the arguments of the refractive index $n_\sigma(\omega, \boldsymbol{\kappa}, z, \boldsymbol{r})$ are suppressed. In the weak anisotropy limit

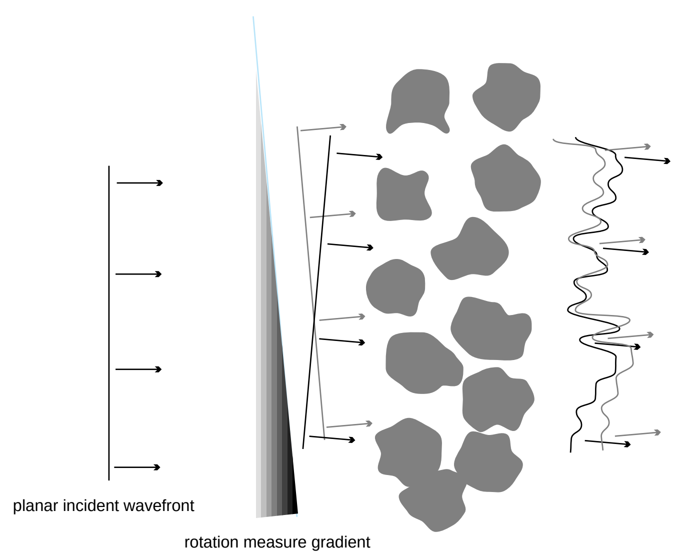

FIG. 3.—Schematic of the Faraday wedge from the viewpoint of physical optics. A RM gradient causes a difference in the ray paths of the left- and right-hand circularly polarized wave fronts. Upon arrival at the Earth, the scintillation pattern of one wave front is slightly displaced with respect to the other, leading to variability in the CP.

one writes the refractive index in terms of polarization-dependent and -independent terms $n_\sigma = n + \sigma\Delta n/2$ with $\Delta n = n_+ - n_-$. For a screen a distance L from the observer, the relative displacement of the centers of the images in the two modes on the observer's plane is $L\Delta\zeta$, with

$$\Delta\zeta = -\frac{\Delta n}{n}\frac{\partial}{\partial r}\int_0^{\Delta z} dz\, n + \frac{\partial}{\partial r}\int_0^{\Delta z} dz\, \Delta n\,. \quad (18)$$

The refractive indices depend on the magnetoionic parameters, $X = (\nu_p/\nu)^2$, $Y = \nu_B/\nu$, where ν_p is the plasma frequency and ν_B is the electron cyclotron frequency. One has $X \ll 1$, $Y \ll 1$ here, with $n = 1 - \frac{1}{2}X$, $\Delta n = XY \cos\theta$, where θ is the angle between the wave-normal direction and $\boldsymbol{B}$. Then the dominant contribution to $\Delta\zeta$ is due to the gradient of $\int_0^{\Delta z} dz\, XY \cos\theta = \Delta\mathrm{RM}\,\lambda^3/2\pi$, where ΔRM is the contribution of the path length Δz to the RM. We also introduce the RM phase, $\phi_{\mathrm{RM}} = \lambda^2$ RM, which is the relative phase difference between the two wave fronts. The displacement between the left- and right-circularly polarized images on the observer's screen reduces to (further details are given in Appendix B)

$$\Delta\boldsymbol{x}(\boldsymbol{r}) = \frac{L\lambda}{2\pi}\nabla\phi_{\mathrm{RM}}\,, \quad (19)$$

where $\nabla_\perp$ denotes the gradient in the x-y plane.

