Preferential acceleration of heavy ions from thermal velocities

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The acceleration of ions from thermal velocities is analyzed to determine conditions under which heavy ions can be preferentially accelerated. Two accelerating mechanisms involving high- and low-frequency hydromagnetic waves respectively are considered. Preferential acceleration of heavy ions occurs for high-frequency waves if the frequency spectrum falls off faster than $(\text{frequency})^{-1}$. For the low-frequency waves heavy ions are less effectively accelerated than lighter ions. However, very heavy ions can be preferentially accelerated, the abundances of the very heavy ions being enhanced by a factor A_i over the thermal abundances. Acceleration of ions in the envelope of the Crab nebula is considered as an example.

INTRODUCTION

Any model for the acceleration of galactic cosmic rays must account for both the total energy requirements and the heavy-ion-rich chemical composition of cosmic rays. The chemical composition can only be due to either an anomalous composition in the source or a preferential acceleration of heavy ions.

Consider a model in which cosmic rays are accelerated in supernova envelopes. For the Crab nebula the power input into relativistic electrons is of the order 10^{38} ergs/s. The power input into ions cannot be determined directly; however, it may be of the same order as that into electrons. Considering that many supernova envelopes exist at any given time, it is plausible that the energy requirements of 10^{39}–10^{40} ergs/s into cosmic rays can be met. Therefore, supernova envelopes may be significant sources of cosmic rays. In this article we point out ways in which the chemical abundance of cosmic rays can differ from that of the thermal gas in the source due to preferential acceleration.

It is convenient to distinguish three stages in the acceleration of a cosmic-ray ion: (a) from thermal to several times thermal velocity, (b) from several times thermal velocity to several times the thermal velocity of electrons, and (c) to higher energies.

It is in stage (c) that the major part of the energy is supplied. The acceleration is probably by plasma waves (Tsytovich 1964) and by the Fermi mechanism.

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It is in stages (a) and (b) that the chemical composition is determined. This is because the acceleration mechanisms which operate usually accelerate different ions at different rates, and because the effects of the Coulomb interactions are important. The Coulomb interactions determine a collision frequency which depends on the charge, mass, and velocity of the ion. There are two maxima in the effect of the Coulomb interactions, one near the thermal velocity of protons due to interactions with thermal protons, and one near the thermal velocity of electrons due to thermal electrons. Clearly the former is important in stage (a) and the latter in stage (*b*).

Although the acceleration mechanisms which act at higher energies do not act on thermal ions (stage (a)), there are still many mechanisms which can act. Two such mechanisms which are likely to be important are considered here to illustrate two different ways by which the abundances of heavy ions can be changed in the acceleration.

RESONANT ACCELERATION

The most direct way in which a strong selection effect can occur is when the acceleration depends on the gyrofrequency of the ions. This occurs for resonant acceleration by hydromagnetic waves (Tsytovich 1963; Melrose 1968*a*).

An ion resonates with a wave when the wave frequency is a harmonic of the gyrofrequency of the ion in the rest frame of the center of gyration of the ion. Then the ion experiences a systematic acceleration by the rotating electric field of the wave.

Hydromagnetic waves with frequencies near the gyrofrequencies of the ions can be generated by microinstabilities. For example such waves can be generated when fast ions (velocities greater than the Alfven velocity) move into a region where the magnetic field is changing. The changing field causes the particles to have anisotropic pitch-angle distributions which can relax back towards an isotropic distribution by generating Alfven and magnetoacoustic waves. The waves generated are confined to a small cone about the direction of the magnetic field (Kennel and Wong 1967). The wave frequency depends on the velocity of the ion. For protons with velocity v_3 along the field line greater than the Alfven velocity v_A, the wave frequency is

$$\omega \cong \Omega v_A / v_3 < \Omega,$$

where Ω is the gyrofrequency of protons.

In the regions of magnetic inhomogeneity (say of order 10^{15} cm for the Crab nebula) such waves can be generated by superalfvenic ions. They can then accelerate subalfvenic ions by resonating with them. This so-called cyclotron damping (Stix 1962) is likely to be the major damping mechanism for the waves. Energy is being transferred from fast ions to slow ions via the waves.

