Particle acceleration by a fast ordinary mode in an electron–positron plasma

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The possibility of nonresonant particle acceleration in an electron–positron plasma of a pulsar magnetosphere is investigated. A mechanism is proposed in which modulations of a fast superluminal (with phase velocity exceeding the speed of light) longitudinal ordinary mode (caused by a beat wave of two transverse electromagnetic waves propagating along the magnetic field) stimulate nonresonant quasilinear diffusion leading to a redistribution of plasma particles in pitch angle. The resulting perpendicular momenta of the particles lead to synchrotron radiation which is in the γ-ray range. © *2000 American Institute of Physics.* [S1070-664X(00)01304-5]

I. INTRODUCTION

The most developed models of a pulsar magnetosphere[1–3] are based on the idea proposed in Ref. 4 and the hypothesis of Ref. 5 that the plasma is formed by electrons and positrons. The e^-e^+ pairs are created by γ quanta in the strong external magnetic field $\mathbf{B}_0$. The perpendicular (with respect to the field $\mathbf{B}_0$) component of the momentum of the created particles rapidly disappears due to synchrotron losses, and the particles move in one dimension along the magnetic field lines.

Notable features of the pulsar magnetospheric plasma are the absence of gyrotropy (because the masses of electrons and positrons are equal), the relativistic character of particle motion, and the one-dimensional particle distribution function. The plasma particles move with relativistic velocities along the external magnetic field lines and can generate the natural modes of such a plasma. It is thought that pulsar radio emission results from the relativistic particles exciting the natural wave modes of the plasma, with some mechanism allowing conversion into escaping radiation. Thus most of the properties of the radio emission are formed in the pulsar magnetosphere.

Naturally, in the magnetospheric plasma the electromagnetic modes generated include not only those that can leave the magnetosphere, but also include Langmuir waves. In this paper we are concerned with nonlinear effects associated with superluminal (phase speed $v_{ph}>c$) Langmuir modes that can result from nonlinear wave–wave interactions in the plasma. These waves experience no Landau damping, so with even a modest generation mechanism their energy density can easily build up to a level where nonlinear processes become possible. Of particular interest for us here is the possibility that the nonlinear processes lead to pitch-angle scattering of the relativistic particles. If the pitch angle is such that the perpendicular momentum of the particles greater exceeds mc (where m is their mass), then they radiate synchrotron radiation, which produces the high-energy radiation. Thus we are concerned with the possibility that nonlinear processes allow the energy stored in superluminal Langmuir waves to be finally converted into very high energy photons through this sequence of processes.

To describe nonlinear wave processes in a pulsar magnetosphere, it is not sufficient to use only the approximation of weak collective fields $|E|^2 \ll mc^2 n\gamma$, where m is the electron (positron) mass, $n \leqslant 10^{15}-10^{16}\,\mathrm{cm}^{-3}$ is the number density of the main plasma particles, and $\gamma = 1/\sqrt{1-v^2/c^2}$ is the Lorentz factor; it is also necessary to take into account the (small) parameter of the drift approximation $(\omega_{pe}/\omega_B) \ll 1$, where $\omega_{pe}=(4\pi ne^2/m)^{1/2}$ is the electron plasma frequency, $\omega_B = eB_0/mc$ is the cyclotron frequency, and $B_0 \simeq 10^{12}$ G is the pulsar's magnetic field near the surface of the neutron star. The magnetic field, assumed to be dipolar, decreases away from the star proportional to the cube of the distance. Thus the parameter ω_p/ω_B increases with the increasing distance, but it is much less than unity in the whole magnetospheric region confined by the light cylinder (the latter is the surface on which the velocity of particles corotating with the magnetic field is equal to the speed of light).

We are concerned with nonlinear processes for slightly oblique waves [i.e., waves propagating with small angle $\Theta \sim k_\perp/k_z$ with respect to the magnetic field $\mathbf{B}_0=(0,0,B_0)$]. In the reference frame of the observer, this approximation is quite general since the plasma of interest moves toward the observer with Lorentz factor $\gamma_p \gg 1$, and the angle of the observed radiation can be estimated as $\Theta \leqslant 1/2\gamma_p$. It is well known that the decay $l \to l' + l''$ of a Langmuir wave into two Langmuir waves is forbidden in such a plasma; it was demonstrated[6] that the decay $l \to 2t$ into two transverse waves is possible (as well as the reverse process; the latter was applied in Ref. 7 to explain the pulsar radio emission). Three wave decay processes and calculation of the wave spectra were investigated in Ref. 8, and nonlinear scattering of Langmuir waves on plasma particles (applied to pulsar magnetosphere) was considered in Ref. 9. Analysis of these processes shows that in pulsar magnetospheres there is ex-

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cessive energy stored in the superluminal Langmuir waves. Conversion of the wave energy into energy of plasma particles and possible consequences of the process is the subject of the present investigation.