4.2. *Refraction at a Faraday Prism*

An alternative model for a Faraday wedge is a prism. Let the prism have an apex angle of ψ and let its refractive index be $n - n_0$ greater than the refractive index (n_0) in the surroundings. For simplicity, suppose that the prism is oriented nearly perpendicular to the line of sight, so that we are interested only in rays at small angles relative to the direction perpendicular to the axis of the prism (see Fig. 3). Then a ray incident at an angle θ_{in} emerges at an angle θ_{out} given by

$$\theta_{\mathrm{out}} = \theta_{\mathrm{in}} - 2(n - n_0)\tan(\psi/2)\,. \quad (20)$$

The mean angular deviation, $\Delta\theta = \theta_{\mathrm{out}} - \theta_{\mathrm{in}}$, of a ray at the prism and the difference, $\Delta\zeta$, between the emerging ray in the two CPs are then given by

$$\Delta\theta = 2(n - n_0)\tan(\psi/2)\,, \quad \Delta\zeta = 2\Delta n \tan(\psi/2)\,, \quad (21)$$

respectively.

Let us compare the results from equations (18) and (21). Retaining only the final term in equation (18), assuming the region to be uniform along the z-direction and with a uniform gradient with a scale length $L_\perp$ in the perpendicular direction, equation (18) gives $\delta\zeta = \Delta n \Delta z/L_\perp$. Thus the two results coincide for $2\tan(\psi/2) = \Delta z/L_\perp$. More generally, one can conclude that a Faraday wedge leads to a deviation of the rays with opposite CPs by an angle $\Delta\zeta$ which is of the order of the difference, Δn, in the refractive indices of the two modes within the wedge, times a geometric factor [$\Delta z/L_\perp$ or $2\tan(\psi/2)$] which is less than or of order unity.

4.3. *Requirements on the Model*

A Faraday wedge leads to CP due to a displacement, Δx, between ripples in the wave fronts corresponding to opposite CPs. Suppose that the ripples have a scale length r_{scint} and that the modulation index, m_I, of the intensity due to these ripples. Then one should observe fluctuations in CP with an rms degree of CP of

$$\frac{V_{\mathrm{rms}}}{I} \sim m_I \begin{cases} \Delta x/r_{\mathrm{scint}} & \text{for } \Delta x \ll r_{\mathrm{scint}}\,, \\ 1 & \text{for } \Delta x \gtrsim r_{\mathrm{scint}}\,, \end{cases} \quad (22)$$

on a characteristic timescale r_{scint}/v, where v is the speed at which the pattern moves across the observer's plane. For diffractive scattering one has $r_{\mathrm{scint}} \sim r_{\mathrm{diff}}$ and $m_I \sim 1$, and for refractive scattering one has $r_{\mathrm{scint}} \sim r_{\mathrm{ref}}$ and $m_I \sim (r_{\mathrm{diff}}/r_{\mathrm{ref}})^{(\beta-4)/2}$, with $(\beta - 4)/2 = -1/6$ for a Kolmogorov spectrum $\beta = 11/3$. For a source of angular size θ_s to exhibit scintillations requires $\theta < r_{\mathrm{diff}}/L$ and $\theta < r_{\mathrm{ref}}/L$ for diffractive and refractive scattering, respectively.

A semiquantitative estimate for when these conditions are satisfied is deduced from equation (16) as follows. With $\phi_{\mathrm{RM}} = \mathrm{RM}\lambda^2$, we require $\Delta x \gtrsim r_{\mathrm{diff}}$ for diffractive scintillations and $\Delta x \gtrsim r_{\mathrm{ref}}$ for refractive scintillations. Hence we require

$$\frac{\nabla_\perp \mathrm{RM}\lambda^3}{2\pi} \gtrsim \begin{cases} r_{\mathrm{diff}}/L \gtrsim \theta_s & \text{(diffractive)}, \\ r_{\mathrm{ref}}/L \gtrsim \theta_s & \text{(refractive)}. \end{cases} \quad (23)$$

