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ELECTROSTATIC BREMSSTRAHLUNG

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Abstract. Inverse Compton radiation due to scattering off electron plasma waves, called 'electrostatic bremsstrahlung' by Colgate (1967), is discussed with the view of resolving a discrepancy which exists between various authors on the mean frequency radiated.

1. Introduction

Inverse Compton radiation when the unscattered waves are electron plasma waves is called 'electrostatic bremsstrahlung' by Colgate (1967). Colgate treats this process by analogy with synchrotron radiation but does not attempt to justify the analogy. The same problem is treated by Gailitis and Tsytovich (1964) whose method is analogous to the one used in an accompanying article, Melrose (1970) hereinafter called I. Gailitis and Tsytovich estimate the mean frequency radiated from, see Equation (11) of I,

$$\omega - \mathbf{k} \cdot \mathbf{v} = \omega' - \mathbf{k}' \cdot \mathbf{v},$$

which for $\omega' = \omega_p$ (= plasma frequency) and $v_\phi = \omega_p / k'$ (= the phase velocity of the plasma waves) reads

$$\omega (1 - \beta \cos \theta) = \omega_p \left(1 - \frac{\beta}{\beta_\phi} \cos \theta' \right), \quad \beta_\phi = \frac{v_\phi}{c}, \tag{1}$$

where we use the notation of I. For $\beta_\phi \ll 1$ and $\beta \approx 1$, Gailitis and Tsytovich note that the maximum frequency radiated is given by

$$\omega = \frac{\omega_p}{\beta_\phi} \gamma^2 .$$

They argue that this is the typical frequency radiated.

Neither Colgate (1967), whose analogy is not well-founded when extended to the frequency radiated, nor Gailitis and Tsytovich (1964) calculate the mean frequency radiated; both sets of authors conjecture on the value of this mean frequency. Our purpose in the present article is to calculate the mean frequency radiated.

No major astrophysical application of 'electrostatic bremsstrahlung' seems to be considered in the literature. This may be due in part to the effect not being well-known. The total power radiated per relativistic electron is, apart from a numerical factor, the same as in synchrotron radiation or inverse Compton radiation for the same energy density in electron plasma waves, the magnetic field, or electromagnetic waves respectively. It is conceivable that the energy density in electron plasma waves could

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be a significant fraction of either of these other energy densities under extreme circumstances. The major distinction between these processes is in the mean frequency radiated. The mean frequency radiated in each of these processes is a basic frequency times the Lorentz factor squared of the particle. In synchrotron radiation the basic frequency is the non-relativistic gyrofrequency of the particle and in inverse Compton radiation the basic frequency is the mean frequency of the unscattered waves. We wish to find the corresponding basic frequency for 'electrostatic bremsstrahlung'.

2. Scattering Formulas

We use the formalism outlined in I to treat 'electrostatic bremsstrahlung'. Because electron plasma waves can exist only in the presence of a background medium, the scattering by individual particles necessarily includes effects related to the shielding of the particles. These shielding effects do not affect the scattering to lowest order in γ^{-1} ($\gamma =$ Lorentz factor), i.e. to lowest order in γ^{-1} only Compton scattering contributes. It is intuitively obvious that shielding effects are small for ultrarelativistic particles; this is shown to be the case by Gailitis and Tsytovich (1964). Here we include only Compton scattering but emphasize that our results apply only to ultrarelativistic particles.

Electron plasma waves with phase velocities well in excess of the thermal velocity of electrons have

$$\frac{W_E}{W_T} = \tfrac{1}{2}, \quad \omega' = \omega_p, \quad \mathbf{e}' = \boldsymbol{\kappa}'. \tag{2}$$