The paper is organized as follows: In Sec. II, we briefly review the main results of the linear theory of collective wave and particle processes in a relativistic electron–positron plasma; the motion of a test particle is investigated in Sec. III; Sec. IV is devoted to a study of the dynamics of the nonlinear waves, and in Sec. V we consider particle acceleration and the possibility of increasing pitch-angles because of the nonresonant interaction, resulting in synchrotron radiation. Application of the results to the observed features of the pulsar γ-emission is discussed in Sec. VI.

II. WAVES IN PULSAR MAGNETOSPHERES

The curvature of the pulsar magnetic field can be neglected for wave propagation at wave lengths λ which satisfy the inequality

$$\frac{\lambda}{B}\frac{\partial B}{\partial r}\ll 1, \tag{1}$$

i.e., when the wave length is much less than the scale length of the inhomogeneity of the magnetic field. We may then introduce Cartesian coordinates with the z-axis directed along the magnetic field, and the x-axis directed along the perpendicular component $\mathbf{k}_\perp$ of the wave vector $\mathbf{k}$. We assume the plasma being cold and nonrelativistic in its rest frame. The dispersion equation factorizes into two independent equations. One describes a purely transverse electromagnetic mode with the dispersion[10]

$$\omega_x = kc\left(1-\frac{1}{8}\frac{\omega_p^2}{\omega_B^2}\frac{1}{\gamma_0^3}\right), \tag{2}$$

where $\omega_p=[4\pi(n_e+n_{e+})e^2/m]^{1/2}=\sqrt{2}\omega_{pe}$ is the "combined" plasma frequency (i.e., taking into account contributions of plasma electrons and positrons), and γ_0 is the Lorentz factor of plasma particles moving with velocity v_0 along the field lines. Below, we consider a strongly magnetized relativistic plasma, $\omega_p \ll \tilde{\omega}_B$, where $\tilde{\omega}_B=\omega_B/\gamma_0$.

The second factor of the dispersion equation describes two modes of mixed longitudinal-transverse character. The lower-frequency subluminal mode is analogous to the Alfvén wave, and the higher-frequency mode which can be superluminal, $v_{ph}>c$, is the ordinary o-mode. Expressions for the dispersion of these modes can be easily written in some limits. We consider the case $kc\ll\omega_p$ for waves propagating almost parallel to the magnetic field $|\mathbf{k}_\perp|\ll k_z$. Thus for the o-mode we have

$$\omega_o^2 \simeq \frac{\omega_p^2}{\gamma_0^3}+3k_z^2c^2+|\mathbf{k}_\perp|^2c^2. \tag{3}$$

Graphically, the dispersion relations (2) and (3) are presented in Fig. 1.

For parallel propagation there is also a point $\omega_p \approx k_0 c$, see Fig. 2, where the dispersion relations for all three modes coincide and proper consideration of their nonlinear properties must take this into account. The electric field of the x-mode is perpendicular to the plane of vectors $\mathbf{k}$ and $\mathbf{B}$; the electric fields of the o-mode and Alfvén mode are in the plane. For frequencies close to the plasma frequency ω_p: the latter modes are almost longitudinal, and for frequencies far from ω_p, both modes are almost transverse.

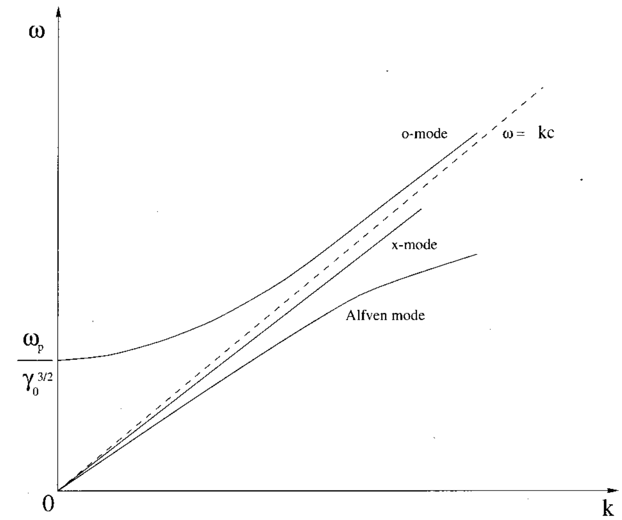

FIG. 1. Spectra of waves in an electron-positron plasma for oblique propagation.

