Oscillating pair creation in pulsar magnetospheres

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Abstract. A model for a pulsar magnetosphere is formulated based on oscillatory pair creation associated with large-amplitude oscillations of an electric field, $E_\parallel$, parallel to the magnetic field. The amplitude is large enough for a typical particle (electron or positron) to become highly relativistic before slowing down, reversing its direction of motion and becoming highly relativistic propagating in the opposite direction. The maximum particle energy is above the threshold for effective pair creation, and pair creation is restricted to phases near this maximum.

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INTRODUCTION

A central problem in pulsar electrodynamics concerns the distribution of electric fields and charges along the magnetic field lines that thread the polar cap regions [1]. The problem may be described in terms of two competing effects. On the one hand, if there is no adequate source of free charges in the magnetosphere (it is "charge starved") then the vacuum electric field associated with the rotating magnetized star has a large component, $E_\parallel$, parallel to the magnetic field. This $E_\parallel$ accelerates any free particles to extreme energies leading ultimately to copious pair production, providing the free charges needed to screen $E_\parallel$. On the other hand, if there are sufficient charges to screen $E_\parallel$ at the surface of the star, so that the charge density there is equal to the corotational ("Goldreich-Julian") value, these charges can escape freely, and the Goldreich-Julian density cannot be maintained at greater heights without an additional source of free charges in the magnetosphere [2, 3, 4]. To resolve this problem, various models have been formulated in which a vacuum-like $E_\parallel$ develops but is restricted to an "inner gap" [5] and an "outer gap" [6] where particle acceleration occurs to above the threshold for effective pair creation. We refer to the most widely favored models as "gap-plus-PFF" models. In such models the pairs are created (above the inner gap) in a pair formation front (PFF), which is static in the corotating frame, and its thickness is small compared with other dimensions in the problem [5, 7]; screening of $E_\parallel$ occurs above the PFF due to a surface charge on the PFF. A gap-plus-PFF model involves a delicate balance between the inductively driven $E_\parallel$, which is required to be present in gaps to produce pairs, and its electrostatic screening at a PFF (in the corotating frame) by the pairs it produces.

Despite the relatively wide acceptance of gap-plus-PFF models, there have been persistent questions about their viability. Indeed, some early authors argued that the pair creation must be time-variable. For example, Sturrock [3] argued that non-stationary pair

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creation leads to sheets of outflowing pairs from which the radio emission emanates, and Ruderman & Sutherland [4] invoked localized, transient bursts of pair creation called sparking. Recent criticisms of the standard gap-plus-PFF models have been summarized by Michel [8], who favored a radically different steady-state model in which electrostatic fields form due to large-scale charge separation in the magnetosphere. The oscillatory model discussed here and the charge-separation model may be viewed as opposite extremes, being purely inductively and purely electrostatically based, respectively.

In the present paper we argue that models for PFFs are unstable and we reconsider older models in which pair creation is oscillatory. Specifically, we consider a model in which $E_\parallel$ oscillates with large amplitude, with pair creation occurring only near the phases of the oscillations where the Lorentz factors of the particles are maximum. The electrons and positrons are highly relativistic over most of the phase of the oscillations, becoming briefly nonrelativistic around the phase where they reflect. First, however, it is relevant to review salient features of gap-plus-PFF models.

GAP-PLUS-PFF MODELS

In a corotating model there is assumed to be a supply of free charges that is adequate to screen the parallel component of the vacuum field, $E_{\parallel\mathrm{vac}}$, which for an aligned rotator is given by

$$E_{\parallel\mathrm{vac}} = \frac{2\Omega B_* R_*}{r^4}\,\frac{\cos^3\theta}{(3\cos^2\theta+1)^{1/2}}, \tag{1}$$

where B_* is the magnetic field at the surface of the star, with angular speed Ω and radius R_*, and where r is the radial distance in units of R_* and θ is the polar angle. Provided there is an adequate source of charge, the charges not only screen the vacuum field, but also set up the corotation electric field, $\mathbf{E} = -(\boldsymbol{\Omega}\times\mathbf{r})\times\mathbf{B}$. This requires a charge density equal to the Goldreich-Julian value,

