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Mode Coupling in the Solar Corona. I
Coupling near the Plasma Level

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Abstract

The theory of mode coupling in the radiation from solar type I storms is extended to treat coupling when the frequency f is near the plasma frequency f_p. It is found that Cohen's coupling ratio $Q = (f/f_t)^4$, where f_t is the transition frequency ($f \gg f_t$ for strong coupling), is to be multiplied by a factor $(1-f_p^2/f^2)^{5/2}$ for QT regions, i.e. coupling is relatively suppressed close to the plasma level. The implications on the handedness of solar radio radiation are discussed. The possibility of depolarization due to mode coupling is considered briefly.

1. Introduction

There is evidence from observational data that coupling between magnetoionic modes in the solar corona is more effective than predicted by existing theories with 'reasonable' choices of parameters. We use the terminology of Cohen (1960), in which *strong coupling* refers to the limiting case where there is no change in the state of polarization of radiation due to propagation effects, and *weak coupling* to the other limiting case where waves remain in the same magnetoionic mode (i.e. the polarization continuously adjusts to the local polarization for the mode in question). Cohen defined a transition frequency f_t such that for $f \gg f_t$ ($f \ll f_t$) the coupling is strong (weak). He estimated that the coupling should be weak for radio waves in the solar corona except at so-called QT (quasi-transverse) regions, where the direction of ray propagation and the direction of the magnetic field are orthogonal. In practice a QT region could be regarded as an interface separating a region where the angle between the ray and the magnetic field is acute from a region where it is obtuse. A transition frequency could be ascribed (for a given ray) to each such QT region. For $f \gg f_t$ the handedness of the polarization would not change on crossing such a QT region but for $f \ll f_t$ it would reverse, i.e. LH would become RH and vice versa. Cohen estimated that f_t is in the microwave range, $f_t \gtrsim 10^3$ MHz, for QT regions in the corona. This would imply that for metre-wave radiation the handedness should reverse every time a QT region is crossed. It is this conclusion which is in conflict with the observational data. The evidence that either no polarization reversals occur or that such reversals are very rare is summarized briefly in Section 4 below; it has been discussed in more detail by Melrose (1973).

Cohen's (1960) formula for the transition frequency is equivalent to

$$f_t^4 \approx 5L_B f_p^2 f_H^3/c\,, \tag{1}$$

where all frequencies are in megahertz and L_B is the characteristic distance in kilometres over which the direction of the magnetic field varies (see equation (35) below).

For a plasma frequency $f_p \sim 100$ MHz and the 'reasonable' estimates of a magnetic field of $B \sim 1$ G ($f_H \sim 3$ MHz) with a characteristic distance $L_B \sim 10^5$ km, equation (1) gives $f_t \sim 10^3$ MHz, as found by Cohen. For f_t to be much less than 100 MHz would require very weak ($B \ll 1$ G) and/or very tangled ($L_B \ll 10^5$ km) magnetic fields. Alternatively, the handedness of metre-wave radiation could be preserved on crossing a QT region if equation (1) gave a drastic overestimate of the transition frequency (e.g. because of the inapplicability of the assumptions under which it was derived).

In this paper and in the following Part II (Melrose 1974, present issue pp. 43–52) certain of the assumptions made in deriving equation (1) are relaxed. The important assumptions made by Cohen (1960) were: (i) that the inhomogeneous corona can be regarded as smoothly varying and stratified, i.e. all plasma parameters depend on only one coordinate, z say; (ii) that there is vertical incidence, i.e. propagation along the z axis (because of the development of the theory in connection with the ionosphere, 'vertical' incidence has become accepted terminology for normal incidence on the stratified medium); and (iii) that the frequencies are high ($f \gg f_p, f_H$). In this paper the assumption (iii) is relaxed to allow $f \gtrsim f_p \gg f_H$, while in Part II the assumption of vertical incidence is relaxed. It is shown that neither assumption causes Cohen's estimate of f_t to be too high. The possibility that assumption (i) is inapplicable is discussed qualitatively in Section 4 below.

In Section 2 mode coupling is treated following Clemmow and Heading (1954). The appropriate generalizations of Cohen's (1960) formulae are derived in Section 3. The polarization of metre-wave solar radio bursts is discussed in Section 4. A limitation imposed by coupling between 'upgoing' and 'downgoing' waves is derived in Appendix 1.

2. The Coupled Equations

In this section coupled equations describing the propagation of magnetoionic waves in a stratified medium are written down following Clemmow and Heading (1954) (see also e.g. Budden and Clemmow 1957; Budden 1961, Ch. 18; Heading 1961; Budden 1972). The equations are evaluated explicitly for vertical incidence.