If the angular deviation, $\Delta\zeta$, between the rays in the two modes is dominated by a single structure (single Faraday wedge), which contributes Δ RM to the total RM, then one has $\nabla_\perp \phi_{\mathrm{RM}} \sim \Delta\mathrm{RM}\lambda^2/L_\perp$, where $L_\perp$ is a distance that characterizes the gradient in RM. The condition in equation (23) includes the requirement $\Delta\mathrm{RM}\lambda^2/2\pi L_\perp \gtrsim \theta_s$. Assuming that the Faraday wedge has a density n_e and a magnetic induction B, ΔRM is proportional to $n_e B$ times the line-of-sight distance, Δz through the structure. Then equation (23) reduces to $\mathrm{RM}\lambda^3/2\pi L_\perp \gtrsim \theta_s$, and on inserting numerical values, this becomes

$$\lambda^3\left(\frac{n_e}{1\ \mathrm{m}^{-3}}\right)\left(\frac{B}{1\ \mathrm{G}}\right)\left(\frac{\Delta z}{L_\perp}\right) \gtrsim 10^9\left(\frac{\theta_s}{1\ \mathrm{mas}}\right). \quad (24)$$

Physically, the dependence on $\Delta z/L_\perp$ is due to the requirement that refraction cause a large enough separation between the rays in the two CPs. The region acts like a prism with a small angle at its apex; the deviation of the ray is zero when this angle is zero (the prism reduces to a slab), and the deviation increases as this angle increases.

The condition in equation (24) is not easily satisfied: it requires an exceptionally dense, strongly magnetized region along the line of sight, and it is more easily satisfied at longer wavelengths.

5. APPLICATION TO PULSARS

The degree of CP observed in most pulsars is relatively small (e.g., Han et al. 1999). However, the degree of CP usually quoted refers to the pulse-averaged quantity. Studies of single pulses are possible for a small subset of pulsars, and the CP in single pulses can be very much greater than the pulse-averaged CP (Manchester et al. 1975). Thus the pulse-averaged CP appears to be the mean value of a CP that can vary greatly on a timescale of the order of the pulse period. There is no satisfactory explanation for the pulse-averaged CP (e.g., Radhakrishnan & Rankin 1990; Kazbegi, Machabeli, & Melikidze 1991; Radhakrishnan 1992). It is usually assumed that this is imposed either by the emission process or by propagation in the pulsar magnetosphere. In this section we discuss the possibility that this rapidly varying component in the CP might be due to the Faraday wedge effect associated with diffractive scintillations induced by propagation through the ISM.

5.1. *The Vela Pulsar*

We apply the foregoing ideas to the Vela pulsar, for which there is direct evidence for RM fluctuations. Three parameters are required to estimate the magnitude of the CP generated by the Faraday wedge: the magnitude of the RM gradient, the slope of the power spectrum of density inhomogeneities, β, and the refractive scale, $r_{\rm ref}$. We list our best estimates of these parameters, and then use these to estimate the degree of CP expected according to the foregoing theory.

Hamilton, Hall, & Costa (1985) reported a linear change in RM across the Vela pulsar for the interval 1970–1985. The best fit to the RM gradient is 0.73 rad $\rm m^{-2}\ yr^{-1}$, which translates into a spatial RM gradient of $2.3 \times 10^{-11} v_{\rm km\,s^{-1}}^{-1}$ rad $\rm m^{-3}$, where $v_{\rm km\,s^{-1}}$ is the speed of the wedge transverse to the line of sight in kilometers per second. We assume the Kolmogorov value of $\beta = 11/3$ for the power spectrum of density inhomogeneities for this pulsar. This is consistent with some measurements, although we note that the measurements of refractive flux variations are more consistent with β closer to 3.9 (Johnston, Nicastro, & Koribalski 1998). The size of the scattering disk, as measured by Gwinn et al. 1997, is $\approx$1.0 AU at around 2.3 GHz. (The actual measurement band was from 2.273 to 2.801 GHz.) The refractive length, $r_{\rm ref}$, scales as $\nu^{-[1+2/(\beta-2)]}$.