$$\rho = eN_B, \qquad N_B = \frac{\varepsilon_0}{e}\left[-2\boldsymbol{\Omega}\cdot\mathbf{B} + (\boldsymbol{\Omega}\times\mathbf{r})\cdot\mathrm{curl}\,\mathbf{B}\right], \tag{2}$$

everywhere in the magnetosphere. In (2) the term involving $\mathrm{curl}\,\mathbf{B}$ is unimportant near the star, and may be neglected. In models based on space-charge limited flow [5] the charge density is equal to the Goldreich-Julian value immediately above the stellar surface. The outflowing particles constitute a current, and conservation of current requires J proportional to B, and for particles with speeds close to the speed of light this requires N proportional to B. However, the corotation condition (2) requires N_B proportional to B_z, and these two requirements are incompatible with $N = N_B$ in general. As a result an $E_\parallel$ develops in a gap, leading to acceleration of 'primary' particles to high energy.

The primary particles (assumed to be electrons in the following discussion) emit photons due to curvature radiation (CR) or inverse Compton scattering (ICS) of thermal photons from the stellar surface. As the energy of the primaries increases with increasing height, so does the energy of the photons they emit. Photons are emitted nearly along the magnetic field lines and their angle of propagation increases with height due to the curvature of the field lines. A photon can decay into a pair when the component of its

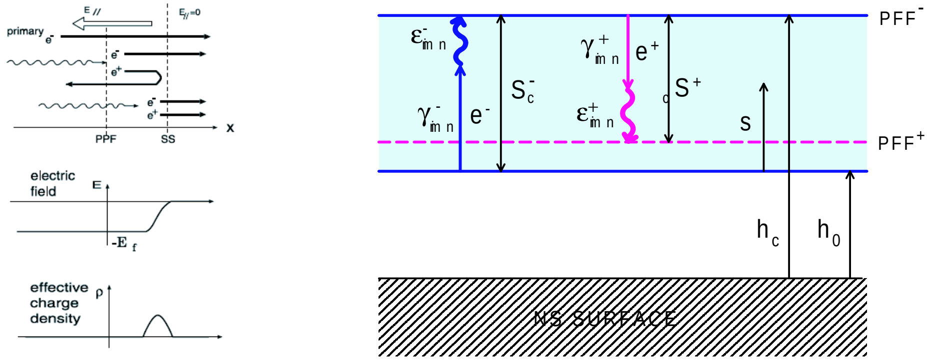

FIGURE 1. Left: The model proposed in [10] for the PFF, with the neutron star (NS) surface to the left. In the upper panel the reflection of positrons is indicated, and in the other two panels the effect of the reflecting positrons on the electric field and the charge density are illustrated schematically. Right: The model of two PFFs as illustrated in [11].

energy perpendicular to the magnetic field exceeds $2m_ec^2$. Pair creation occurs above a specific height that defines the lower boundary of the PFF, and it is assumed that screening of $E_\parallel$ occurs over a small range of heights, such that the PFF may be regarded as a thin surface.

Electrostatic screening of $E_\parallel$ above the PFF requires an effective surface charge on the PFF. It is possible for this charge to be due to the positrons having a lower energy that the electrons, due to acceleration by $E_\parallel$, so that they move more slowly and spend longer within any given region. However, this effect is weak and a detailed investigation leads to an unacceptably high density of pairs for effective screening [9]. Reflection of positrons within the PFF greatly increases the efficiency of the screening, as illustrated in figure 1. On a larger scale the PFF is approximated by a surface, and this excess of positrons over electrons corresponds to a net positive surface charge on the PFF. In a steady-state model (in the corotating frame), the surface charge on the PFF produces an electric field above the PFF that is equal and opposite to $E_\parallel$ in the absence of the PFF, giving a net $E_\parallel = 0$ above the PFF. Note that this surface charge also implies an $E_\parallel$ below the PFF; this $E_\parallel$ is equal in magnitude and opposite in sign to that above the PFF. Thus, if the surface charge on the PFF screens $E_\parallel$ just above the PFF it also double $E_\parallel$ just below the PFF.