The fraction of the energy of the waves which goes into each ionic species determines the number of each species accelerated. Because the wave frequency is less than the proton gyrofrequency, the protons cannot resonate with the waves. Hence proton acceleration is suppressed.

Analyzing the acceleration by such waves (confined to a small cone) by the method of Tsytovich (1966), the systematic increase in energy of an ion with charge $Z_i e$, mass $A_i M$, and gyrofrequency

$$\Omega_i = \frac{Z_i}{A_i}\frac{eB}{Mc},$$

where e, M, and $\Omega = eB/Mc$ are the charge, mass, and gyrofrequency of protons, one finds (Melrose 1968a)

$$\dot{E} = \frac{Z_i^2}{A_i} 2\pi^2 \frac{e^2}{Mc^2} v_A^2 \rho(\Omega_i), \quad (1)$$

where $\rho(\omega)$ is the spectral energy density of the waves. E can also be interpreted as the power absorbed per ion by thermal ions of species i (Z_i, A_i).

If heavy ions are to be preferentially accelerated, then $\rho(\omega)$ must have a maximum at a frequency lower than the gyrofrequency of the heaviest ion (for Fe+ the maximum of $\rho(\omega)$ must occur for $\omega < \Omega/56$). The power absorbed per ion increases with ion mass A, if $\rho(\omega)$ has a power-law spectrum $\rho(\omega)$ a ω^{-n} with n $>$ 1. Then the power absorbed per ion varies as E $\propto Z_i^{2-n} A_i^{n-1}$.

The total number of thermal ions accelerated in stage (a) is roughly proportional to $A_i^{n-1} n_i$ (setting $Z_i = 1$ for thermal ions) where n_i is the thermal number density of species. The spectrum of the waves, and so n, depends on the energy spectrum of the superalfvenic ions generating the waves.

RANDOM DISTRIBUTION OF ALFVEN WAVES

A random distribution of Alfven waves can accelerate ions by quite a different process. If the wave trains have a correlation length L, then there is a random electric field parallel to the magnetic field associated with the finiteness of L. This random field is of the order of the ratio of the gyroradius of the ion to L times the electric field of the wave (Schatzman 1967). Such a field accelerates the particles by a statistical process. Assuming that the gyrofrequency of the ions is very much greater than the frequency of the waves and that the particle velocities are much less than the Alfven velocity, the coefficient for systematic acceleration can be derived from the diffusion equations found by Asseo and Berthomieux (1966) and Sturrock (1966). We find (Melrose 1968b) that

$$\dot{E} = \frac{A_i M V_A^2}{L}\left(\frac{\delta B}{B}\right)^2 v_3. \quad (2)$$

This formula is similar to that for the Fermi acceleration, but it is difficult to interpret this analogy.

For the same energy density in the waves the magnitudes of the accelerations (1) and (2) coincide for typical frequencies of the waves in the second process of order

$$w \cong (v_3/v_A)\Omega.$$

The acceleration must be weak in the sense that only ions from far out in the Maxwellian tail can be accelerated. If this is not so, then the wave motion damps out very quickly until the acceleration is weak. There therefore exists a threshold velocity for each ion, v_{ci}, above which acceleration occurs. For $v < v_{ci}$ the effect of the Coulomb collisions dominates.

The fraction of each ionic component accelerated in unit time is determined by Coulomb interactions. These tend to maintain a Maxwellian distribution and so feed ions into the depleted tail to balance the loss due to the acceleration of ions with $v > v_{ci}$. The rate at which ions are fed across the surface v_{ci} in velocity space determines the fraction of each component accelerated in unit time.