In estimating the Faraday rotation phase change across the scattering disk, we choose an observing frequency of 600 MHz, where the effect of Faraday rotation is strong. Scaling the refractive length to this frequency, one has $r_{\rm ref} \approx 19$ AU. Combining this with our estimate of the RM gradient, the total RM change across the scattering disk, $r_{\rm ref} \nabla_\perp \phi_{\rm RM}$, is $15.9 v_{\rm km\,s^{-1}}^{-1}$. To calculate the rms degree of CP, we use this estimate of $r_{\rm ref}$ in equation (16) for diffractive scintillation, giving $[\langle V^2 \rangle(z)/\langle I^2 \rangle(0)]^{1/2} = 15\%$ (4%), assuming that $v_{\rm km\,s^{-1}}$ is 100 (500).

We conclude that measurement of scintillation-induced CP is feasible for the Vela pulsar, and that it is likely to be feasible for other pulsars at low frequency. Although they are not discussed in detail here, there are several other pulsars known to exhibit variability in RM, including the Crab pulsar (Rankin et al. 1988) and PSR 1259−63 (Johnston et al. 1996), which exhibits RM variations of $\sim$200 rad $\rm m^{-2}$ on timescales of 0.5 hr. An observational test of the scintillation-induced CP model requires statistics on the variance in the CP in individual pulses, and the way this variance changes with frequency of observation.

6. APPLICATION TO EXTRAGALACTIC SOURCES

A small but significant degree of CP ($\lesssim$0.1% to a few percent) is observed in some compact extragalactic radio sources (e.g., Roberts et al. 1975; Weiler & de Pater 1982; de Pater & Weiler 1982; Komesaroff et al. 1984). The suggested interpretations include the intrinsic polarization associated with synchrotron radiation (Legg & Westfold 1968), and partial conversion of linear into circular polarization due to ellipticity of the natural wave modes of the cold background plasma (Pacholczyk 1973) or of the relativistic electron gas itself (e.g., Sazonov 1969; Jones & O'Dell 1977a, 1977b). However, none of these suggested interpretations has proved satisfactory in accounting for (1) the frequency dependence, (2) the temporal variations, and (3) the magnitude of the observed CP. Here we explore the possibility that the CP observed in compact extragalactic sources is scintillation-induced CP. We consider the requirements for scintillations to produce 0.1% CP due to scintillation either in our Galaxy or in the host object.

6.1. *Extragalactic Sources*

Scintillation-induced CP relies on birefringent refraction, which is determined by the gradient in RM. To estimate the degree of CP using equation (16) requires an estimate of the parameter Δx, which is determined by equation (14) with equation (2). Our estimate for the CP then reduces to

$$\sqrt{\frac{\langle V^2 \rangle(z)}{\langle I^2 \rangle(0)}} \sim \alpha \left(\frac{r_{\rm F}}{r_{\rm diff}}\right)^{\beta-2} , \qquad (25)$$

where we ignore an observed slight excess in the relative fluctuations in RM in our Galaxy compared with the relative fluctuations in the electron density (Minter & Spangler 1996).

We estimate the degree of CP at 5 GHz. For a screen at distance of $L = 1$ kpc, the Fresnel scale is $r_{\rm F} = (\lambda L/2\pi)^{1/2} = 5 \times 10^8$ m. The length scale $r_{\rm diff}$ is estimated from the frequency, ν_t, at which the scattering becomes strong, at $r_{\rm diff} = r_{\rm F}$, and from the fact that $r_{\rm diff}$ scales proportional to $\nu^{2/(\beta-2)}$. We assume a Kolmogorov spectrum of density inhomogeneities, $\beta = 11/3$. For an object located off the Galactic plane one typically has $\nu_t \approx 7$ GHz (Walker 1998), yielding $r_{\rm diff} \approx 3 \times 10^8$ m at 5 GHz. Taking $\langle B_G \cos\theta \rangle = 3 \mu$G, one has $\alpha \approx 2.5 \times 10^{-9}$, which yields $V_{\rm rms} = 6 \times 10^{-9} \langle I^2 \rangle^{1/2}$. Hence, we conclude that the contribution of Galactic RM fluctuations to the CP is negligible.