Low-frequency modes analogous to the ion-acoustic wave in an electron-ion plasma, are absent in an electron-positron plasma. Thus when considering nonlinear effects in the wave propagation, the only possibility for amplitude modulations of the o-mode is due to nonresonant excitation of a beat wave. This was first pointed out in Ref. 11, see also Refs. 12–14, and considered in detail in Ref. 15, where the possibility of modulations of the superluminal o-mode by

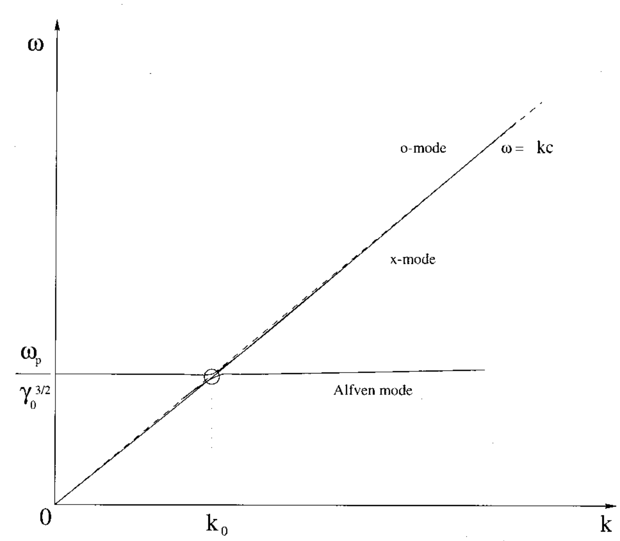

FIG. 2. Spectra of waves in an electron-positron plasma for parallel propagation.

transverse waves investigated (by ''transverse'' in Ref. 15 implies not only the x-mode, but also the higher-frequency, compared with ω_p, o-mode, when its dispersion is close to the vacuum case). In this interaction, the longitudinal superluminal component of the perturbation appears as a result of the interaction of two transverse waves with

$$\left|\frac{\omega^t-\omega'^t}{k_z-k_z'}\right|>c. \tag{4}$$

Substituting (2) into (4), and using $|\mathbf{k}|\simeq k_z(1-\mathbf{k}_\perp^2/2k_z^2)$, this inequality implies

$$1-\frac{1}{2}\left[\frac{\mathbf{k}_\perp^2}{k_z(k_z-k_z')}-\frac{\mathbf{k}_\perp'^2}{k_z'(k_z-k_z')}\right]>1. \tag{5}$$

To satisfy (5) for $k_z>k_z'$, it is required that $|\mathbf{k}_\perp'|>|\mathbf{k}_\perp|$ as well as

$$\frac{|\mathbf{k}_\perp'|}{k_z}\frac{|\mathbf{k}_\perp'|-|\mathbf{k}_\perp|}{k_z-k_z'}>1. \tag{6}$$

III. MOTION OF PARTICLES

The equation of motion for a test particle moving together with the plasma (i.e., in our reference frame $p_{0z}=0$) in the external magnetic field and the fields of the incident and scattered waves is

$$\frac{d\mathbf{p}}{dt}=e\left[\mathbf{E}+\mathbf{E}'+\frac{1}{c}\mathbf{v}\times(\mathbf{B}_0+\mathbf{B}+\mathbf{B}')\right]. \tag{7}$$

There are two small parameters in the problem. First, there is the smallness of the wave fields as compared to the external magnetic field, $(\mathbf{E},\mathbf{E}',\mathbf{B},\mathbf{B}')\ll\mathbf{B}_0$. Second, there is the smallness of the wave energies compared to the plasma particle thermal energy, $(|\mathbf{E}|^2,|\mathbf{E}'|^2)\ll mc^2n\gamma_T$. Furthermore, we split the test particle momentum into three parts,

$$\mathbf{p}=\mathbf{p}_{0\perp}+\mathbf{p}_1+\mathbf{p}_2, \tag{8}$$

where $\mathbf{p}_{0\perp}$ corresponds to the unperturbed motion of the particle in the external magnetic field $\mathbf{B}_0$, $\mathbf{p}_1\ll\mathbf{p}_{0\perp}$ is the linear perturbation of $\mathbf{p}_{0\perp}$ due to the waves, and $\mathbf{p}_2\ll\mathbf{p}_1$ is the nonlinear perturbation of the particle motion.

Expansion of the Lorentz-factor $\gamma=[1+(\mathbf{p}_{0\perp}+\mathbf{p}_1)^2/m^2c^2]^{1/2}$ to first order gives

$$\frac{\mathbf{v}}{c}\simeq\frac{\mathbf{p}_{0\perp}}{mc\gamma_{0\perp}}+\frac{\mathbf{p}_1}{mc\gamma_{0\perp}}-\frac{(\mathbf{p}_0\cdot\mathbf{p}_1)\mathbf{p}_0}{mc\gamma_{0\perp}^3}, \tag{9}$$

where $\gamma_{0\perp}=[1+\mathbf{p}_{0\perp}^2/m^2c^2]^{1/2}$. After substitution of this expression into Eq. (7), which is expanded in the same way, we find a system of coupled equations for $\mathbf{p}_{0\perp}$, $\mathbf{p}_1$, and $\mathbf{p}_2$. In the zeroth approximation, we have

$$\frac{d\mathbf{p}_{0\perp}}{dt}=\frac{e}{mc}\mathbf{p}_{0\perp}\times\mathbf{B}_0. \tag{10}$$

If the test particle is an electron, the solution of (10) is

$$p_{0x}=p_{0\perp}\cos\tilde{\omega}_Bt,\quad p_{0y}=-p_{0\perp}\sin\tilde{\omega}_Bt, \tag{11}$$

where $\tilde{\omega}_B=\omega_B/\gamma_{0\perp}$. For a positron, the sign of the y-component of the momentum is opposite.