On summing over the polarizations of the scattered electromagnetic waves we have, in place of (28) and (29) of I,

$$w(\mathbf{p}, \mathbf{k}, \mathbf{k}') = \frac{(2\pi)^3 q^4}{m^2} \frac{|A|^2}{\omega\omega_p} \delta\left\{\omega(1 - \boldsymbol{\kappa}\cdot\boldsymbol{\beta}) - \omega_p\left(1 - \frac{\boldsymbol{\kappa}'\cdot\boldsymbol{\beta}}{\beta_\phi}\right)\right\}, \tag{3}$$

$$\begin{aligned} |A|^2 &= (\delta_{il} - \kappa_i\kappa_l)\,\kappa'_j\kappa'_m A_{ij}(\mathbf{k}, \mathbf{k}', \mathbf{v})\, A_{lm}(\mathbf{k}, \mathbf{k}', \mathbf{v}) \\ &= \frac{\omega_p^2}{\omega^2} \frac{(1 - \beta^2)}{(1 - \boldsymbol{\kappa}\cdot\boldsymbol{\beta})^4} \left[(1 - \boldsymbol{\kappa}\cdot\boldsymbol{\beta})^2 \{1 - (\boldsymbol{\kappa}'\cdot\boldsymbol{\beta})^2\} - (1 - \beta^2) \right. \\ &\qquad \left. \times (\boldsymbol{\kappa}\cdot\boldsymbol{\kappa}' - \boldsymbol{\kappa}'\cdot\boldsymbol{\beta})^2\right], \end{aligned} \tag{4}$$

if we use Equation (14) of I.

The power radiated in scattered waves, the power gained by the unscattered waves, the rate photons are scattered and the energy density in the unscattered waves are given by

$$P = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} \hbar\omega w(\mathbf{p}, \mathbf{k}, \mathbf{k}')\, N'(k'), \tag{5}$$

$$P' = -\int \frac{d^3\mathbf{k}}{(2\pi)^3} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} \hbar\omega_p w(\mathbf{p}, \mathbf{k}, \mathbf{k}')\, N'(k'), \tag{6}$$

$$R = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} w(\mathbf{p}, \mathbf{k}, \mathbf{k}')\, N'(k'), \tag{7}$$

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$$W_p = \int \frac{d^3\mathbf{k}'}{(2\pi)^3} \hbar\omega_p N'(k') = \hbar\omega_p n, \tag{8}$$

respectively; where n is the number density of unscattered photons (quanta of electron plasma waves) and where we assume that the unscattered waves are isotropically distributed. Because $\hbar\omega_p$ is a constant (6) and (7) imply

$$P' = -\hbar\omega_p R. \tag{9}$$

For simplicity we assume that all the electron plasma waves have the same phase velocity, i.e. we regard β_ϕ as a constant. This is not an essential assumption. We discuss the cases $\beta_\phi > \beta$ and $\beta_\phi \ll \beta$ separately; we assume $\beta \approx 1$ throughout. We consider the case $\beta_\phi > \beta$ first as this is the simpler case.

THE CASE $\beta_\phi > \beta$.

For $\beta_\phi > \beta$, the δ-function in (3) gives

$$\omega(1 - \beta\cos\theta) = \omega_p\left(1 - \frac{\beta}{\beta_\phi}\cos\theta'\right) > 0$$

for all values of $\cos\theta'$, i.e. there is no restriction on the values of $\cos\theta'$ allowed. As we are concerned with relativistic particles we have $\beta \lesssim 1$ and so $\beta_\phi > \beta$ implies $\beta_\phi \gtrsim 1$. Thus for electron plasma waves with phase velocities in excess of the velocity of light there is no restriction on the scattering due to angular effects.

Following the derivation of (35) and (36) from (30) and (31) in I respectively, we find from (5), (6) and (8)

$$P = 2\pi\left(\frac{q^2}{mc^2}\right)^2 cW_p(1-\beta^2)\int_{-1}^{+1} d\cos\theta\, g_1(\beta, \cos\theta),$$

$$P' = -2\pi\left(\frac{q^2}{mc^2}\right)^2 cW_p(1-\beta^2)\int_{-1}^{+1} d\cos\theta\, g_2(\beta, \beta_\phi, \cos\theta),$$

$$g_1(\beta, \cos\theta) = \frac{1}{(1-\beta\cos\theta)^5}\left[(1-\beta\cos\theta)^2\left(1-\tfrac{1}{3}\beta^2\right) - \tfrac{1}{3}(1-\beta^2)(1-2\beta\cos\theta+\beta^2)\right],$$