Method of Clemmow and Heading

Clemmow and Heading (1954) used a matrix notation in writing the rate of change of the components of the field vectors from Maxwell's equations. After choosing the coordinate axes such that there is no variation with respect to y and Fourier transforming in time and in distance along the x axis (that is, $\partial/\partial y \to 0$, $\partial/\partial x \to -\mathrm{i}(\omega/c)\sin\psi$, $\partial/\partial t \to \mathrm{i}\omega$), they wrote

$$\mathbf{e}' = -\mathrm{i}(\omega/c)\mathbf{T}\,\mathbf{e}\,. \tag{2}$$

Here $\mathbf{e}$ is the column matrix with elements E_x, $-E_y$, B_x and B_y (Gaussian units are used throughout the present paper), $\mathbf{T}$ is a 4×4 matrix and the prime denotes differentiation with respect to z. With the techniques used here and in Part II no explicit expression for $\mathbf{T}$ is required. The other components, namely E_z and B_z, are related algebraically to those retained.

The characteristic equation

$$\det(\mathbf{T} - q\,\mathbf{1}) = 0\,, \tag{3}$$

where $\mathbf{1}$ is the unit 4×4 matrix, is the Booker quartic equation. Let the four solutions of (3) be written as $q = q_i$, with $i = 1, ..., 4$, and let the corresponding four characteristic column matrices be $\mathbf{v}_i$. In both this paper and Part II these solutions are constructed from the known results of magnetoionic theory—it is not necessary to solve for the q_i or $\mathbf{v}_i$ explicitly.

Suppose one has solved for the four characteristic column vectors $\mathbf{v}_i$. Define the 4×4 matrix $\mathbf{R}$ such that its ith column is $\mathbf{v}_i$, and then define a new column matrix $\mathbf{f}$ by

$$\mathbf{f} = \mathbf{R}^{-1}\mathbf{e}. \tag{4}$$

(The method breaks down if $\mathbf{R}$ is singular, i.e. if any two of the solutions are the same.) The original equation (2) becomes

$$\mathbf{f}' + \mathrm{i}(\omega/c)\mathbf{Q}\,\mathbf{f} = -\mathbf{R}^{-1}\mathbf{R}'\mathbf{f}, \tag{5}$$

$\mathbf{Q}$ being the diagonal matrix with elements q_1, q_2, q_3 and q_4. The elements of $\mathbf{f}$ can be interpreted as amplitudes of the four modes. The matrix $-\mathbf{R}^{-1}\mathbf{R}'$ in equation (5) can then be interpreted as a coupling matrix.

It is desirable to define a new set of amplitudes so that no autocoupling appears, i.e. so that the new coupling matrix has no diagonal elements. This can be achieved as follows. Write

$$-\mathbf{R}^{-1}\mathbf{R}' = \mathbf{D} + \mathbf{\Gamma}, \tag{6}$$

where $\mathbf{D}$ is the matrix consisting of the diagonal elements of $-\mathbf{R}^{-1}\mathbf{R}'$, introduce a new diagonal matrix $\mathbf{d}$ which satisfies

$$\mathbf{d}' = -\mathbf{D}\,\mathbf{d} \tag{7}$$

and write

$$\mathbf{g} = \mathbf{d}\,\mathbf{f}, \qquad \boldsymbol{\gamma} = \mathbf{d}\,\mathbf{\Gamma}\,\mathbf{d}^{-1}. \tag{8}$$

The coupled equations (6) become

$$\mathbf{g}' + \mathrm{i}(\omega/c)\mathbf{Q}\,\mathbf{g} = \boldsymbol{\gamma}\,\mathbf{g}. \tag{9}$$

Vertical Incidence

For vertical incidence it is straightforward to write down the four solutions q_i and the characteristic column matrices $\mathbf{v}_i$. The labelling of the solutions may be chosen such that one has

$$q_1 = \mu_{\mathrm{o}}, \qquad q_2 = \mu_{\mathrm{x}}, \qquad q_3 = -\mu_{\mathrm{o}}, \qquad q_4 = -\mu_{\mathrm{x}}, \tag{10}$$

where μ_σ with $\sigma = \mathrm{o}$ or x is the refractive index for one of the magnetoionic modes. The four solutions define four modes which may be labelled o↑, x↑, o↓ and x↓ for short, with ↑ (↓) referring to upgoing (downgoing) waves.