The reflected positrons propagate back through the gap towards the star, and are accelerated to high energy. Their emission of downward propagating photons can lead to pair creation at a lower PFF [11], as illustrated in figure 1. Reflected electrons at this lower PFF propagate upward back through the gap and effectively become additional primary particles. The possibility of a steady-state model involving two such PFFs has been considered [11], and it was found that no steady-state solutions exist if pair creation is dominated by ICS, and that a steady-state model may be possible at heights 0.5–1 R_* above the surface when CR is the dominant process.

STABILITY OF A PFF

The stability of a steady-state gap-plus-PFF model is obscured in most discussions by assuming the boundary condition $E_\parallel = 0$ above the PFF (and below the lower PFF in the model with two PFFs). This boundary condition precludes the possibility that $E_\parallel$ might oscillate about zero (or some non-zero value) above the PFF. The following arguments suggest that PFFs may be intrinsically unstable and that the system tends to oscillate with large amplitude rather than approach the assumed quasi-static configuration.

The boundary condition $E_\parallel = 0$ above the PFF requires a charge density on the PFF that provides an electric field above the PFF equal and opposite to that in the absence of the PFF. For the sake of discussion, let us suppose that in the absence of the PFF one would have $E_\parallel = E_{\parallel \rm vac} < 0$. Then $E_\parallel = 0$ above the PFF requires the surface charge density on the PFF to be $-2\varepsilon_0 E_{\parallel \rm vac}$. This postulated charge density also produces an electric field below the PFF, and this field is in the same direction as $E_{\parallel \rm vac}$. Hence, the postulated charge density implies $E_\parallel = 2E_{\parallel \rm vac}$ just below the PFF.

Consider a perturbation on this postulated steady state in which $E_{\parallel \rm vac}$ changes. Suppose $E_{\parallel \rm vac}$ changes by $\delta E_{\parallel \rm vac} < 0$ so that it increases in magnitude. First suppose that the PFF remains at its initial height. Then, below the PFF this change enhances the energy of the accelerated particles and hence it increases the rate of pair creation. Above the PFF it leads to $E_\parallel = \delta E_{\parallel \rm vac} < 0$ which enhances the fraction of positrons that are reflected. The surface charge density on the PFF would increase, further enhancing $E_\parallel$ below the PFF, suggesting instability. However, the PFF can move to a lower height to offset this change. There is a new steady-state configuration, and this involves the PFF moving to a slightly different height, and the charge density on it adjusting to the required new value, $-\varepsilon_0(E_{\parallel \rm vac} + \delta E_{\parallel \rm vac})/2$ to compensate exactly for the postulated change. Thus, for the system to adjust to the change, the charge density on the PFF and the location of the PFF must adjust simultaneously to a new steady state. Any overshooting in the adjustment to a postulated change leads to oscillations of the system, and stability requires that such oscillations tend to damp. However, the feedback between the processes below the PFF (acceleration and emission of pair-producing photons) and those above the PFF (determining the fraction of positrons reflected) are indirect, and there seems no dynamical reason for the PFF to adjust smoothly towards a steady state. The effects of a change in $E_{\parallel \rm vac}$ has quite different effects above and below the PFF, and this suggests that the PFF tends to break up rather than adjust smoothly to a new configuration. Nevertheless, let us suppose that the PFF does adjust by moving and changing its surface charge density, and consider the implications of such motion.

Consider a perturbation that causes the location of the PFF to move with speed $v_{\rm PFF}$. Let the thickness of the PFF be δz, which is assumed small, so that the charge density within the PFF is $2\varepsilon_0 E_{\parallel \rm vac}/\delta z$. The associated current density is $J_\parallel = 2\varepsilon_0 E_{\parallel \rm vac} v_{\rm PFF}/\delta z$. The parallel component of the induction equation is

$$\frac{\partial E_\parallel}{\partial t} = -\frac{J_\parallel - J_{\parallel 0}}{\varepsilon_0}, \qquad J_{\parallel 0} = \mathbf{b}\cdot{\rm curl}\,\mathbf{B}, \tag{3}$$