Next consider RM variations internal to an extragalactic source. The theory (developed in Paper I) needs to be modified slightly if the Faraday wedge is assumed to be located in the host object because the nonplanarity of the wave front cannot be ignored. The modifications for a spherical wave front involve making the substitutions (Goodman & Narayan 1989)

$$z \rightarrow \frac{z_1 z_2}{z_1 + z_2} , \qquad (26)$$

$$\boldsymbol{r} \rightarrow \frac{z_1}{z_1 + z_2} \boldsymbol{r} , \qquad (27)$$

where z_1 is the distance from the source to the scattering screen and z_2 the distance from the screen to the observer. Using equation (4.9) in (5.21) in Paper I, this implies $\langle V^2 \rangle^{1/2} \approx 2^{-1/2} [z_1 \nabla_\perp \lambda^2 {\rm RM}/(2\pi r_{\rm diff}/\lambda)] \langle I^2 \rangle^{1/2}$.

For the sake of discussion we choose the specific object 3C 345, for which there is an estimated dependence (Matveenko et al. 1996) of ${\rm RM} = 3500(R/3.79\ h^{-1}\ {\rm pc})^{-3}$ rad $\rm m^{-2}$ on radial distance R from the core, where $R = 3.79$ h^{-1} pc corresponds to an angular size of 1 mas. Consider the RM gradient required to produce 0.1% CP at 5 GHz. Since the scattering is likely to be concentrated around the AGN core we take an effective distance of $z_1 = 100$ pc between the source and screen and $r_{\rm diff} = 10^7$ m. One then requires $\nabla_\perp {\rm RM} \approx 3.6 \times 10^{-11}$ rad $\rm m^{-3}$. We conclude that variations in the large-scale RM are large enough to produce this degree of CP within the central 0.85 $h^{-3/4}$ pc of the core.

The distance between the source and the screen is assumed to be $z_1 = 100$ pc. There is no direct evidence for this parameter, and our choice is based on a plausibility argument in view of the known scattering parameters of Sgr A*. Sgr A* is an AGN-like object at our own Galactic center which appears to exhibit refractive scintillation (e.g., Zhao & Goss 1993), presumably due to turbulence in a medium ~ 100 pc from the source (Backer 1978). Our plausibility argument is simply that compact extragalactic sources are likely to have some similarities to Sgr A*, and that it is plausible that they scintillate as a result of turbulence in the surrounding interstellar medium ~ 100 pc from the source.

The foregoing estimates apply when the source exhibits diffractive scintillation. CP can also result from refractive scintillation, for which the source size requirement is less stringent. However, the requirement on the separation of the two senses of CP is more stringent: the Faraday wedge must cause a separation that is a significant fraction of the scale on which refractive flux variations occur. Thus, one requires (see eq. [16]) $\langle V^2 \rangle^{1/2} \approx m_{\rm ref}\ r_{\rm diff}\ \lambda^2 \nabla_\perp {\rm RM}$, where $m_{\rm ref} \approx (r_{\rm diff}/r_{\rm F})^{2-\beta/2}$ is the refractive modulation index. For example, the RM gradient in 3C 345 used above is sufficiently large that a centrally located object small enough to exhibit refractive scintillation would produce greater than 0.1% CP at 5 GHz provided the RM gradient is greater than 3×10^{-8} rad m^{-3}. According to the RM model used above, such gradients are encountered within the central 0.15 $h^{-3/4}$ pc of the core.

Similar data available on large RM gradients on milliarcsecond scales near the cores of other extragalactic objects (Roberts et al. 1990; Udomprasert et al. 1997; Cawthorne et al. 1993; Taylor 1998) suggest that CP due to this effect is possible for a broad class of extragalactic sources.