Using the polarization vectors known from magnetoionic theory it is straightforward to write down the ratio $E_x : -E_y : B_x : B_y$, where the z direction is the direction of wave propagation, and so to write down an expression for $\mathbf{R}$. The arbitrary factors multiplying each column may be chosen for convenience; a convenient choice gives

$$\mathbf{R} = \begin{bmatrix} R_{\mathrm{o}} & R_{\mathrm{x}} & R_{\mathrm{o}} & R_{\mathrm{x}} \\ -1 & -1 & -1 & -1 \\ -\mu_{\mathrm{o}} & -\mu_{\mathrm{x}} & \mu_{\mathrm{o}} & \mu_{\mathrm{x}} \\ \mu_{\mathrm{o}} R_{\mathrm{o}} & \mu_{\mathrm{x}} R_{\mathrm{x}} & -\mu_{\mathrm{o}} R_{\mathrm{o}} & -\mu_{\mathrm{x}} R_{\mathrm{x}} \end{bmatrix}, \tag{11}$$

where R_σ is the ratio E_x/E_y for upgoing waves in the mode σ. When the unit vector along the background magnetic field is written

$$\boldsymbol{b} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta), \tag{12}$$

the explicit expression for R_σ reads

$$R_\sigma = (\cos\phi\, T_\sigma - \mathrm{i}\sin\phi)/(\sin\phi\, T_\sigma + \mathrm{i}\cos\phi), \tag{13}$$

where $T_\sigma(\omega,\theta)$ is in the notation used by Melrose and Sy (1972, Appendix I). The identities

$$R_\mathrm{o} R_\mathrm{x}^* \equiv -1 \equiv R_\mathrm{o}^* R_\mathrm{x}, \tag{14}$$

where the asterisk denotes complex conjugation, follow from $T_\mathrm{o} T_\mathrm{x} = -1$.

It might be commented that for downgoing waves the ratio E_x/E_y is formally R_σ^* and not R_σ. However, for downgoing waves the angle between $\boldsymbol{k}$ and $\boldsymbol{B}$ is $\pi-\theta$ rather than θ, and because $T_\sigma(\omega,\theta)$ is an odd function of $\cos\theta$ one has

$$R_\sigma^*(\omega, \pi-\theta) = R_\sigma(\omega,\theta).$$

This identity has been used in the third and fourth columns of the matrix (11), as has the corresponding identity

$$\mu_\sigma(\omega,\pi-\theta) = \mu_\sigma(\omega,\theta)$$

for the refractive indices.

The calculation of $\mathbf{R}'$ and of $\mathbf{R}^{-1}$ is straightforward. The nondiagonal elements of $-\mathbf{R}^{-1}\mathbf{R}'$, that is, the elements of $\boldsymbol{\Gamma}$, read

$$\Gamma_{12} = \Gamma_{34} = \frac{\mu_\mathrm{o}+\mu_\mathrm{x}}{2\mu_\mathrm{o}}\frac{R_\mathrm{x}'}{R_\mathrm{x}-R_\mathrm{o}}, \qquad \Gamma_{21} = \Gamma_{43} = \frac{\mu_\mathrm{o}+\mu_\mathrm{x}}{2\mu_\mathrm{x}}\frac{R_\mathrm{o}'}{R_\mathrm{o}-R_\mathrm{x}}; \tag{15a}$$

$$\Gamma_{14} = \Gamma_{32} = \frac{\mu_\mathrm{o}-\mu_\mathrm{x}}{2\mu_\mathrm{o}}\frac{R_\mathrm{x}'}{R_\mathrm{x}-R_\mathrm{o}}, \qquad \Gamma_{41} = \Gamma_{23} = \frac{\mu_\mathrm{x}-\mu_\mathrm{o}}{2\mu_\mathrm{x}}\frac{R_\mathrm{o}'}{R_\mathrm{o}-R_\mathrm{x}}; \tag{15b}$$

$$\Gamma_{13} = \Gamma_{31} = \mu_\mathrm{o}'/2\mu_\mathrm{o}, \qquad \Gamma_{24} = \Gamma_{42} = \mu_\mathrm{x}'/2\mu_\mathrm{x}. \tag{15c}$$

Here the elements are grouped according to those which describe the following three classes of coupling: (*a*) between unlike parallel modes, o↑ and x↑ or o↓ and x↓; (*b*) between unlike antiparallel modes, o↑ and x↓ or o↓ and x↑; and (*c*) between like antiparallel modes, o↑ and o↓ or x↑ and x↓. Class (*a*) is the one of interest here. Class (*c*) is associated with reflection (see Appendix 1).

The elements of the coupling matrix $\boldsymbol{\gamma}$ in equation (9) are given by

$$\gamma_{ij} = (d_i/d_j)\Gamma_{ij} \qquad (i,j = 1,\dots,4 \text{ not summed}), \tag{16}$$

with

$$d_1 = d_3^* = \mu_\mathrm{o}^{\frac{1}{2}}\exp\left(\int\frac{R_\mathrm{o}'\,\mathrm{d}z}{R_\mathrm{o}-R_\mathrm{x}}\right), \qquad d_2 = d_4^* = \mu_\mathrm{x}^{\frac{1}{2}}\exp\left(\int\frac{R_\mathrm{x}'\,\mathrm{d}z}{R_\mathrm{x}-R_\mathrm{o}}\right). \tag{17}$$

