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Variation in the Polarization Across Bends in the Spectra of Self-Absorbed Synchrotron Sources

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The accepted interpretation of the low frequency turnovers in the spectra of many extragalactic radio sources is that they are due to the effects of synchrotron self-absorption, **i.e.** to the source becoming optically thick. It has been shown[1, 2] that the signs of the degrees of both linear and circular polarization for a homogeneous source at frequencies where it is optically thick are opposite to those at frequencies where it is optically thin.

In this paper I report the results of calculations relating to the variation of the degrees of polarization with frequency as a homogeneous source becomes optically thick.

THE TRANSFER EQUATION

The equation describing the transfer of polarized radiation through a homogeneous source can be written in matrix form in terms of the Stokes parameters **I**, Q, *U*, V. When considering self-absorption in a homogeneous source any plane of polarization is determined by the direction of the background magnetic field H. One is free to orient the axes so that *U* vanishes identically. The transfer equation with only synchrotron emission and absorption included then reduces to [1]

$$\frac{\partial}{\partial l}\begin{bmatrix} I \\ Q \\ V \end{bmatrix} = \varepsilon \begin{bmatrix} 1 \\ r_{lo} \\ r_{co} \end{bmatrix} - \frac{\gamma}{c}\begin{bmatrix} 1 & \zeta & \delta \\ \zeta & 1 & 0 \\ \delta & 0 & 1 \end{bmatrix}\begin{bmatrix} I \\ Q \\ V \end{bmatrix} \qquad (1)$$

where 1 denotes length along the ray path, ε is the volume **emissivity** summed over states of polarization and γ is the absorption coefficient per unit time averaged over states of **polarization.**

For the purposes of the calculations reported here only the frequency dependences of ε and γ plus specific values for the remaining quantities are required. For a power law spectrum,

$$N(E) = KE^{-a}, \qquad (2)$$

one has

$$\varepsilon \propto f^{-(a-1)/2}, \quad \gamma \propto f^{-(a+4)/2} \qquad (3)$$

and

$$r_{lo} = \frac{-(a+1)}{a+7/3}, \quad \zeta = \frac{-3(a+2)}{3a+10},$$

$$r_{co} = \sqrt{3}\cot\theta(\sin\theta)^{\frac{1}{2}}\,\frac{a+1}{a}\,\frac{2+a+g(\theta)}{3a+7} \times$$

$$\times \frac{\Gamma\left(\frac{3a+8}{12}\right)\Gamma\left(\frac{3a+4}{12}\right)}{\Gamma\left(\frac{3a+7}{12}\right)\Gamma\left(\frac{3a-1}{12}\right)}\left(\frac{f}{f_H}\right)^{-\frac{1}{2}}, \qquad (4)$$

$$\delta = f_2(a)\, r_{co}.$$

In (4), f_H is the gyrofrequency of non-relativistic electrons and θ is the angle between the line of sight and the magnetic field. The function f_2 (a) was defined and tabulated by **Melrose.**[1]

In finding the eigenvalues and **eigenfunctions** of the matrix in (1), it is **useful** to note that a number of small terms should be neglected as they would be inconsistent with other approximations. Specifically only **terms** of up to first order in (Lorentz factor of radiating **electron**)$^{-1}$ are retained in treating synchrotron radiation[1, 3]; because r_{co}, 6 and V (when there is no external source of circularly polarized radiation) are all of first order, products of these quantities amongst themselves are to be neglected. Consequently, (1) can be replaced by

$$\frac{\partial}{\partial l}\begin{bmatrix} I+Q \\ I-Q \\ \zeta V - \delta Q \end{bmatrix} = \varepsilon \begin{bmatrix} 1+r_{lo} \\ 1-r_{lo} \\ \zeta r_{co} \mp \delta r_{lo} \end{bmatrix} -$$

$$-\frac{\gamma}{c}\begin{bmatrix} 1+\zeta & 0 & 0 \\ 0 & 1-\zeta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} I+Q \\ I-Q \\ \zeta V - \delta Q \end{bmatrix}. \qquad (5)$$

SIMPLE MODELS

I have considered three simple models for synchrotron sources in solving (5). In each of these models the magnetic field is assumed to be uniform. The models are the following:

1. Plane Parallel Slab or Cubic Model

The source is bounded by two infinite parallel planes orthogonal to the line of **sight** and separated by a distance L. The relevant results are unaffected by taking the planes to be of any finite extent, **e.g.** for a cubic model.