The timescale of variability in the CP depends on the relative speed, v_s, of the screen and the source and on the relative speed, v_o, of the observer and the screen. The speed at which the pattern moves across the observer's plane is $v_s(z_1 + z_2)/z_1$, and the observer moves across this plane at v_o. The pattern scale at the observer is $r_{\rm diff}(z_1 + z_2)/z_1$ for diffractive scintillation and $r_{\rm ref}(z_1 + z_2)/z_1$ for refractive scintillation. The timescale for variations in V is determined by the pattern scale of the oppositely circularly polarized regions divided by the relevant speed. For a point source, diffractive scintillation would be observed on a timescale $r_{\rm diff}/v_s$, and refractive scintillation would be observed on a timescale $r_{\rm ref}/v_s$, where it is assumed that the relevant speed is that of the screen relative to the source. This is the case for $z_1 \ll z_2$, since the apparent speed of the scattering material across the line of sight dominates the motion of the observer across the pattern.

Taking $v_s = 100$ km s^{-1} with the parameters assumed above, the timescale of diffractive variations is 10^2 s and the refractive timescale is 3×10^4 s. Based on these estimates, diffractive variations should exhibit reversals in sign of the CP on a timescale of a few minutes, which does not explain the observed variability over hours to days (Komesaroff et al. 1984). Although the refractive timescale is closer to the observed timescale of CP variations in extragalactic sources, evidence for reversals in the sense of the CP on this timescale is lacking.

7. CONCLUSIONS

It was shown in Paper I that propagation of radio waves through a magnetized inhomogeneous plasma leads to fluctuations in circular polarization (CP), even if the scintillating source is itself unpolarized. Although the mean CP induced by propagation through the medium is zero, the variance, $\langle V^2 \rangle$, is nonzero.

The generation of this CP is attributed to a gradient in RM, called a Faraday wedge, leading to a lateral displacement at the observer's plane of the wave fronts for opposite CPs. Inhomogeneities in the ISM introduce corrugations into the wave front, and the displacement of these at the observer's plane leads to alternate regions of opposite CP. As the ISM moves across the line of sight to a source, an observer samples these alternate patches of opposite CPs. The mean CP is zero, and the variance is nonzero. The effect is significant provided that the lateral displacement of the corrugations in the wave fronts for the opposite CPs is a significant fraction of (or larger than) the typical size of the corrugations. This scintillation-induced CP varies on the timescale associated with the scintillations. For a source undergoing diffractive scintillations, the CP varies on the diffractive timescale, and for a source undergoing refractive scintillations, the CP varies on the refractive timescale.

Pulsars exhibit both diffractive and refractive scintillations, and one expects them to exhibit scintillation-induced CP on a timescale associated with the diffractive scintillation. The observed CP in pulsars can be several tens of percent, but it may be that some of this CP results from birefringence in the pulsar magnetosphere itself, which we do not consider here. Based on data on RM variations associated with the Vela pulsar, we predict that scintillation-induced CP is likely to be observable. We suggest here that the pulse-to-pulse variation in CP observed in some pulsars is due to this effect.

There is a strong case that the small ($\sim$0.5%–0.05%) degree of CP exhibited by some compact extragalactic sources is due to scintillation-induced CP. The high RMs (e.g., Udomprasert et al. 1997) and proposed strong magnetic fields (Rees 1987) near the cores of AGNs are sufficient to generate the observed degree of CP in association with refractive scintillation.

A prediction is that scintillation-induced CP should reverse sign randomly, such that the average value is zero. There is some evidence for this for pulsars, for which the pulse-averaged CP is small compared with the relatively high CP in individual pulses. However, there is no strong evidence for reversals in the sense of CP for extragalactic sources. To determine whether or not this is compatible with the theory requires further consideration of the nature of RM fluctuations. One obvious point concerns the distinction between mean RMs and the variance of the RM. If the length scale over which the magnetic field changes orientation is short compared to the path length through the medium, the variance in the difference between the phases of the left- and right-hand polarized wave fronts may be very much larger than the mean value. This point is exemplified by studies of the ISM which seek to relate the mean electron density along a line of sight to its scattering properties. Based on the variation of $\langle n_e \rangle$ observed in the Galaxy, $\langle (\delta n_e)^2 \rangle$ is predicted to vary by 1.3 orders of magnitude; however, the observed variation is considerably larger at 4.2 orders of magnitude (Cordes, Weisberg, & Borkiakoff 1985).