If the downgoing waves are now ignored by defining $g_3 = g_4 = 0$, equation (9) becomes

$$\begin{bmatrix} g_\mathrm{o}' \\ g_\mathrm{x}' \end{bmatrix} + \mathrm{i}(\omega/c)\begin{bmatrix} \mu_\mathrm{o} & 0 \\ 0 & \mu_\mathrm{x} \end{bmatrix}\begin{bmatrix} g_\mathrm{o} \\ g_\mathrm{x} \end{bmatrix} = \begin{bmatrix} 0 & \chi_\mathrm{x} \\ \chi_\mathrm{o} & 0 \end{bmatrix}\begin{bmatrix} g_\mathrm{o} \\ g_\mathrm{x} \end{bmatrix}, \tag{18}$$

with

$$\chi_\sigma = \tfrac{1}{2}\{(\mu_o+\mu_x)/(\mu_o\mu_x)^{\frac{1}{2}}\}\psi_\sigma, \qquad (19)$$

where

$$\psi_o = \frac{R_o'}{R_o-R_x}\exp\left(\int dz\,\frac{R_x'+R_o'}{R_x-R_o}\right), \qquad (20a)$$

$$\psi_x = \frac{R_x'}{R_x-R_o}\exp\left(\int dz\,\frac{R_o'+R_x'}{R_o-R_x}\right) \qquad (20b)$$

are the coupling parameters introduced by Cohen (1960). The coupling parameters (19) or (20) can be shown to satisfy

$$\chi_o^* = -\chi_x, \qquad \psi_o^* = -\psi_x. \qquad (21)$$

3. Transition Frequency

The coupled equations (18) are now used to evaluate a coupling ratio Q and a transition frequency f_t for arbitrary $f > f_p$ (with the following proviso, however); this generalizes the analogous result derived by Cohen (1960) for $f \gg f_p, f_H$. The proviso is that only waves above the cutoff frequency of the x-mode at $X = 1-Y$ (where $X = f_p^2/f^2$ and $Y = f_H/f$) are of interest for the coupling considered here. For $f_H \ll f_p$ this cutoff occurs at $f_p+\frac{1}{2}f_H$. Only frequencies $f > f_p+\frac{1}{2}f_H \gg f_H$ are considered.

Coupling Ratio

Following Budden (1952) and Cohen (1960), one can conveniently introduce the amplitudes

$$u_\sigma = g_\sigma \exp\left((i\omega/2c)\int dz\,(\mu_o+\mu_x)\right). \qquad (22)$$

Then equation (18) becomes

$$\begin{bmatrix} u_o' \\ u_x' \end{bmatrix} = \begin{bmatrix} -(i\omega/2c)\Delta\mu & \chi_x \\ \chi_o & (i\omega/2c)\Delta\mu \end{bmatrix}\begin{bmatrix} u_o \\ u_x \end{bmatrix}, \qquad (23)$$

with

$$\Delta\mu = \mu_o-\mu_x. \qquad (24)$$

The coupling ratio can then be defined as

$$Q = |\chi_o\chi_x/(\omega\Delta\mu/2c)^2|^{\frac{1}{2}}. \qquad (25)$$

The physical significance of Q can be illustrated as follows. (An alternative physical description of the coupling has been given recently by Titheridge (1971), who considered only the case of a real coupling coefficient, i.e. $\alpha = 0$ in equation (27b) below.) Firstly note that equation (23) with the properties (21) implies

$$|u_o|^2+|u_x|^2 = \text{const.}, \qquad (26)$$

which has an obvious interpretation as expressing energy conservation. Let $\Delta\phi$ be the phase difference between u_o and u_x and write

$$\rho = |u_o|/(|u_o|^2+|u_x|^2)^{\frac{1}{2}}, \qquad \chi_o = a\exp i\alpha. \qquad (27a, b)$$

Then equation (23) gives

$$\rho' = -(1-\rho^2)^{\frac{1}{2}} a \cos(\Delta\phi+\alpha), \tag{28a}$$

$$\Delta\phi' = -\frac{\omega}{c}\Delta\mu + \frac{1-2\rho^2}{\rho(1-\rho^2)^{\frac{1}{2}}} a\sin(\Delta\phi+\alpha). \tag{28b}$$

For $a \ll \frac{1}{2}\omega\,\Delta\mu/c$, that is, for $Q \ll 1$, the phase difference between the two waves changes much more rapidly than their relative amplitude. This is Faraday rotation. For $\rho = 1$ initially, the ratio $|u_x|/|u_o|$ oscillates with an amplitude of order Q. Thus to lowest order in Q the two modes propagate independently.