Assuming that v_{ci} is greater than the thermal velocity of protons for all species of ions, the fraction accelerated can be determined by the method of Gurevich (1960). Neglecting the anisotropic character of the acceleration (2), one finds that the threshold velocity in this case is

$$v_{ci} = \tfrac{1}{2}A_i^{-\frac{1}{2}}(kT/M)^{\frac{1}{2}}\beta^{-\frac{1}{2}} \tag{3}$$

and the fraction of each component accelerated in unit time is

$$\frac{1}{n_i}\frac{dn_i}{dt} = \sqrt{\frac{2}{\pi}}\, Z_i^2 A_i^{\frac{1}{2}} \nu_0 \exp(-A_i) \times \exp\left[-\frac{Z_i}{2\beta^{\frac{1}{2}}}\left(\tfrac{1}{2}\pi - \tan^{-1}2A_i\beta^{\frac{1}{2}}\right)\right], \tag{4}$$

where

$$\beta = \frac{V_A^2}{4L}\left(\frac{\delta B}{B}\right)^2 \frac{1}{\nu_0}\left(\frac{M}{kT}\right)^{\frac{1}{2}} \tag{5}$$

and

$$\nu_0 = \frac{4\pi n e^4 \ln \Lambda}{M^{\frac{1}{2}}(kT)^{3/2}} \tag{6}$$

where n is the electron number density and $\ln \Lambda$ the Coulomb logarithm.

The factor $Z_i^2 A_i^{\frac{1}{2}} \nu_0$ in (4) is the collision frequency for the ion; $\exp(-A_i)$ is proportional to the number of ions of atomic mass A_i with velocity greater than the thermal velocity of protons in a thermal distribution. The final exponential is related to the flux of ions in velocity space. Gurevich (1960) does not include terms $\exp(-A_i)$ and that involving the inverse tangent. These appear from a more precise treatment of the Coulomb interactions (Melrose 1968*b*).

For very massive ions the formulas break down because the acceleration can always dominate over the Coulomb interactions. In this case the Coulomb interactions have a negligible effect. The ions are accelerated approximately in the ratio of the power absorbed per ion times the initial abundance. From (2) it follows that very heavy accelerated ions will have abundances $A_i n_i$.

This mechanism can be quite efficient. In fact it may be efficient enough to accelerate virtually all the ions in the Crab nebula for quite reasonable values of the parameters involved. To demonstrate this, set

$$\frac{T}{n_i}\frac{dn_i}{dt} \cong 1$$

where

$$T \cong 3 \times 10^{10}\ \text{s}$$

is the lifetime of the Crab. This gives an estimate for the β required once the collision frequency is known. Taking $n \cong 1\ \text{cm}^{-3}$, $T \cong 5 \times 10^4\ °\text{K}$, this gives $\beta \cong 10^{-2}$. With the Alfven velocity

$$V_A \cong 10^8\ \text{cm/s},$$

this gives

$$(\delta B/B)^2 \cong 10^{-18}L.$$

L can be estimated by noting that variations over a few weeks to a few months are observed in the Crab (Oort and Walraven 1956). Hence

$$L = 10^{14}\text{–}10^{15}\ \text{cm}$$

Consequently a fluctuating field of a few percent of the steady field could accelerate virtually all the ions in the Crab in 3×10^{10} s.

With the value $\beta \cong 10^{-2}$ the fraction of each component accelerated decreases with increasing A_i until $A_i \cong 20$. Thereafter the Coulomb interactions are ineffective and the abundances of the accelerated ions vary as $A_i n_i$.

CONCLUSION

Whenever the Coulomb interactions are important the fraction of each ionic component accelerated drops off with increasing ion mass. However, lor massive ions preferential acceleration can occur for both the mechanisms considered. This comes about because the variation in the strength of the Coulomb interactions with ion mass and velocity is such that for massive enough ions the Coulomb interactions are less effective than the acceleration for velocities near the thermal velocity of the ion.

This is analogous to a similar effect which occurs for velocities near the thermal velocity of the electrons. This can also enhance the abundance of the heavy ions (Korchak and Syrovatskii 1959; Ginzburg and Syrovatskii 1961).

For the resonant acceleration the heavier ions are only preferentially accelerated if the frequency spectrum of the waves falls off fast enough.

If ions are accelerated in supernova envelopes, then one would expect the composition of the accelerated ions to differ from that of the unaccelerated ions by an underabundance in heavy ions.

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