For the same reason, the mean RM along a given line of sight may not reflect the degree of scintillation-induced CP expected.

The generation of CP due to ultrafine-scale structure in the magnetic field also needs to be addressed. If the observed density fluctuations in the ISM are the result of magnetohydrodynamic turbulence, one expects similarly fine-scale structure in the magnetic field. This also is expected to boost the importance of scintillation-induced CP in the ISM. Further detailed modeling of this effect is warranted.

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APPENDIX A

EVALUATION OF $g(\beta)$

We wish to evaluate the integral

$$\langle D(r_{ij})\rangle = \int d^2\boldsymbol{x}_i\, d^2\boldsymbol{x}_j\, p(\boldsymbol{x}_i) p(\boldsymbol{x}_j) \left(\frac{|\boldsymbol{x}_i - \boldsymbol{x}_j|}{r_{\rm diff}}\right)^{\beta-2}$$

$$= \left(\frac{1}{r_{\rm diff}}\right)^{\beta-2} \int dx_i\, dx_j\, dy_i\, dy_j\; p(x_i)p(x_j)p(y_i)p(y_j)[(x_i - x_j)^2 + (y_i - y_j)^2]^{(\beta-2)/2} . \quad \text{(A1)}$$

Using the expansion

$$(x + y)^\gamma = \sum_{i=0}^{\infty} \left[\prod_{j=0}^{i-1} (\gamma - j)\right] x^{\gamma - i} y^i , \quad \text{(A2)}$$

equation (A1) becomes a series of products of two-dimensional integrals:

$$\langle D(r_{ij})\rangle = \left(\frac{1}{r_{\rm diff}}\right)^{\beta-2} \sum_{m=0}^{\infty} \left[\prod_{k=0}^{m-1} \left(\frac{\beta-2}{2} - k\right)\right] \int dx_i\, dx_j\; p(x_i)p(x_j)(x_i - x_j)^{\beta-2-2m} \int dy_i\, dy_j\; p(y_i)p(y_j)(y_i - y_j)^{2m} . \quad \text{(A3)}$$

An observer sees speckles over the entire scattering disk of radius $r_{\rm ref}$, and this provides a cutoff to the distribution $p(\boldsymbol{x}_i)$. For the purposes of calculation, we approximate the outer boundary of the speckle pattern as a square of area $r_{\rm ref}$. We then have

$$p(\boldsymbol{x}_i) = \begin{cases} 1/r_{\rm ref}^2 & |x_i| < r_{\rm ref}/2, |y_i| < r_{\rm ref}/2, \\ 0 & \text{otherwise.} \end{cases} \quad \text{(A4)}$$

Making the substitutions $u_x = x_i - x_j$, $u_y = y_i - y_j$, $v_x = x_i + x_j$ and $v_y = y_i + y_j$, we then have

$$\langle D(r_{ij})\rangle = \frac{1}{4 r_{\rm ref}^4} \left(\frac{1}{r_{\rm diff}}\right)^{\beta-2} \sum_{m=0}^{\infty} \left[\prod_{k=0}^{m-1} \left(\frac{\beta-2}{2} - k\right)\right] \int_A du_x\, dv_x\; u_x^{\beta-2-2m} \int_A du_y\, dv_y\; u_y^{2m} , \quad \text{(A5)}$$

where the integration area is taken into account as follows:

$$\int_A du_x\, du_y = \int_0^{r_{\rm ref}} du_x \int_0^{r_{\rm ref}-u} dv_y + \int_0^{r_{\rm ref}} du_x \int_{u-r_{\rm ref}}^0 dv_y + \int_{-r_{\rm ref}}^0 du_x \int_{-r_{\rm ref}-u}^0 dv_y + \int_{-r_{\rm ref}}^0 du_x \int_0^{r_{\rm ref}+u} dv_y . \quad \text{(A6)}$$