On the other hand, for $a \gg \omega\,\Delta\mu/c$, that is, for $Q \gg 1$, one can neglect the term involving $\Delta\mu$ in equation (28b) and integrate to find

$$\rho(1-\rho^2)^{\frac{1}{2}}\sin(\Delta\phi+\alpha) = \text{const.},$$

where $\alpha =$ const. has been assumed for simplicity. With equation (26) this implies

$$|u_\pm|^2 = \text{const.}, \qquad u_\pm = u_o \pm i u_x \exp(-i\alpha),$$

which also follows directly from equation (23) with $\Delta\mu = 0$. These conservation laws imply that the initial polarization is preserved.

Evaluation of Q

The explicit evaluation of Q is indicated in Appendix 2. The result is

$$Q = \frac{c}{\omega}\frac{(\mu_o+\mu_x)^2}{(\mu_o\mu_x)^{\frac{1}{2}}}\frac{1-X-Y^2+XY^2\cos^2\theta}{XY^2\sin^2\theta}\left(\frac{T_\sigma^2-1}{T_\sigma^2+1}\right)^2 \times\left\{(\phi')^2+\frac{T_\sigma^2}{(T_\sigma^2+1)^2}\left(\frac{Y'}{Y}+\frac{X'}{1-X}+(2\cot\theta+\tan\theta)\theta'\right)^2\right\}^{\frac{1}{2}}. \tag{29}$$

For $Y \ll 1-X$ and $X \ll 1$ one can justify the following approximation in (29),

$$\{(\mu_o+\mu_x)^2/(\mu_o\mu_x)^{\frac{1}{2}}\}(1-X-Y^2+XY^2\cos^2\theta) \approx 4(1-X)^{3/2}. \tag{30}$$

Further simplification occurs in the QL (quasi-longitudinal) and QT limits. The QL (QT) limit applies for $\theta \ll \theta_0$ ($\theta \gg \theta_0$) for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$ with θ_0 given by

$$\sin^2\theta_0 = 2Y^{-2}[\{(1-X)^4+Y^2(1-X)^2\}^{\frac{1}{2}}-(1-X)^2]. \tag{31a}$$

For $Y \ll 1-X$ equation (31a) simplifies to

$$\cos\theta_0 \approx Y/2(1-X). \tag{31b}$$

Even at $Y = 1-X$ equation (31b) is adequate for semiquantitative purposes (e.g. at $Y = 1-X$ equation (31b) gives $\cos\theta_0 = 0{\cdot}5$ whereas (31a) gives $\cos\theta_0 = \sqrt{2}-1 = 0{\cdot}414$; for $Y < 1-X$ the errors are smaller than this).

In the QL limit one sets $|T| = 1$ in equation (29) except in the factor T^2-1, which is rewritten using the quadratic equation that T satisfies (see Appendix 2). One finds

$$Q \approx \frac{c}{\omega}\frac{\tan^2\theta}{X(1-X)^{\frac{1}{2}}}\left\{(\phi')^2+\frac{1}{4}\left(\frac{Y'}{Y}+\frac{X'}{1-X}+(2\cot\theta+\tan\theta)\theta'\right)^2\right\}^{\frac{1}{2}}. \tag{32}$$

In the QT limit one has

$$T_o = -Y\sin^2\theta/(1-X)\cos\theta = -T_x^{-1}. \tag{33}$$

In this case equation (29) with the approximation (30) reduces to

$$Q \approx \frac{4c}{\omega}\frac{(1-X)^{5/2}}{XY^3\sin^3\theta}\left\{\left(\theta'(1+2\cot^2\theta)+\frac{Y'\cot\theta}{Y}+\frac{X'\cot\theta}{1-X}\right)^2+\frac{(\phi')^2Y^2\sin^2\theta}{(1-X)^2}\right\}^{\frac{1}{2}}. \tag{34}$$

The limiting cases (32) and (34) overlap for $\theta \approx \theta_0$.

The results derived by Cohen (1960) are reproduced by replacing the factors $(1-X)^{-\frac{1}{2}}$ and $(1-X)^{5/2}$ in equations (32) and (34) respectively by unity. (Cohen has an unimportant error in the power of $\sin\theta$ in his counterpart of equation (34).)

In the QL limit, coupling is ineffective for radio waves in the corona. Although for $1-X \ll 1$ equation (32) predicts that Q becomes large like $(1-X)^{-3/2}$, large Q in this case does not imply strong coupling. As shown in Appendix 1, coupling between upgoing and downgoing waves in this limit is more important than coupling between the different modes. Heading (1961) and Budden (1972) pointed out that large coupling parameters need not imply effective coupling, and their comments are appropriate to the coupling in a QL region for $1-X \ll 1$.

In the QT limit, the factor $(1-X)^{5/2}$ in equation (34) implies that the coupling becomes less effective as the plasma frequency is approached.