2. *Uniform* Spherical Model

The source is spherical with a uniform distribution of radiating electrons; the source is unresolved so that the Stokes parameters are averaged over the source.

3. Gaussian *Model*

As in model 2 except that the density of radiating electrons depends on radial distance r as exp $(-\ r^2/L^2)$, L = constant.

Each model leads to a solution of the form

$$I = \tfrac{1}{2}I_0\,\frac{3a+10}{3a+7}\left[\frac{3a+5}{3a+8}\,g(\alpha_1) + g(\alpha_2)\right],$$

$$Q = \tfrac{1}{2}I_0\,\frac{3a+10}{3a+7}\left[\frac{3a+5}{3a+8}\,g(\alpha_1) - g(\alpha_2)\right],$$

$$V = r_{co}\left[\frac{3a+10}{3(a+2)}f_2(a)\,Q + I_0\,g(\alpha_3)\times\right.$$

$$\left.\times\left\{1-\frac{3a+10}{3a+7}\;\frac{a+1}{a+2}f_2(a)\right\}\right], \quad (6)$$

with

$$I_0 = \frac{\varepsilon c}{\gamma},\quad \alpha_1 = \frac{3a+8}{3a+10}\;\frac{2\gamma L}{c},$$

$$\alpha_2 = \frac{4}{3a+10}\;\frac{\gamma L}{c},\quad \alpha_3 = \frac{\gamma L}{c}. \quad (7)$$

For models 1,2 and 3 one has

$$g(\alpha) = 1 - e^{-\alpha}, g(\alpha) = 1 - \frac{2}{\alpha^2}\left\{1-(1+\alpha)\,e^{-\alpha}\right\}$$

$$\text{and } g(\alpha) = \sum_{n=1}^{\infty}\frac{(-)^{n-1}\alpha^n}{n(n!)},$$

respectively, with L identified with the diameter of the source for model 2.

For all three models the shapes of the curves describing the dependences of I, Q/I and V/I on frequency depend only on the value of spectral index a.

RESULTS

The variation of I, V/I (both in arbitrary units) and Q/I with frequency was calculated numerically for model 1 with a $= 1.0, 1.5, 2.0, \ldots, 4.0$. The results were displayed as curves by the computer and photographs were taken. Figure 1 is an example of such a photograph. In Figure 1, the scale of Q/I is the actual scale, with

$$\frac{Q}{I} = \begin{cases} \dfrac{a+1}{a+7/3}\,(\log f \gg 1) \\[2ex] -\dfrac{3}{6a+13}\,(\log f \ll 1); \end{cases}$$

$\log f = 0$ corresponds to the maximum of I, while all other scales are arbitrary. The variation of the degree of polarization V/I can be regarded as the $f^{-\frac{1}{2}}$ dependence of r_{c0} multiplied by a function equal to unity for log $f \gg 1$ and equal to $-\{f_2(a) - f_1(a)\}$, as defined in reference 1, for $\log f \ll 0$.

Table 1 gives the ratios of the frequencies at which the two degrees of polarization pass through zero (i.e. where the plane of linear polarization jumps through 90° and where the handedness of the circularly polarized component flips) to the frequency at which the intensity is a maximum for a in the range $1.5 \leq a \leq 4.0$. [The case a $= 1.0$ is omitted because I approaches a constant for $\log f$)) 0, i.e. there is no maximum of the spectrum for a ≤ 1.]