In the first approximation, the equation of motion can be written in the form

$$\frac{dp_{1x}}{dt}=e(E_x+E_x')+\tilde{\omega}_B(p_{1y}\cos\tilde{\omega}_B+p_{1x}\sin\tilde{\omega}_B)\cos\tilde{\omega}_B, \tag{12}$$

$$\frac{dp_{1y}}{dt}=e(E_y+E_y')-\tilde{\omega}_B(p_{1x}\sin\tilde{\omega}_B+p_{1y}\cos\tilde{\omega}_B)\sin\tilde{\omega}_B. \tag{13}$$

The solution of (12) and (13) is given by

$$p_{1x}=\frac{e|\mathbf{E}_\perp|\sin\omega t}{2(\omega+\tilde{\omega}_B)}+\frac{e|\mathbf{E}_\perp'|\sin\omega' t}{2(\omega'+\tilde{\omega}_B)}, \tag{14}$$

$$p_{1x}=-\frac{e|\mathbf{E}_\perp|\sin\omega t}{2(\omega+\tilde{\omega}_B)}+\frac{e|\mathbf{E}_\perp'|\sin\omega' t}{2(\omega'+\tilde{\omega}_B)}. \tag{15}$$

We also assume that

$$E_x=|\mathbf{E}_\perp(\mathbf{r},t)|\cos\omega t,\quad E_y=|\mathbf{E}_\perp(\mathbf{r},t)|\sin\omega t, \tag{16}$$

as well as

$$E_x'=|\mathbf{E}_\perp'(\mathbf{r},t)|\cos\omega' t,\quad E_y'=-|\mathbf{E}_\perp'(\mathbf{r},t)|\sin\omega' t, \tag{17}$$

where the wave amplitudes $\mathbf{E}_\perp^{(\prime)}(\mathbf{r},t)$ are slow functions of position and time. For positrons, we have similar solutions with the change $e\to-e$ and therefore $\omega_B\to-\omega_B$ as well.

Taking into account that the parallel component of $\mathbf{p}_2\times\mathbf{B}_0$ is zero, we have

$$\frac{dp_{2z}}{dt}=\frac{e}{mc\gamma_{0\perp}}[\mathbf{p}_1\times(\mathbf{B}+\mathbf{B}')]_z. \tag{18}$$

From Maxwell's equations, we have

$$B_x=\frac{E_y}{\cos\Theta},\quad B_y=-\frac{E_x}{\cos\Theta}, \tag{19}$$

where we use the wave dispersion equation $\omega\simeq|\mathbf{k}|c$ and introduce the angle Θ between the external magnetic field and the wave vector: $\cos\Theta=k_z/|\mathbf{k}|$. Substituting (14), (15), and (19) into Eq. (18), we obtain the nonlinear longitudinal perturbation equation

$$\frac{dp_{2z}}{dt}=\frac{e^2\mathbf{E}_\perp^2}{mc\omega_p}\sin(\Delta\omega t)\left[\frac{\omega_p}{\tilde{\omega}_B+\omega}\frac{1}{\cos\Theta}-\frac{\omega_p}{\tilde{\omega}_B+\omega'}\frac{1}{\cos\Theta'}\right], \tag{20}$$

where we introduce $\Delta\omega=\omega^t-\omega'^t$. The electric field E_{2z} is the result of the nonlinear interaction of the waves t and t' with the plasma particles.

Consider the limit $\omega^t\gg\tilde{\omega}_B$ (note we have $\omega_p\ll\tilde{\omega}_B$). In this case plasma particles do not have time to complete one Larmor cycle, and generation of the wave t' is due to reradiation of the wave t by the particle whose unperturbed motion is effectively rectilinear. For simplicity, we do not consider the possibility of generation on higher cyclotron harmonics. In this case, the frequency of the radiated wave ω'^t is close to the frequency of the incident wave ω^t. For $\cos\Theta\simeq\cos\Theta'\approx1$ and $|\mathbf{E}_\perp|\approx|\mathbf{E}_\perp'|$, from Eq. (20) we obtain in the dimensionless variables

$$E\rightarrow\frac{eE}{mc\omega_p},\quad \mathbf{r}\rightarrow\frac{\omega_{*0}\mathbf{r}}{c},\quad t\rightarrow\omega_{*0}t,\quad p\rightarrow p/mc \tag{21}$$

the following equation:

$$\frac{dp_2}{dt}=\frac{\omega_{*0}\widetilde{\omega}_B}{\omega^2}\frac{|\mathbf{E}_\perp^t|^2}{\gamma_{0\perp}}\sin(\Delta\omega t), \tag{22}$$

where $\omega_{*0}=\omega_p/\gamma_{0\perp}^3$.