Transition Frequency

Following Cohen (1960) the transition frequency can be defined by assuming that the term in equation (34) involving

$$|\theta'| = L_B^{-1} \tag{35}$$

dominates and by then writing

$$Q = (1-X)^{5/2} f^4/f_t^4, \tag{36}$$

with f_t given by equation (1). For $f \gg f_p$ equation (36) implies $Q = 1$ at $f = f_t$. However, for f close to f_p the factor $(1-f_p^2/f^2)^{5/2}$ causes Q to depend on f in a more complicated way.

Suppose we seek the conditions under which f_t is equal to αf_p with α (>0) some predetermined constant. (For example, radio waves generated at the plasma frequency may encounter a QT region before the level at which the plasma frequency is half that at the initial level. If there is to be no reversal of polarization f_t must be less than $2f_p$ in this case.) Using equations (1) and (36) we find that the condition $f_t \leqslant \alpha f_p$ implies

$$f_H^3 \leqslant g(\alpha) f_p^2/L_B \tag{37}$$

with

$$g(\alpha) = (\alpha^2-1)^{5/2}/5\alpha. \tag{38}$$

A plot of $g(\alpha)$ is given in Fig. 1.

The inequality (37) would place an upper limit on the magnetic field strength or an upper limit on L_B if it could be argued that no polarization reversal occurs on crossing a QT region at a frequency close to the plasma frequency.

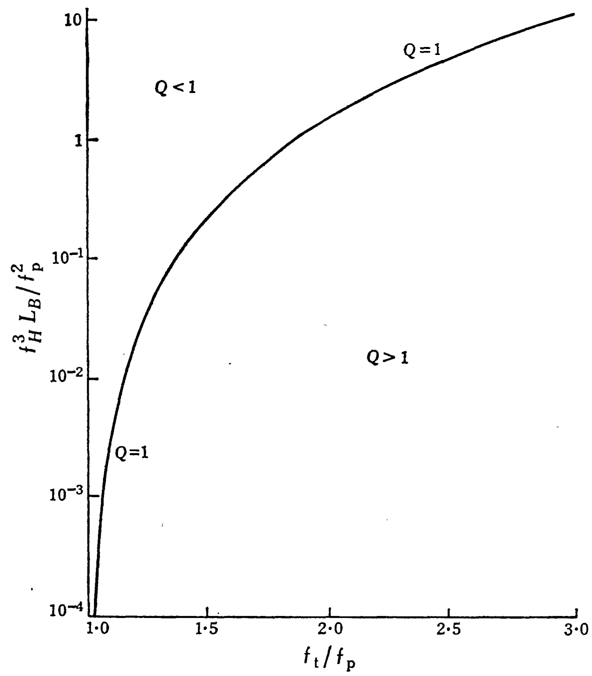

Fig. 1. Plot of $g(\alpha) = f_H^3 L_B/f_p^2$ as a function of f_t/f_p, with the frequencies f_H and f_p in megahertz and L_B in kilometres, which gives $Q = 1$. The regions of strong ($Q > 1$) and weak ($Q < 1$) coupling are indicated.

4. Polarization of Solar Radio Bursts

In this section the polarization of metre-wave solar noise storm radiation is discussed briefly. Evidence in favour of a depolarizing agent acting in the corona is indicated. Mode coupling is then discussed with this evidence in mind.

Polarization of Type I Storms

The motivation for the present investigation involved a discrepancy between theory and observation relating to the polarization of type I solar radio storms (Melrose 1973). The discrepancy can be summarized as follows.

(1) If weak coupling obtains in a storm centre situated in a bipolar magnetic field it would be predicted that (i) the two feet of the bipolar region would be polarized in the same sense (Piddington and Minnett 1951) contrary to observation, and (ii) this sense should reverse at CMP (central meridian passage) (Martyn 1946; Piddington and Minnett 1951; Takakura 1961; Zhelezniakov 1970, p. 131) again contrary to observation. Thus the coupling would appear to be strong.

(2) The identification of the emission as ordinary mode from the dominant spot (Payne-Scott and Little 1951) presupposes that mode coupling is strong at every QT region between the source and Earth.

(3) On the other hand, the proposal by Cohen (1961) that polarization reversals in microwave bursts could be explained in terms of mode coupling with $f_t \sim 10^3$ MHz would suggest that f_t is well above the observed frequencies for type I storms (which come from active regions as do the microwave bursts), i.e. that the coupling should be weak in QT regions above storm centres for metre-wave radiation.

The thought behind the present investigation was that if mode coupling were stronger for $f \approx f_p$ than would be estimated by extrapolating from the case $f \gg f_p$, then one could hope to resolve the above discrepancy between Cohen's (1961) theory and the observation of metre-wave radiation. The result of the present investigation is that mode coupling for $f \approx f_p$ is weaker than would have been estimated from the case $f \gg f_p$. Various possible ways in which the above discrepancy might be resolved have been discussed by Melrose (1973).