where $\mathbf{b}$ is a unit vector along the magnetic field. If the imposed perturbation on $E_\parallel$ discussed above is periodic with a frequency $\Omega_{\rm osc}$, then the motion of the PFF would cause an inductive perturbation in $E_\parallel$ of order $2E_{\parallel \rm vac} v_{\rm PFF}/\delta z\, \Omega_{\rm osc}$. If this inductive change is the dominant electric effect, then it is inappropriate to treat the screening as an electrostatic effect. The inductive change is the dominant one for $v_{\rm PFF} > \delta z\, \Omega_{\rm osc}\, \delta E_{\parallel{\rm vac}}/E_{\parallel{\rm vac}}$, which is necessarily satisfied for sufficiently small δz. Conversely, if the system oscillates, then the model based on electrostatic screening due to a thin PFF cannot be valid. Indeed the concept of a PFF ceases to be valid if the system oscillates significantly because the inductive electric field then dominates.

Although the foregoing arguments are only qualitative, they lead us to two conclusions. First, the postulated electrostatic screening above the PFF in a gap-plus-PFF model is of questionable validity due to it appearing to be unstable to perturbations. Second, inductive effects are neglected in discussions of the PFF, whereas it seems that inductive effects dominate in response to a temporal perturbation, causing the system to oscillate, rather than to settle down to a quasi-static, electrostatically screened configuration.

OSCILLATORY MODEL

An oscillatory model for a pulsar magnetosphere involves similar processes to steady-state models: pair creation provides the source for the additional charges needed to maintain corotation, and reflection of positrons and electrons occurs, somewhat analogous to a model involving two PFFs. An important difference is that the separate locations where these processes occur in a steady-state model are replaced by separate phases of the oscillation. A mathematical distinction is that the oscillating $E_\parallel$ associated with the pairs is an inductive field, determined by the induction equation with an oscillating current. For simplicity, the oscillations considered here are purely temporal, and then Poisson's equation does not appear explicitly in the theory. On the other hand, in models for screening at a PFF, Poisson's equation plays a central role in determining the charge density that provides the screening above the PFF; the induction equation plays no role by fiat: the quasi-static assumption precludes any inductive contribution to $E_\parallel$. Purely spatial oscillations appear in some steady state models [12], but these are not directly related to the temporal oscillations discussed below. More generally, the oscillations should be regarded as large-amplitude waves [13] which vary in both time and space. An important distinction is between cases where the phase speed is greater (superluminal) or less (subluminal) than the speed of light. Temporal (spatial) oscillations are related through a Lorentz transformation to superluminal (subluminal) waves, e.g., [14]. The temporal oscillations considered here should generalize to superluminal wave-like motions, but we have yet to explore this generalization.

Sawtooth solution

A simple analytic model for the large-amplitude oscillations is possible under conditions that our numerical calculations suggest are relevant in practice. The simple analytic model requires that the amplitude, $E_{\rm osc}$ say, of the oscillations be large enough that typical electrons and positrons are highly relativistic through most of the oscillation, being nonrelativistic only briefly as they reverse their direction of propagation (along the magnetic field lines) each half period. The amplitude must also be small enough that pair creation has only a minor effect on the oscillation. The current density, $J_\parallel$, reverses its sign each half period. Except near the phase where the motion of the particles reverses sign, the electrons and positrons propagate in different directions, and the current density is approximately twice that due to either the positrons or electrons, and is approximately independent of time ($v_\pm \approx c$). Hence, $E_\parallel$ increases approximately linearly with time over one half period, its derivative then reverses rapidly as the mean velocities of the electrons and positrons pass through zero, $E_\parallel$ then decreases approximately linearly with time over the next half period. Thus $E_\parallel$ has a sawtooth profile. Suppose that pair creation is ignored, and that the oscillating particle current density is much larger than the constant global current, $|J_\parallel| \gg |J_0|$. Then over one period, T, of oscillation, $-T/4 < t < 3T/4$, this sawtooth profile may be described by

$$E_\parallel \approx E_{\rm osc} \begin{cases} 4t/T & \text{for} -T/4 < t < T/4, \\ (2T-4t)/T & \text{for} \quad T/4 < t < 3T/4, \end{cases} \tag{4}$$

with the amplitude of the oscillations approximated by

$$E_{\rm osc} \approx \frac{|J_\parallel| T}{4\varepsilon_0} \approx \frac{eN_+cT}{2\varepsilon_0}, \quad |J_\parallel| \gg |J_0|. \tag{5}$$

The amplitude $E_{\rm osc}$ can be estimated by setting the corresponding amplitude, $\Gamma_{\rm max}$, for the Lorentz factor equal to the threshold value, $\Gamma_{\rm th}$, for effective pair creation.

