EFFECTS OF AN AMBIENT MAGNETIC FIELD ON THE PROPERTIES OF LANGMUIR WAVES

(*Extended Summary*)

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1. Introduction

In an isotropic plasma Langmuir waves are strictly longitudinal, and their approximate dispersion relation is

$$\omega^2 \approx \omega_p^2 + 3k^2 V_e^2, \tag{1}$$

where ω_p is the plasma frequency and V_e is the thermal speed of electrons. In a magnetized plasma it is usually assumed that 'generalized' Langmuir waves are longitudinal, and it then follows that their dispersion relation is given by setting the longitudinal part of the dielectric tensor equal to zero. For a weakly magnetized plasma (i.e. for $\omega_B \ll \omega_p$, where ω_B is the electron gyrofrequency) the resulting dispersion relation is

$$\omega^2 \approx \omega_p^2 + 3k^2 V_e^2 + \omega_B^2 \sin^2 \theta. \tag{2}$$

However, it can be shown that the only waves which are strictly longitudinal in a magnetized plasma are propagating either parallel ($\theta = 0$ or π) or perpendicular ($\theta = \pi/2$) to the ambient magnetic field.

Two questions arise. First, under what conditions is the longitudinal approximation valid? Second, when it is not valid, how does one determine the properties of the relevant waves? I shall answer these two questions, starting with the latter, and then mention some possible implications for type III bursts.

2. *Z*-mode Waves

'Generalized' Langmuir waves should be regarded as waves in a modified form of the Z-mode, i.e. the lower frequency branch of the extraordinary mode of magnetoionic theory. (This view was adopted by Dunkel (1974) and it was stated explicitly by Melrose (1975).) The relevant modification is the inclusion of the effects of finite thermal motions of the electrons.

Let the dielectric tensor $\varepsilon_{ij}(\mathbf{k}, \omega)$ be expanded in powers of $k^2 V_e^2/\omega^2$:

$$\varepsilon_{ij}(\mathbf{k}, \omega) = \varepsilon_{ij}^{(0)}(\omega) + \varepsilon_{ij}^{(1)}(\mathbf{k}, \omega) + \dots. \tag{3}$$

Let $n^{(0)}(\omega, \theta)$ and $\mathbf{e}^{(0)}(\omega, \theta)$ be the refractive index and the (unimodular) polarization vector, respectively, for waves in any mode in the cold plasma limit, i.e. for $\varepsilon_{ij} = \varepsilon_{ij}^{(0)}$. A perturbation expansion allows one to determine the thermal

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correction $n^{(1)}(\omega, \theta)$ to the refractive index:

$$[n^{(1)}]^2 = \frac{e_i^{(0)*} e_j^{(0)} \varepsilon_{ij}^{(1)}}{1 - (\boldsymbol{\kappa} \cdot \mathbf{e}^{(0)})^2}, \tag{4}$$

where $\varepsilon_{ij}^{(1)}$ is to be evaluated at $k = n^{(0)}\omega/c$, and where $\boldsymbol{\kappa}$ is a unit vector along $\boldsymbol{\kappa}$.

3. The Longitudinal Approximation

If the thermal corrections are small (i.e. for $[n^{(1)}]^2 \ll [n^{(0)}]^2$), then one is to regard the waves as Z-mode waves, to a first approximation, and thermal corrections may be included by means of this perturbation approach. On the other hand, if $[n^{(1)}]^2$ turns out to be much greater than $[n^{(0)}]^2$, then, because this occurs when $\mathbf{e}^{(0)}$ is very nearly equal to $\boldsymbol{\kappa}$ in (4), one is to regard the waves as longitudinal to a first approximation, and the dispersion relation (2) applies.

It is straightforward to calculate $n^{(1)}$ using (4) and known properties of Z-mode waves, and then to set $n^{(1)} = n^{(0)}$ to determine the range of validity of the longitudinal approximation. The following condition results: the longitudinal approximation is valid only for

$$v_\phi^2 \ll V_0^2, \qquad V_0 = \frac{\sqrt{3}\, V_e}{\sin\theta} \frac{\omega_p}{\omega_B}, \tag{5}$$

where $v_\phi (= \omega/k)$ is the phase speed of the waves. For typical parameters in the solar corona, $(V_0/c) \sin\theta$ is slightly less than unity, and for typical parameters near the orbit of the Earth it is slightly greater than unity. Consequently, one expects the longitudinal approximation to be invalid only for $v_\phi \gtrsim c$, i.e. for supraluminous waves in Lerche's (1968) terminology.

4. Discussion

There are two significant differences between the actual properties of modified Z-mode waves, and the (nonphysical) properties of the hypothesized longitudinal waves for $v_\phi \gg V_0$.

First, the group properties of Z-mode waves are quite complicated, while those of the hypothesized longitudinal waves are simple. Dunkel (1974) pointed out that, because of their group properties, Z-mode waves could be trapped in field-aligned depressions in density. It appears possible that a partial trapping in regions of slightly enhanced density might also be possible. These possibilities warrant a detailed investigation, especially in connection with the suggestion that the Langmuir waves are not seen by spacecraft *in situ* because the waves are confined to blobs, rather than being distributed uniformly throughout the volume filled by the stream of electrons.

The second significant difference relates to waves with $v_\Phi > c$: the modified Z-mode waves have $\omega < \omega_p$ for $v_\Phi > c$, whereas the hypothesized longitudinal waves always have $\omega > \omega_p$. An immediate implication is that Z-mode waves with $v_\Phi > c$ cannot lead to fundamental plasma emission. Suppose that the waves produced by the stream were to evolve in such a way that their phase speeds increase. (This could occur through nonlinear effects or through their propagation into regions of enhanced plasma density.) Then, provided the refractive index remains greater than $\sqrt{3}/2 = 0.87$ (i.e. $v_\Phi < 2c/\sqrt{3}$), second harmonic plasma emission would occur in the absence of any fundamental emission whatsoever. This is one conceivable explanation for the absence of fundamental plasma emission from type III streams in the interplanetary medium.

References

Dunkel, N.: 1974, dissertation, Stanford University.
Lerche, I.: 1968, Phys. Fluids **11,** 2459.
Melrose, D. B.: 1975, *Austral. J. Phys.* **28,** 101.

Discussion

Dunckel: The Z-mode is extremely narrow, width about $\frac{1}{2}$ gyrofrequency.

Melrose: Lerche's superluminous waves are Z-mode.

D. Smith: There is a limitation on θ. What happens to the curves if we start at $\theta = 0$?

Melrose: In the limit $\theta = 0$ the critical speed is infinite which means that the longitudinal approximation is valid always.