Using

$$\int_A du_x\, dv_x\; u_x^{\beta-2-2m} = \frac{4 r_{\rm ref}^{\beta-2m}}{(\beta - 2m)(\beta - 2m - 1)} , \quad \text{(A7)}$$

provided $\beta - 2m \neq 1$ and $\beta - 2m \neq 0$, and

$$\int_A du_y\, dv_y\; u_y^{2m} = \frac{4 r_{\rm ref}^{2+2m}}{(2 + 2m)(2m + 1)} , \quad \text{(A8)}$$

we obtain

$$\langle D(r_{ij})\rangle = 4\left(\frac{r_{\rm ref}}{r_{\rm diff}}\right)^{\beta-2} \sum_{m=0}^{\infty} \left[\prod_{k=0}^{m-1} \left(\frac{\beta-2}{2} - k\right)\right] \left[\frac{1}{(2m+2)(2m+1)}\right] \times \left[\frac{1}{(\beta - 2m)(\beta - 2m - 1)}\right] \quad \text{(A9)}$$

$$\equiv g(\beta) \left(\frac{r_{\rm ref}}{r_{\rm diff}}\right)^{\beta-2} . \quad \text{(A10)}$$

APPENDIX B

DERIVATION OF RAY-BENDING DUE TO A FARADAY WEDGE

Here we derive the result of the ray-bending due to the Faraday wedge model discussed in § 4. The relative displacement of the right- and left-polarized wave fronts is derived using Hamiltonian equations for a ray. These are

$$\frac{d\boldsymbol{x}}{dt} = \frac{\partial \omega_\sigma(\boldsymbol{k}, \boldsymbol{x}, t)}{\partial \boldsymbol{k}}, \qquad \frac{d\boldsymbol{k}}{dt} = -\frac{\partial \omega_\sigma(\boldsymbol{k}, \boldsymbol{x}, t)}{\partial \boldsymbol{x}}, \qquad \frac{d\omega}{dt} = \frac{\partial \omega_\sigma(\boldsymbol{k}, \boldsymbol{x}, t)}{\partial t}. \tag{B1}$$

In a stationary medium, assumed here, the third equation is not relevant. In the paraxial approximation one separates $\boldsymbol{x}$ into z, $\boldsymbol{r}$, with the z-axis along $\boldsymbol{k} = k\boldsymbol{\kappa}$. The relation $k = n\omega/c$ is used after combining the first and second of equations (B1) to yield an expression for the angular deviation of the ray in terms of the refractive index variations on the scattering screen,

$$\frac{d(n_\sigma \boldsymbol{\kappa})}{dz} = \frac{\partial n_\sigma}{\partial \boldsymbol{r}}, \tag{B2}$$

where the arguments of $n_\sigma(\omega, \boldsymbol{\kappa}, z, \boldsymbol{r})$ are suppressed. The change, $\delta\boldsymbol{\kappa}_\sigma$, across a wedge of thickness Δz is given by

$$\delta\boldsymbol{\kappa}_\sigma = \frac{1}{n_\sigma}\int_0^{\Delta z} dz\, \frac{\partial n_\sigma}{\partial \boldsymbol{r}}. \tag{B3}$$

In the weak anisotropy limit one writes the refractive index in terms of polarization-dependent and -independent terms $n_\sigma = n + \sigma\Delta n/2$, with $\Delta n = n_+ - n_-$, to simplify this to

$$\delta\boldsymbol{\kappa}_\sigma = \frac{1}{n}\int_0^{\Delta z} dz\left[\left(1 - \frac{\sigma\Delta n}{2n}\right)\frac{\partial n}{\partial \boldsymbol{r}} + \frac{\sigma}{2}\frac{\partial \Delta n}{\partial \boldsymbol{r}}\right], \tag{B4}$$

neglecting terms of order $(\Delta n)^2$. Thus, if the screen is a distance L from the observer, the relative displacement of the centers of the images in the two modes on the observer's plane is $L\Delta\zeta$, as given by equation (18).

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