Bulk motions of the particles

In a simple fluid model, the electrons and positrons are described by cold, relativistic flowing fluids with number densities $N_\pm$ and 4-velocities, $U_\pm = \Gamma_\pm v_\pm$, that have components only along the field lines, which is along the z-axis. The continuity equations for the two fluids are

$$\frac{\partial N_\pm}{\partial t} + \frac{\partial (v_\pm N_\pm)}{\partial z} = \tfrac{1}{2} Q, \tag{6}$$

where Q describes the rate of pair creation. The equations of motion for the two fluids may be written as equations for the 4-momentum densities, $m_e N_\pm U_\pm$. Then, using (6) to rewrite the derivative of the number densities, the equations of motion become

$$\left(\frac{\partial}{\partial t} + v_\pm \frac{\partial}{\partial z} \right) U_\pm = \mp \frac{eE_\parallel}{m_e c} + \frac{q'_\pm}{m_e c^2} - q_\pm U_\pm, \tag{7}$$

where the term involving $q'_\pm$ describes the effect of losses due to emission of photons (that create pairs), and with $q_\pm = Q/2N_\pm$.

In the absence of pair creation, the 4-speed, $U_\pm = \Gamma_\pm v_\pm$, varies with time as $dU_\pm/dt = \mp eE_\parallel/m_e$. With (4) this implies that $U_\pm$ oscillates between $\pm U_{\rm max} \approx \pm \Gamma_{\rm max}$. Over the half period $-T/4 < t < T/4$ this oscillation may be described by

$$\Gamma_\pm \approx \Gamma_{\rm max} \frac{(T/4)^2 - t^2}{(T/4)^2}, \qquad \Gamma_{\rm max} \approx \frac{eE_{\rm osc}}{8m_e c} T \approx \left(\frac{\Omega_{p+} T}{4} \right)^2, \tag{8}$$

except near the turning points at $t = \pm T/4$ where the particles briefly become nonrelativistic. The maximum Lorentz factor, $\Gamma_{\rm max}$, is determined by

$$\Gamma_{\rm max} \approx \frac{eE_{\rm osc}}{8m_e c} T \approx \left(\frac{\Omega_{p+}T}{4}\right)^2, \tag{9}$$

with $\Omega_{p+}^2 = e^2 N_+/\varepsilon_0 m_e$.

Inclusion of pair creation

Pair creation can be due to photons emitted by either CR or ICS. For present purposes we seek a simple description that retains the essential features of pair creation. The rate of pair creation is a strong function of the energy of the photons, and the energy of the photons is a strongly increasing function of the energy of the radiating particles. This suggests a simplification, which we adopt, that there is an effective threshold, $\Gamma_{\rm th}$, such that pair creation is negligible for $\Gamma < \Gamma_{\rm th}$. The threshold for pair creation due to CR depends on the radius of curvature, $R_{\rm c}$, of the field lines,

$$\Gamma_{\rm th} > \left(\frac{4m_e c R_{\rm c}}{3\hbar}\right)^{1/3}, \tag{10}$$

and because of the cube root not too great an error should be introduced by replacing the inequality by an equality. The threshold for ICS depends on the spectrum of thermal photons from the stellar surface, cf. [15, 16]. A complication for ICS is that the thresholds for upward and downward propagating particles are different [11], due to these encountering the thermal photons by being overtaken by them and by hitting them head on, respectively. Here we ignore this and other complications, and assume a single threshold with a simple analytic form corresponding to CR.

The rate of production of pairs is proportional to the number density of pairs already present and above threshold. Above threshold the terms Q in (6) and $q'_\pm$ in (7) are assumed to have the forms

$$Q = \alpha_+ N_+ + \alpha_- N_-, \qquad q'_\pm = \frac{(\alpha_+ N_+ \varepsilon_{\gamma+} + \alpha_- N_- \varepsilon_{\gamma+})c}{N_\pm}, \tag{11}$$

with $\alpha_\pm = 10^2 \Gamma_\pm$ for $\Gamma_\pm > \Gamma_{\rm th}$, and where $\varepsilon_{\Gamma\pm} \approx (3\hbar c \Gamma_\pm^3 / 2R_{\rm c})$ are the energies of the CR photons emitted by particles with Lorentz factors $\Gamma_\pm$.