It might also be commented that the argument in point (3) above could be used against Cohen's (1961) interpretation of the polarization reversals for microwave bursts (the argument being that $f_t \leqslant 100$ MHz is implied from the observed properties of type I storms). Recent observations of microwave bursts tend to favour other explanations of the polarization reversal (see e.g. Énomé *et al.* 1969; Tanaka and Énomé 1970; Naito and Takakura 1973).

Depolarization

There is evidence for depolarization of metre-wave radiation due to propagation effects. Noise storms and type I bursts near the limb appear to come from higher in the corona and to be less polarized than those near the central meridian (Morimoto and Kai 1961; Le Squeren 1963). Suzuki (1961) suggested that the decrease in the degree of polarization as the limb is approached is due to effects of mode coupling.

A strong case can be made in favour of a depolarizing agent acting on type II bursts. The absence of significant polarization (Komesaroff 1958; Stewart 1966) would imply $B < 10^{-2}$ G if the emission mechanism were the same as in type III bursts (Melrose and Sy 1972). On the other hand, if the exciting agency is a hydromagnetic shock with a Mach number of 2–3 (Smith 1971), the statistical analysis by Weiss (1965) implies typical field strengths $B \sim 1$ G. These assumptions regarding the nature of the emission mechanism and of the exciting agency would be compatible only if the observed radiation had been depolarized.

A similar, but less convincing, case can be made in favour of depolarization of type III bursts.

Depolarization due to Mode Coupling

The possibility that mode coupling might lead to depolarization, as was suggested by Suzuki (1961), does not seem to have been discussed in detail. One cause of depolarization was pointed out by Zhelezniakov and Zlotnik (1963). They showed that radiation initially 100% polarized in one mode becomes partially linearly polarized on crossing a QT region. The degree of linear polarization r_l they derived is given by

$$r_l = 2\{e^{-2\delta}(1-e^{-2\delta})\}^{\frac{1}{2}}, \tag{39}$$

with

$$2\delta = \pi/2Q.$$

Differential Faraday rotation can depolarize this linear component. The amount of depolarization which results depends on the magnitude of $Q \propto (f/f_t)^4$. Zhelezniakov and Zlotnik presented a plot of r_l as a function of f/f_t. Some numerical values showing the extremes of the frequency range $f_1 \leqslant f_t \leqslant f_2$ over which r_l exceeds a given value, with f_t redefined such that equation (39) gives $r_l = 1$ at $f = f_t$, are set out below.

r_l	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1
f_1/f_t	1·00	0·86	0·81	0·77	0·74	0·71	0·68	0·65	0·62	0·58
f_2/f_t	1·00	1·20	1·33	1·46	1·60	1·78	2·00	2·34	2·87	4·08

From these values one infers, for example, that a reduction in the degree of polarization by a factor greater than 2 ($= r_l^{-1}$ and thus requiring $r_l = \frac{1}{2}$) occurs over a frequency range $0 \cdot 7 f_t < f < 1 \cdot 8 f_t$, that is, over a bandwidth of order f_t.

There is an indication that depolarization occurs close to the plasma level, e.g. in type II bursts and type I bursts near the limb. To have effective mode coupling at a QT region close to the plasma level requires

$$5 L_B f_H^3 < f_p^2 , \tag{40}$$

where f_H and f_p are in megahertz and L_B is in kilometres. If the condition (40) is satisfied there exists a region with $Q \approx 1$ near the plasma level (see Fig. 1). Because the QT approximation applies over a relatively large range of angles for $X \approx 1$ (see equation (31a)), and because $Q \sim 1$ implies substantial depolarization, such regions could be effective in depolarizing radiation emitted near the plasma frequency.

Furthermore, it would not be necessary for the angle between $\boldsymbol{k}$ and $\boldsymbol{B}$ to pass through $\frac{1}{2}\pi$ for the coupling to be important; the terms involving ϕ' and X' in equation (34) can also be important for $X \approx 1$ (but $1 - X$ must not be so small that the inequality (A4) of Appendix 1 is violated). In particular, large amplitude Alfvén waves, which lead to twists in the magnetic field lines (i.e. $\phi' \neq 0$), could cause some coupling between magnetoionic modes close to the plasma level. Such an effect, along with effects of other random variations in the properties of the medium, are outside the scope of the theory used here. The suggestion that certain types of randomness in the medium might lead to effective mode coupling does not seem to have been considered.

5. Conclusions

The results of the present investigation can be summarized as follows.

(1) When the condition $f \gg f_p$ is relaxed Cohen's (1960) coupling ratio Q is to be replaced by $\mu^5 Q$ in a QT region and by $\mu^{-1} Q$ in a QL region, where $\mu = (1 - f_p^2/f^2)^{\frac{1}{2}}$ is the refractive index ($f_p \gg f_H$ is assumed).