For further convenience, we rewrite Eq. (22) as

$$\frac{dp_2}{dt}=a\sin(\Delta\omega t), \tag{23}$$

where $\Delta\omega=(\omega^t-\omega'^t)/\omega_{*0}$,

$$a=\frac{\omega_{*0}\widetilde{\omega}_B}{\omega^2}\frac{|\mathbf{E}_\perp^t|^2}{\gamma_{0\perp}}. \tag{24}$$

From Eq. (23), we have the solution

$$p_{2z}(t)=\frac{a}{\Delta\omega}(1-\cos\Delta\omega t). \tag{25}$$

IV. NONLINEAR DYNAMICS OF THE *O*-MODE

Detailed investigation of the instability leading to the nonlinear modulation of the *o*-mode by two high-frequency transverse waves was done in Ref. 15, where we refer the interested reader. Here, we briefly remind the reader of the main results of the study.

Force (22) demonstrates nonlinear coupling of longitudinal components E^l with transverse components E^t. To describe the nonlinear dynamics, we start from Maxwell equations and consider a wave packet propagating at a small angle with respect to the external magnetic field. Separating the low-frequency and high-frequency transverse components, $\mathbf{E}=\mathbf{E}^l+\mathbf{E}^t$, where $E_z^t=0$, we assume $\omega^t\gg\omega^l$.

Introducing slowly changing wave amplitudes and density perturbations δn, we obtain

$$\frac{\partial E_{(x,y)}^t}{\partial t}=0, \tag{26}$$

and

$$\begin{aligned}&2i\frac{\omega^l}{\omega_{*0}}\frac{\partial E_{(x,y)}^l}{\partial t}-i\frac{k_0^lc}{\omega_{*0}}\frac{\partial E_z^l}{\partial(x,y)}=\frac{\delta n}{n_0}E_{(x,y)}^l,\\ &2i\frac{\omega^l}{\omega_{*0}}\frac{\partial E_z^l}{\partial t}-i\frac{k_0^lc}{\omega_{*0}}\left(\frac{\partial E_x^l}{\partial x}+\frac{\partial E_y^l}{\partial y}\right)=\frac{\delta n}{n_0}E_z^l.\end{aligned} \tag{27}$$

Equations (26) are written in the zeroth approximation for the expansion in the parameter ω^l/ω^t. The nonlinear terms on the right-hand sides of Eqs. (27) are determined by the amplitudes of the high-frequency waves $E_\perp^t$.

The expression for $\delta n/n_0$ is found by averaging the continuity equation over the high frequency

$$\frac{\partial}{\partial t}\frac{\delta n}{n_0}=\frac{\partial}{\partial z}p_{2z}. \tag{28}$$

Note that $\partial\delta n/\partial x=\partial\delta n/\partial y=0$, and p_{2z} is defined by (25).

Excluding the term with E_z^l, we find from the first equation of the system (27)

$$\frac{\partial}{\partial t}(\nabla\times\mathbf{E}^l)_z=\frac{\delta n}{n_0}(\nabla\times\mathbf{E}^l)_z. \tag{29}$$

Thus we obtain

$$(\nabla\times\mathbf{E}^l)_z=C\exp\left[\int^t\left(\frac{\delta n}{n_0}\right)dt\right], \tag{30}$$

where C is a constant. Since there is an instability of the curl field, below we assume that $\nabla_\perp\cdot\mathbf{E}_\perp^l=0$. Thus in the presence of the density perturbation there is exponential growth of the transverse fields. We note that in the drift approximation the density modulation as well as the change of the momentum p_2, caused by the high-frequency fields E^t (when $\mathbf{k}\|\mathbf{B}_0$), are parallel to the axis $\mathbf{z}\|\mathbf{B}_0$. However, the growth of the fields due to the parallel density modulations may be in any direction: $E_{(x,y,z)}^l\propto\exp(-ik_zz-i\mathbf{k}_\perp\cdot\mathbf{r}_\perp)$.