NUMERICAL RESULTS

Our numerical results are based on integrating the fluid equations that describe the model on a uniform computational grid with 10^4 cells. In all the numerical calculations the inner boundary is at the stellar surface, and the outer boundary is above the pair creation region. The results are insensitive to this upper boundary condition. In all cases, the threshold for effective pair creation is set at $\Gamma_{\rm th} = 10^6$.

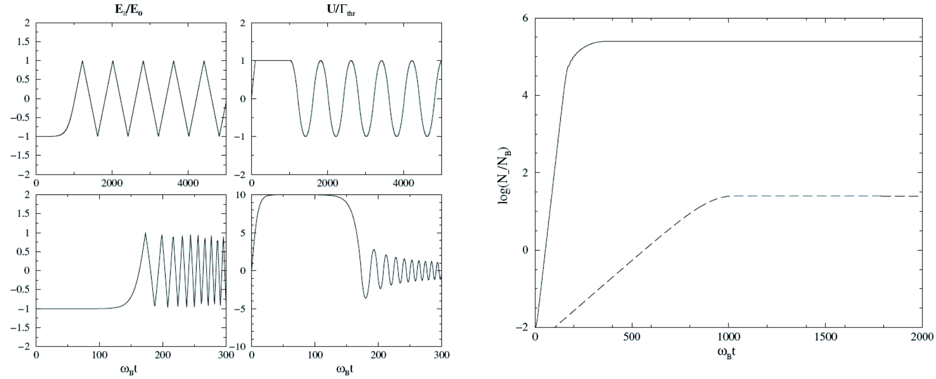

FIGURE 2. Left: Evolution of the $E_\parallel$ (left panels) and the 4-velocity (right panels) in a large-amplitude oscillation with an uniform initial electric field $\tilde{E}_0 = -10^4$ (upper panels) and $\tilde{E}_0 = -10^6$ (lower panels). The 4-velocity saturates at $U = c\Gamma_{\rm th}$, where pair creation becomes effective, and the system oscillates after the charge screening overshoots. Right: Evolution of the pair density, showing an exponential increase until the screening overshoots, after which it remains roughly constant as the system oscillates.

The equations are rendered dimensionless, with the dimensionless variables denoted by a tilde. A number density is normalized to the Goldreich-Julian value. $\tilde{N} = N/N_B$. The plasma frequency corresponding to the Goldreich-Julian density, $\Omega_B^2 = e^2 N_B/\varepsilon_0 m_e$, is used to define the unit of time, $1/\Omega_B$, and the unit of length, $L = c/\Omega_B$. The dimensionless electric field is $\tilde{E}_\parallel = eLE_\parallel/m_e c^2$. In these units the vacuum electric field (1) becomes

$$\tilde{E}_{\parallel\rm vac} \approx 10^6 \left(\frac{B_*}{10^{12}\,\rm G}\right)^{1/2} \left(\frac{P}{1\,\rm s}\right)^{1/2} \left(\frac{R_*}{10\,\rm km}\right). \tag{12}$$

The estimated frequency of the oscillations is then $\Omega_{\rm osc} \approx \Omega_B \tilde{E}_{\rm osc}/8\Gamma_{\rm max}$, where $\Gamma_{\rm max} \approx \Gamma_{\rm th}$ is assumed.