$$g_2(\beta, \beta_\phi, \cos\theta) = \frac{1}{(1-\beta\cos\theta)^4}\left[\beta_\phi^2(1-\beta\cos\theta)^2 + \frac{(1-\beta^2)\beta_\phi^2}{2\beta^2}\{3\cos^2\theta - 1 - 4\beta\cos\theta + 2\beta^2\} + \frac{1}{2}\frac{\beta_\phi}{\beta}\left\{(1-\beta_\phi^2)(1-\beta\cos\theta)^2 - \frac{(1-\beta^2)\beta_\phi^2}{2\beta^2}(3\cos^2\theta - 1 - 4\beta\cos\theta + 2\beta^2) - \frac{(1-\beta^2)}{2}\sin^2\theta\right\}\ln\frac{\beta_\phi+\beta}{\beta_\phi-\beta}\right].$$

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The remaining integrals are straightforward but tedious; we find

$$P = \frac{8\pi}{9}\left(\frac{q^2}{mc^2}\right)^2 cW_p \frac{3-\beta^2}{1-\beta^2} \approx \frac{16\pi}{9}\left(\frac{q^2}{mc^2}\right)^2 cW_p\gamma^2 \quad (\gamma \gg 1), \tag{10}$$

$$P' = -\frac{8\pi}{3}\left(\frac{q^2}{mc^2}\right)^2 cW_p f(\beta_\phi, \beta), \tag{11}$$

$$f(\beta_\phi, \beta) = \beta_\phi^2 + \tfrac{1}{2}(1-\beta_\phi^2)\frac{\beta_\phi}{\beta}\ln\frac{\beta_\phi+\beta}{\beta_\phi-\beta}. \tag{12}$$

The function $f(\beta_\phi, \beta)$ is slowly varying for $\beta_\phi \gtrsim 1$. We are concerned only with the case $\beta \approx 1$. Then for $\beta_\phi = 1$ and $\beta_\phi \gtrsim 10$ the function (12) is close to unity; for $1 \lesssim \beta_\phi \lesssim 10$ the function (12) varies between these limits of unity and has a minimum of about $\frac{2}{3}$.

For $\gamma \gg 1$ the mean frequency radiated follows from (9), (10), (11) and

$$P = \hbar \langle\omega\rangle R.$$

We find

$$\langle\omega\rangle \approx \tfrac{2}{3}\frac{\omega_p\gamma^2}{f(\beta_\phi, \beta)} \approx \omega_p\gamma^2 \quad (\beta_\phi \gtrsim \beta). \tag{13}$$

In view of the slowly varying property of $f(\beta_\phi, \beta)$ when one averages over a distribution of waves with $\beta_\phi > \beta$ the average value of $f(\beta_\phi, \beta)$ to be inserted in (13) must lie between $\frac{2}{3}$ and unity.

THE CASE $\beta_\phi \ll \beta$.

For $\beta_\phi \ll \beta$ not all values of $\cos\theta'$ are permitted by the δ-function in (3). To have

$$\omega(1-\beta\cos\theta) = \omega_p\left(1 - \frac{\beta}{\beta_\phi}\cos\theta'\right) > 0$$

we require

$$\cos\theta' < \frac{\beta_\phi}{\beta} \approx 0.$$

Thus for $\beta_\phi \ll \beta$ only about half the electron plasma waves can be scattered by a given particle. If one repeats the derivation of (10) for this case and ignores terms of order β_ϕ/β, then the resulting expression

$$P = \frac{4\pi}{9}\left(\frac{q^2}{mc^2}\right)^2 cW_p \frac{3\beta^2}{1-\beta^2} \approx \frac{8\pi}{9}\left(\frac{q^2}{mc^2}\right)^2 cW_p\gamma^2 \quad (\gamma \gg 1) \tag{14}$$

is just one half of (10) as one might expect.