To obtain an equation for the parallel component E_z^l, we find the mixed derivatives $\partial^2E_{(x,y)}^l/\partial t\partial(x,y)$ from the first equation of (27). Thus differentiating the second equation of (27) with respect to time (i.e., applying $\partial/\partial t$), substituting the resulting expression for the mixed derivatives, and assuming $\nabla_\perp\cdot\mathbf{E}_\perp^l=0$ (which implies that there are no components of the potential electric field perpendicular to the external magnetic field), we obtain

$$\frac{\partial^2E_z^l}{\partial t^2}-K_0^2\Delta_\perp E_z^l+\frac{i}{2}\frac{\partial}{\partial t}\left(\frac{\delta n}{n_0}E_z^l\right)=0, \tag{31}$$

where $K_0=k_0^lc/2\omega_{*0}$. The nonlinear term is proportional to $(E_\perp^t)^2E_z^l$, in our case, we have $(E_\perp^t)^2E_z^l\gg|E_z^l|^3$.

The dispersion equation is given by

$$\omega^2-K_0^2k_\perp^2+\frac{k_za}{\Delta\omega}=0. \tag{32}$$

Thus the instability is possible when

$$\frac{k_za}{\Delta\omega}>K_0^2k_\perp^2. \tag{33}$$

In the approximation considered, the aperiodic growth of the longitudinal potential field is not accompanied by a density modulation since the latter, as the continuity equation (28) implies, is determined by the high-frequency transverse fields. The energy of the high-frequency *t*-modes is assumed to be maintained by external sources, which is reasonable for the plasma in a pulsar magnetosphere where excitation of the transverse modes should be very effective.[16–18]

From Eq. (28), we also find that

$$\begin{aligned}\frac{\delta n}{n_0}&=-i\frac{k_za}{\Delta\omega}\int_0^t(1-\cos\Delta\omega t)dt'\\ &=-i\frac{k_za}{\Delta\omega}\left(t-\frac{1}{\Delta\omega}\sin\Delta\omega t\right).\end{aligned} \tag{34}$$

Substituting this equation into (30) and using the expansion

$$\exp(i\alpha\sin\Delta\omega t)=\sum_{s=-\infty}^{+\infty}J_s(\alpha)e^{is\Delta\omega t}, \tag{35}$$

we obtain

$$(\nabla\times\mathbf{E}^l)_z=C\sum_{s=-\infty}^{+\infty}J_n\left(\frac{k_za}{(\Delta\omega)^2}\right)\exp\left[-i\left(\frac{k_za}{(\Delta\omega)^2}-s\right)\Delta\omega t\right]. \tag{36}$$

From the latter expression, we see that (averaging over sufficiently long time interval $T\gg 1/\Delta\omega$) the transverse perturbation generated by the density modulation is not zero when

$$s=\frac{k_za}{(\Delta\omega)^2}. \tag{37}$$

In this case, $(\nabla\times\mathbf{E}^l)_z$ is determined by the Bessel function with equal index and argument, i.e., $J_\nu(\nu)\sim\nu^{-1/3}$.[19]

V. PARTICLE ACCELERATION

According to the model,[4] a rotating neutron star with corotating magnetic field generates an electric field with a component along $\mathbf{B}_0$. The electric field ''pulls'' primary particles out of the surface of the star and accelerates them up to relativistic velocity. In the curved magnetic field of the pulsar magnetosphere, the relativistic particles emit γ-quanta. When the energy of the γ-quanta exceeds $2mc^2$, pair production starts, $\gamma+B\rightarrow e+e^++B$, and populates the polar-cap regions with secondary pairs. The region where the pair production takes place (and the electric field is effectively screened), is called the pair production front (PPF).[3] The secondary pair plasma moves along the magnetic field lines with relativistic velocity; the primary relativistic beam moves through it. It is assumed that the energy density of the secondary plasma and the primary beam are comparable, $n_p\gamma_p\approx n_b\gamma_b$. The beam density is of the order of the Goldreich–Julian density $n_b\sim n_{GJ}\approx 10^{12}\,\mathrm{cm}^{-3}$, and $\gamma_b\simeq 3\times 10^6$.

As demonstrated above, the instability of superluminal Langmuir waves can develop on density modulations induced by the beat of two transverse waves. The developed modulational perturbation is of ''cigar''-like type, $|\mathbf{k}_\perp-\mathbf{k}'_\perp|>|k_z-k'_z|$. Thus the energy of the Langmuir waves is converted into low-frequency perturbations $\omega-\omega'$. The ponderomotive force acts on plasma particles in the parallel as well as in the perpendicular dierections, and there is nonresonant interaction of the unstable fields with plasma particles. To describe the interaction, we write the nonresonant quasilinear equation[20] which, e.g., was used in Ref. 10 for the relativistic case in the approximation

$$\frac{k_\perp p_\perp}{m\omega_B}\ll 1. \tag{38}$$

We have (back in dimensional variables)