In the calculations illustrated in figure 2 the initial condition is that all quantities are homogeneous within the gap. Two different choices for the initial value, $\tilde{E}_\parallel = \tilde{E}_0$ at $t = 0$, are shown. The left panels correspond to $\tilde{E}_0 = -10^4$ and the right panels to $\tilde{E}_\parallel = -10^6$. These correspond to electric fields well below $\tilde{E}_{\parallel\rm vac}$ and comparable with $\tilde{E}_{\parallel\rm vac}$, respectively. The initial number density chosen is $\tilde{N} = 10^{-2}$, which corresponds to a charge-starved initial condition. The initial current is specified by choosing $U_+ = 0.95c$ and $U_- = 0$, corresponding to $J_\parallel = 6.9 \times 10^{-3} eN_B c$. As shown in figure 2, initially the particles are accelerated rapidly until they reach the threshold for pair creation. Then pair creation begins, and the number density increases exponentially. The electric field eventually reverses sign, and the bulk flow starts to decrease. The electric field and the flow 4-speed then oscillate with slowly varying amplitude. The number density saturates at a value $\tilde{N} \approx \tilde{E}_0^2/2\Gamma_{\rm max}$, $N \approx \varepsilon_0 |E_0|^2/2\Gamma_{\rm max} m_e c^2$, which corresponds to most of the initial energy density in the electric field being converted into pairs.

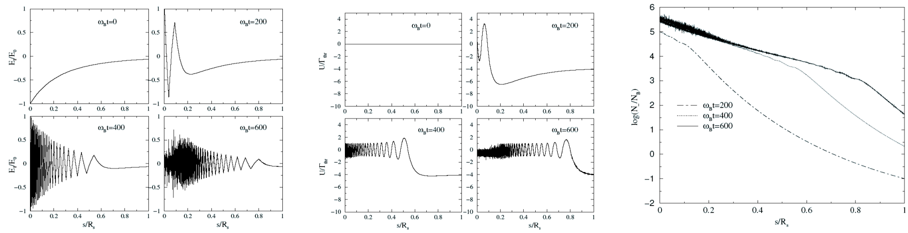

FIGURE 3. Left: Evolution of the system starting from a charge-starved condition with the vacuum electric field decreasing as illustrated in the top left panel. As time increases, oscillations are set up with their frequency and amplitude decreasing with height, as shown. Middle: Evolution of the 4-velocity. Right: The pair density at three different times.

As a second example, we consider the initial electric field to be the vacuum electric field (12), which falls off rapidly with height, r. The specific initial condition chosen is $\tilde{E}_0 = \tilde{E}_{\parallel\rm vac}/(1+s/R_*)^4$, with $\tilde{E}_{\parallel\rm vac}$ given by (12), and with $s/R_* = r-1$ the height above the stellar surface. A spatial dependence is also included in the parameters $\alpha_\pm$ introduced in (11), with $\alpha_\pm = 10^2\Gamma_\pm \exp[-(s/R_*)^2]$ for $\Gamma_\pm > \Gamma_{\rm th}$. As shown in figure 3, a wave pattern then forms near the surface and moves outwards. The oscillations occur with a higher frequency nearer to the star, consistent with $\Omega_{\rm osc}$ being proportional to Ω_B. The 4-speed increases with the maximum Lorentz factor, and saturates just above $\Gamma_{\rm th}$. The density increases with time as the amplitude of the oscillations decreases. The asymptotic form is unclear, and our present calculations are unable to follow the evolution for a sufficiently long time. There is some indication that the system becomes chaotic. One could speculate that after some time the oscillations and the pair creation cease, and the efflux of particles tends to restore the initial vacuum electric field, so that the evolution repeats. More detailed calculations are needed to determine the long-term behavior.

DISCUSSION AND CONCLUSIONS

The oscillatory model proposed here is, in one sense, radically new. The radical difference between an oscillatory model and a gap-plus-PFF models is in the nature of the response to the $E_\parallel$ that unavoidably develops in a pulsar magnetosphere. In an oscillatory model, this response is essentially inductive, whereas in a gap-plus-PFF model it is essentially electrostatic. The radical change is to seek an oscillatory solution, in which $E_\parallel$ is determined by the oscillating current, rather than a steady-state solution, in which $E_\parallel$ is electrostatic. Apart from this change, most of the ingredients in the two types of model are the same: a partially unscreened vacuum electric field that accelerates particles to high energies, gamma-ray emission by these particles, and decay of these gamma rays by one-photon pair creation. In another sense, the oscillatory model in not new, but is rather an older idea that has been unjustifiably ignored. The viewpoint of some early authors, notably [3], is that pair creation must occur in an oscillatory manner. We give some qualitative arguments that suggest that the screening in gap-plus-PFF model is unstable; within the framework of such models, this point is obscured by postulating the existence of a steady-state solution, so that the induction equation is neglected. Our argument is that when inductive effects are included, oscillations seem inevitable. Although we do not discuss the purely electrostatic models consisting of oppositely charged domes and tori explicitly here, discharges between the domes and tori [8] might also lead to oscillatory breakdown of these models when inductive effects are included.