In the derivation of (11) logarithmic factors appear. Because the emitted electromagnetic waves must have frequencies in excess of the plasma frequency we require

$$\omega_p < \omega = \omega_p \frac{\left(1 - \dfrac{\beta}{\beta_\phi}\cos\theta'\right)}{1+\beta},$$

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and so we require that

$$1+\beta \leqq 1-\frac{\beta}{\beta_\phi}\cos\theta' \leqq 1+\frac{\beta}{\beta_\phi}. \tag{15}$$

Then in place of (11) and (12) we find

$$P' = -\frac{4\pi}{3}\left(\frac{q^2}{mc^2}\right)^2 cW_p F(\beta_\phi, \beta), \tag{16}$$

$$F(\beta_\phi, \beta) = \beta_\phi^2 + (1-\beta_\phi^2)\frac{\beta_\phi}{\beta}\ln\frac{\beta_\phi+\beta}{\beta_\phi(1+\beta)}. \tag{17}$$

For $\beta \approx 1$ and $\beta_\phi \ll 1$ (17) reduces to

$$F(\beta_\phi, \beta) \approx -\beta_\phi \ln 2\beta_\phi. \tag{18}$$

The mean frequency radiated in this case, for $\gamma \gg 1$, reduces to

$$\langle\omega\rangle \approx \tfrac{2}{3}\omega_p\gamma^2/(-\beta_\phi \ln 2\beta_\phi) \quad (\beta_\phi \ll \beta). \tag{19}$$

This is smaller by a factor of order $\ln(1/2\beta_\phi)$ than that roughly estimated by Gailitis and Tsytovich (1964).

If there is a distribution of phase velocities then an appropriate average must be performed over (18). If $N(\beta_\phi)$ is the number of waves in the range $d\beta_\phi$ at β_ϕ with $N(\beta_\phi)$ normalized to unity, then the appropriate average is given by integrating (18) times $N(\beta_\phi)$ over β_ϕ.

We emphasize that Equations (10) and (14) neglect the effects of the shielding of the scattering electrons. Shielding effects can only be neglected for $\gamma \gg 1$ and even for $\gamma \gg 1$ lead to corrections to (10) and (14), see Gailitis and Tsytovich (1964).

3. Discussion

The result (10), when compared with the corresponding formula for inverse Compton radiation for $\gamma \gg 1$, see (35) of I, shows that the power radiated in 'electrostatic bremsstrahlung' is one half that in inverse Compton radiation for equal energy densities in the unscattered waves. The appearance of a factor of one half can be understood from the fact that all the energy in electromagnetic waves is electromagnetic whereas only half the energy in electron plasma waves is electrical energy (there is no magnetic energy in these waves). The appearance of yet a further factor of one half in passing from (10) to (14) can be understood from the fact that for $\beta_\phi \ll 1$ only half the electron plasma waves can be scattered by an individual particle.

These results are consistent with the general conclusion that bremsstrahlung off a given energy density in the electromagnetic field by ultrarelativistic particles, when the emission is averaged over any anisotropies, gives the same power radiated irrespective of the form of the electromagnetic energy.

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The stated purpose of the present article is to find the frequency radiated in 'electrostatic bremsstrahlung'. The result of our investigation is that the mean frequency radiated is given by, apart from a factor of order unity,

$$\langle\omega\rangle \approx \omega_p\gamma^2 \begin{cases} 1 & (\beta_\phi \gtrsim \beta), \\ (-\beta_\phi \ln \beta_\phi)^{-1} & (\beta_\phi \ll \beta), \end{cases}$$

where β_ϕ is the ratio of the phase velocity of the electron plasma waves to the velocity of light and where ω_p is the plasma frequency. For $\beta_\phi \ll 1$ the mean frequency radiated falls between $\omega_p\gamma^2$ and $k'c\gamma^2$ where k' is the wave number of the electron plasma waves.

References

Colgate, S. A.: 1967, *Astrophys. J.* **150**, 163.
Gailitis, A. K. and Tsytovich, V. N.: 1964, *Soviet Phys. – JETP* **19**, 1165.
Melrose, D. B.: 1970, *Astrophys. Space Sci.* **6**, 321.

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