$$\begin{aligned}\frac{\partial f}{\partial t}=&\frac{\partial}{\partial p_z}\left[\sum_k\frac{e^2}{2}|E_{kz}|^2\delta(\omega_k-k_zv_z)\frac{\partial f}{\partial p_z}\right]\\&+\left(\frac{\partial}{\partial t}\frac{E_\perp^2}{4B_0^2}\right)\frac{p_z^2}{p_\perp}\frac{\partial}{\partial p_\perp}p_\perp\frac{\partial f}{\partial p_\perp}+\left(\frac{\partial}{\partial t}\frac{B_\perp^2}{4B_0^2}\right)\\&\times\left(p_\perp^2\frac{\partial^2 f}{\partial p_z^2}+\frac{p_z^2}{p_\perp}\frac{\partial}{\partial p_\perp}p_\perp\frac{\partial f}{\partial p_\perp}-2p_\perp\frac{\partial}{\partial p_z}p_z\frac{\partial f}{\partial p_\perp}\right),\end{aligned} \tag{39}$$

where $E_\perp^2=\Sigma_k(|E'_{kx}|^2+|E'_{ky}|^2)$ and $B_\perp^2=\Sigma_k(|B'_{kx}|^2+|B'_{ky}|^2)$.

We should remark on the assumption that $p_\perp$ may be treated as a continuous variable when, for some other purposes, the quantization of the perpendicular motion in the strong magnetic field needs to be taken into account. The quantization involves discrete values of $p_\perp$ separated by $\sim mc(B/B_c)$, with $B_c=4\times 10^{13}\,$G. We are interested in the case $p_\perp\gg mc$, when synchrotron radiation is possible, and then for $(B/B_c)\ll 1$ the quantization of the energy levels is unimportant and $p_\perp$ may be regarded as a continuous variable.

Because the initial distribution function is almost one-dimensional, $p_\perp\ll p_\parallel$, we assume that in (39) $(\partial/\partial p_\perp)\gg(\partial/\partial p_z)$. Taking into account rapid change of the distribution function, assuming the magnetic fluctuations to be dominant, and keeping the largest contribution to the averaged (over momenta, see below) quantities, we obtain

$$\frac{\partial f}{\partial t}\approx\left(\frac{\partial}{\partial t}\frac{B_\perp^2}{4B_0^2}\right)\left(p_\perp^2\frac{\partial^2 f}{\partial p_z^2}+\frac{p_z^2}{p_\perp}\frac{\partial}{\partial p_\perp}p_\perp\frac{\partial f}{\partial p_\perp}-2p_\perp\frac{\partial}{\partial p_z}p_z\frac{\partial f}{\partial p_\perp}\right). \tag{40}$$

Next, we multiply (40) by $p_\perp^2$ and integrate over momenta, and multiply (40) by p_z^2 and integrate over momenta to find

$$\frac{\partial\langle p_\perp^2(t)\rangle}{\partial t}\approx 6\langle p_z^2(t)\rangle\frac{\partial}{\partial t}\left(\frac{B_\perp^2}{4B_0^2}\right) \tag{41}$$

and

$$\frac{\partial\langle p_z^2(t)\rangle}{\partial t}\approx -8\langle p_z^2(t)\rangle\frac{\partial}{\partial t}\left(\frac{B_\perp^2}{B_0^2}\right), \tag{42}$$

where $\langle ...\rangle$ means averaging over particle momenta. Substituting

$$\frac{1}{\langle p_\perp^2(t)\rangle}\frac{\partial\langle p_\perp^2(t)\rangle}{\partial t}=\frac{\partial}{\partial t}\frac{\langle p_\perp^2(t)\rangle}{\langle p_z^2(t)\rangle}+\frac{\langle p_\perp^2(t)\rangle}{\langle p_z^2(t)\rangle}\frac{1}{\langle p_z^2(t)\rangle}\frac{\partial\langle p_z^2(t)\rangle}{\partial t} \tag{43}$$

and using Eq. (42), we obtain the expression for the pitch-angle, assumed small,

$$\psi\approx\left(\frac{\langle p_\perp^2(t)\rangle}{\langle p_z^2(t)\rangle}\right)^{1/2}=\sqrt{\frac{3}{2}}\left(\frac{B_\perp}{B_0}\right). \tag{44}$$

We also have from Eq. (42),

$$\langle p_z^2\rangle = \langle p_{z0}^2\rangle \exp\left(-\frac{8B_\perp^2}{B_0^2}\right), \tag{45}$$

where p_{z0} is the parallel particle momentum at the initial moment, i.e., when $B_\perp = 0$. Since $B_\perp \ll B_0$ everywhere in the magnetosphere, there is no significant change of the average parallel energy. This is expected because the nonresonant interaction redistributed the wave energy amongst all particles, giving relatively little energy to any single particle. The resonant wave–particle interaction is described by the first term on the right-hand side of Eq. (39). This term is not zero only when the resonant condition $\omega_k = k_z v_z$ is satisfied, and this is not possible using the assumptions made here.