The formulation of the oscillatory model, as reported here, is oversimplified in several ways. In particular, a net charge density equal to the Goldreich-Julian value is essential to maintain corotation, and this is ignored in the model in its simplest form; we implicitly assume that the asymmetry associated with the Goldreich-Julian can be treated as a perturbation, but we have yet to show this explicitly. Another deficiency is in identifying the way in which the oscillations are set up. Our numerical solutions show that if one postulates a vacuum-like electric field initially, then the system evolves towards an oscillatory model which is such that the maximum energy of the particles is close to the threshold for effective pair creation. This allows the magnetosphere to adjust the rate of pair creation by modulating the amplitude of the oscillations through a positive feedback mechanism. However, the assumption of an initial vacuum-like electric field is plausible only if the oscillations ultimately cease, and plasma flow out of the magnetosphere tends to establish the vacuum-like conditions required by the postulated initial condition. There is some indication from our calculations that this may occur, but this needs further investigation. Furthermore, in the model developed we have treated the escape of particles to infinity only indirectly, through the apparent outward propagation of the oscillations illustrated in figure 3. Escape of particles can be accommodated explicitly in the model by changing the boundary conditions to include losses, and generalizing the equations to allow the oscillations to vary in both time and space, and hence to propagate. We note that a model for purely temporal oscillations in one frame becomes a model for a propagating wave, with superluminal phase speed, through a Lorentz transformation to another frame[14]. This suggests that allowing outward propagation of the oscillations as a wave should have relatively little effect on the other aspects of the model, but this point also needs to be explored in detail.

An attractive feature of an oscillatory model is that it is insensitive to the sign of the corotation charge density in the polar cap regions. This is because the pairs are created by the oscillating field and this process is symmetric under the change of the sign of the vacuum field and the interchange of electrons and positrons. In contrast, the vacuum gap model of [4] applies only to pulsars where this charge is positive, which applies to only half of all neutron stars. Models based on space-charge limited flow from the surface [5] are generally applied to cases where electrons are drawn off the surface, and one has to argue that the other half of all neutron stars for which ions need to be drawn from the surface lead to pair creation such that there are not two distinct classes of pulsars. The absence of observational evidence for two classes of pulsars, depending on the sign of the Goldreich-Julian charge density, is difficult to reconcile with a vacuum-gap model and. to a lesser extent, for a space-charge-limited-flow model. No such difficulty arises when the pairs result from an oscillating $E_\parallel$.

It is important to identify observational tests that would distinguish the oscillatory model from steady-state models. Identification of signatures is complicated by the lack of consensus on the radio emission mechanism, and on the location of the source regions for the radio emission and for the high-energy emission. Indirect evidence [17] suggests a source for the radio emission at intermediate heights, between the putative inner and outer gaps, but there is no simple model that favors a source at such heights. Even for the high-energy emission, for which the emission mechanisms are well understood, there is no consensus on the height at which the emission occurs, in the inner gap [11] or the outer gap [18]. The oscillatory model allows pair creation to occur over a much wider range of heights, and if one assumes that the emission is located near where the pairs are created, the oscillatory model is consistent with emission over a much wider range of heights than models based on PFFs.

One possible signature for the radio emission that warrants investigation is a direct link between the oscillations and the observed microstructure. However, relativistic effects imply that the observer would see radiation from only a very small fraction of the period of each pulsation, and it seems unlikely that there is any direct relation between the time scales of the observed microstructure and of the oscillations. A feature of the model is the oscillation in the current, and this might lead to an observable effect on the linear polarization [19], for which there is some observational evidence [20]. Another possible signature, that applies to both the radio emission and the high-energy emission, results from downward propagating particles in the hemisphere opposite to the observer. Emission during the phase of the oscillations in which the particles are propagating towards the surface of the star is potentially visible from the opposite hemisphere. We have yet to explore these and other possible signatures in detail.

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