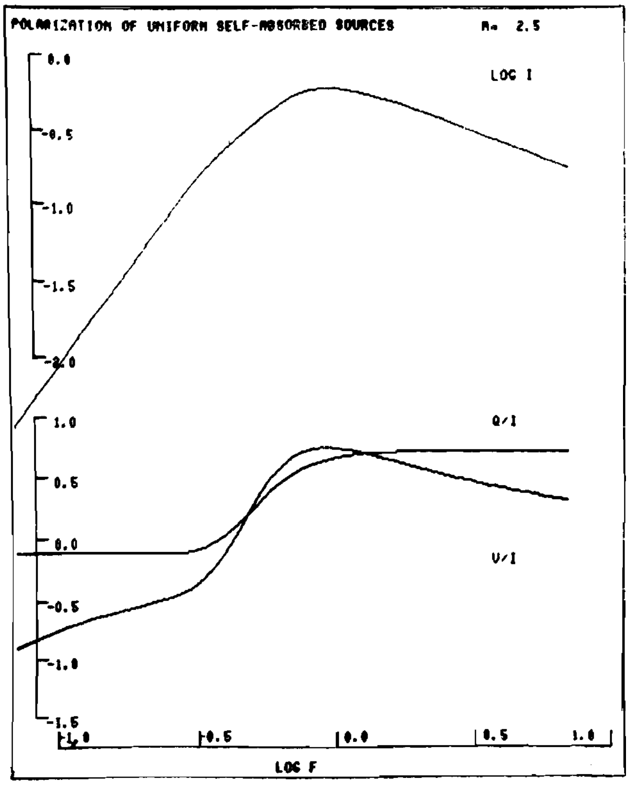

Figure 1. The logarithm of the intensity I in arbitrary units, the degree of linear polarization (Q/I) in actual units and the degree of circular polarization are plotted as function of the logarithm of the frequency f. Log $f = 0$ corresponds to the maximum of log I: data points are separated by intervals of 0.04 in log f. The index of the inverse power law energy spectrum is a $= 2.5$.

TABLE I

Ratio of the Frequency f_l (f_c) at which r_l (r_c) Passes Through Zero to the frequency f_m at which the Intensity is a Maximum for a Range of Values of the Index a of the Inverse Power Law Energy Spectrum

a	f_l/f_m	f_c/f_m
1.5	0.26	0.30
2.0	0.34	0.39
2.5	0.40	0.45
3.0	0.43	0.48
3.5	0.45	0.50
4.0	0.48	0.52

DISCUSSION

With the above model both degrees of polarization reverse sign at about the same frequency which is somewhat less than half the frequency at which the intensity is a maximum. The intensity is between one half and one third of its maximum value at the frequency where reversal occurs.

The observational evidence against the existence of reversals of the degree of circular polarization[4] is as yet insufficient to cast serious doubt on the accepted interpretation of the turnovers as being due to self-absorption. In the next paper J. A. Roberts and R. S. Roger (private communication) show that the shape of the spectra and measured values of V/I for a number of self-absorbed sources can be reproduced with reasonable accuracy by superimposing two or three of the calculated spectra. One could argue that some spherical model would be a more plausible choice for the basic elements in such reconstructions. Calculations for model 2 are being carried out with this in mind; for model 3 I approaches infinity rather than zero at very small frequencies so that model 3 is unacceptable for this purpose.

The calculations were performed by Mrs H. May on the PDP 15/20 computer at CSIRO Division of Radiophysics. Drs J. A. Roberts and R. S. Roger suggested the investigation.

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[2] Pacholczyk, A. G. and Swihart, T. L., *Astrophys. J.*, 170, 405 (1971).
[3] Legg, M. P. C. and Westfold, K. C., *Astrophys. J.*, 154, 499 (1968).
[4] Roberts, J. A., Ribes, J. C., Murray, J. D. and Cooke, D. J., *Nature Phys. Sci.*, 236, 3 (1972).