Thus we find that the nonlinearly generated wave modulations lead to increased pitch-angles. As a particle acquires pitch-angle, it emits synchrotron radiation. Synchrotron radiation is accompanied by a radiation reaction force which can then be taken into account in the kinetic equation (39), similarly to the resonant case.[16,17] However, the contribution of the back reaction to the change of the pitch-angle can be neglected if the relevant plasma instability grows sufficiently fast. The radiation reaction due to synchrotron radiation can be characterized by the energy loss time of a single particle $t_s \approx mc^2\gamma'\psi/P_{\rm syn}$, where[21] $P_{\rm syn} = (1/4\pi)\sigma_T(\gamma')^2 B_0^2\psi$, σ_T is the Thomson cross section, and γ' is the Lorentz factor of radiating particles in the plasma rest frame. This should be compared to the characteristic time scale $t_\perp \sim \psi^2 t_z$ of the energy change associated with the particle's perpendicular motion, cf. Eq. (41): $t_s > \psi^2 t_z$. To estimate t_z, we note that in the equilibrium the parallel diffusion time should equal the growth rate of the relevant plasma instability, i.e., the rate of the energy transfer from the Langmuir condensate to the particles: $t_z \sim (\Delta\omega/k_z a)^{1/2}$. However, the coefficient a defined by (24) can also depend on the synchrotron reaction force, thus making the full self-consistent treatment a very complex problem which is beyond the scope of the present paper. Here, we are interested in the possibility of γ-ray emission due to nonresonant quasilinear diffusion and therefore it is reasonable to assume that t_z should not be longer than the flow time of the plasma, R_{LC}/c, where R_{LC} is the light-cylinder adius, which is typically less than 1 s. Therefore, given the very small pitch angle $\psi \leqslant 10^{-5}$ and $t_z \leqslant 1$ s, the condition $t_s > t_\perp$ can easily be satisfied.

For further estimations, we assume that the wave energy density is of the order of the beam energy density, viz. $B_\perp^2 \sim mc^2 n_b\gamma_b$. We stress here that the amplitude of the growing transverse magnetic field perturbations $B_\perp$ has the characteristic frequency $\Delta\omega$ which is much less than the plasma frequency ω_p. The pulsar magnetic field changes with distance according to

$$B_0 = B_{0p}\left(\frac{R_0}{R}\right)^3, \tag{46}$$

also, we have

$$n_b = n_{0b}\left(\frac{R_0}{R}\right)^3, \tag{47}$$

where R_0 is the radius of the neutron star, $B_{0p} \sim 10^{12}$ G, and $n_{0b} \sim 10^{12}\,\mathrm{cm}^{-3}$. For young, rapidly rotating pulsars (such as Crab and Vela), the radius of the light cylinder is $R_{LC} \sim 10^8$ cm. The estimations show that in the rest frame,

$$\psi \approx \left(\frac{mc^2 n_b'\gamma_b'}{B_0^2}\right)^{1/2}, \tag{48}$$

where $n_b' \approx n_b/2\gamma_b$ and $\gamma_b' \approx \gamma_{bT}$ are the density and Lorentz-factor of the beam in the rest frame of the beam.

Assuming that the width of the beam distribution function γ_T is of the same order as γ_b, for $\gamma_b \sim \gamma_T \sim 10^7$, $n_b \sim 10^{12}\,\mathrm{cm}^{-3}$, $B_0 \sim 10^6$ G (corresponding to $R \simeq 10^2 R_0$), and $R_0 \sim 10^6$ cm, we find that the pitch-angle ψ can be of order 10^{-6}, which is large enough for the particles to emit significant synchrotron radiation. The synchrotron radiation in the rest frame of the beam is given by[21]

$$\omega_0 \approx \omega_B \gamma_b^2 \psi \sim 10^{21}\ \ \mathrm{s}^{-1}. \tag{49}$$

In the reference frame of the observer, the frequency is

$$\omega = \frac{\omega_0}{\gamma}\,\frac{1}{1 - v\cos\Theta/c}, \tag{50}$$

where Θ is the angle between $\mathbf{k}$ (in our case this direction is to the observer) and $\mathbf{B}_0$. For $\Theta \approx 0$ we have $\omega \simeq 2\omega_0\gamma_b$, which corresponds to 10 MeV. Thus the radiation can be in γ-range for some pulsars and the considered process is a potentially important mechanism for the pulsar gamma-ray emission. Detailed application is presented elsewhere.[22]

VI. CONCLUSION

To conclude, we argue that t-modes generated in a pulsar magnetosphere can create beat density modulations along the magnetic field. When the modulation frequency $\Delta\omega$ is much less than the frequency ω of the generated field perturbations, the growth of the parallel potential field E_z^l is accompanied by the growth of the transverse electromagnetic field $\mathbf{E}_\perp$ according to Eq. (30). The energy of the modulations can then be converted into perpendicular momentum of the nonresonant particles and this in turn leads to the synchrotron radiation at high photon energies. Direct application of this mechanism to observational data includes case by case analysis of concrete results and is the subject for further investigations.

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