(2) The enhanced coupling for $\mu \approx 0$, i.e. near the plasma level, in a QL region is partly spurious because reflection effects cannot be ignored for $\mu \approx 0$.

(3) There is no unambiguous evidence of polarization reversals due to propagation effects for metre-wave radiation. If radiation at a frequency $f = \alpha f_p$, where f_p is the local plasma frequency, encounters a QT region this observation implies that $L_B f_H^3/f^2$ is less than the value give by the curve in Fig. 1; for example, for $f \approx 2 f_p = 100$ MHz this would imply $B^3 L_B < 300\ \mathrm{G}^3\,\mathrm{km}$.

(4) Depolarization can occur in a QT region near the plasma level provided that the frequency f_t as given by equation (1) is much less than f_p.

The conditions under which a QT region low in the corona would cause no reversal of the handedness of radiation passing through it are quite restrictive. One would expect to observe occasional reversals in handedness caused by propagation effects, as was suggested by Martyn (1946), Piddington and Minnett (1951) and Zhelezniakov (1970, p. 372).

Another effect which may be observable relates to the suggestion that the lower degree of polarization of type I bursts near the limb might be due to mode coupling (Suzuki 1961). The mechanism by which mode coupling can cause depolarization is through a partial conversion of circular into linear polarization. With a sufficiently narrow bandwidth the linear polarization may be observable.

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Appendix 1. Coupling between Upgoing and Downgoing Waves

Suppose that waves in only one mode, o↑ say, are present initially. For coupling between o↑ and x↑ to be effective requires not only $Q > 1$ but also that Q be greater than the analogous coupling ratio Q_R between o↑ and o↓. Q_R can be estimated as follows. Ignore x-mode waves in equation (9) by setting $g_2 = g_4 = 0$ to find

$$\begin{bmatrix} g_1' \\ g_3' \end{bmatrix} = \begin{bmatrix} -(\mathrm{i}\omega/c)\mu_o & \mu_o'/2\mu_o \\ \mu_o'/2\mu_o & (\mathrm{i}\omega/c)\mu_o \end{bmatrix} \begin{bmatrix} g_1 \\ g_3 \end{bmatrix}. \tag{A1}$$

By analogy with the definition (25) of Q one can define Q_R by

$$Q_R = c\,|\mu_o'|/2\mu_o^2\,\omega. \tag{A2}$$

The analogous coupling ratio between x↑ and x↓ is obtained by replacing o by x in equation (A2).

For $Y \ll 1$ equation (A2) reduces to

$$Q_R \approx c\,|X'|/4\omega(1-X)^{3/2}. \tag{A3}$$

The neglect of the coupling between upgoing and downgoing waves is justified for

$$Q \gg c\,|X'|/4\omega(1-X)^{3/2}\,. \tag{A4}$$

Appendix 2. Evaluation of Q

In the evaluation of Q (see equation (25)) the following results have been used: from Melrose and Sy (1972, equations (A12) and (A13)),

$$|\Delta\mu| = \frac{|\mu_o^2-\mu_x^2|}{\mu_o+\mu_x} = \frac{T_\sigma^2+1}{(\mu_o+\mu_x)\,|T_\sigma^2-1|}\,\frac{XY^2\sin^2\theta}{1-X-Y^2+XY^2\cos^2\theta}\,; \tag{A5}$$

from equations (14), (19) and (20),

$$|\chi_o\chi_x| = \frac{(\mu_o+\mu_x)^2}{4\mu_o\mu_x}\,\frac{|R'_\sigma R^{*\prime}_\sigma|}{(1+|R_\sigma|^2)^2}\,; \tag{A6}$$

from equation (13),

$$\frac{|R'_\sigma|^2}{(1+|R_\sigma|^2)^2} = \frac{(\phi')^2(T_\sigma^2-1)^2+|T'_\sigma|^2}{(T_\sigma^2+1)^2}\,; \tag{A7}$$

and finally, from the relation (Melrose and Sy 1972, equation (A9))

$$T_\sigma^2+\left(\frac{Y\sin^2\theta}{(1-X)\cos\theta}\right)T_\sigma-1 = 0\,, \tag{A8}$$

the result

$$\begin{aligned} T'_\sigma &= \frac{T_\sigma(T_\sigma^2-1)}{T_\sigma^2+1}\left(\ln\frac{Y\sin^2\theta}{(1-X)\cos\theta}\right)' \\ &= \frac{T_\sigma(T_\sigma^2-1)}{T_\sigma^2+1}\left(\frac{Y'}{Y}+\frac{X'}{1-X}+(2\cot\theta+\tan\theta)\theta'\right). \end{aligned} \tag{A9}$$

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