COLLECTIVE PLASMA EFFECTS AND THE ELECTRON NUMBER PROBLEM IN SOLAR HARD X-RAY BURSTS

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Abstract. Due to the relatively high stream densities involved, collective interactions with the ambient plasma are likely to be important for the electrons producing solar hard X-ray bursts. In thick- and thin-target bremsstrahlung models the most relevant process is limitation of the invoked electron beams by ion sound wave generation in the neutralizing reverse current established in the atmosphere. For the thick target model it is shown that typical electron fluxes are near the maximum permitted by stability of the reverse current so that ion-sound wave generation may be the process which limits the electron injection rate. On the other hand the chromospheric reverse current is sufficient to supply the large total number of electrons which have to be accelerated in the corona. For the thin target the low density of the corona severely limits the possible reverse current so that the maximum upward flux of fast electrons is probably much too small to explain X-ray bursts but compatible with observations of interplanetary electrons.

A distinct class of model postulates a small number of electrons confined by resonant scattering in a dense coronal slab surrounding a current sheet with continuous stochastic acceleration offsetting collisional losses. The energetic aspects of such a situation described by Hoyng (1975) are developed here by addition of equations describing the slab geometry in terms of electron diffusion by whistler scattering and of the collisional damping of the accelerating Langmuir waves. Solution of these equations results in values for the field B(70–350 G), density $n_0(2\text{–}5\times10^{12}\ cm^{-3})$, slab dimensions ($10^{18}\ \text{km}^2\times0.3\text{–}3$ km) and relative Langmuir energy density (10^{-3}–10^{-2}) required to produce the observed range of bursts. It is pointed out, however, that there may be no real gain in electron number requirements since the fast electrons in the emitting slab would be constantly swept out along with the frozen-in plasma as dissipation proceeds so that a large total number of electrons is still required. It could in fact be that just such a coronal region is the injection mechanism for the thick-target model.

1. Introduction

A variety of solar hard X-ray burst models, based on the generally accepted collisional bremsstrahlung mechanism, is currently to be found in the literature (see e.g., reviews by Kane, 1974; Lin, 1974; Brown, 1975, 1976). The trend in burst analyses based on these models (e.g., Hoyng *et al.*, 1976) has been toward ever larger estimates of the total number and energy of the source electrons (see e.g., Brown, 1975, 1976). Indeed these numbers seem prohibitively large in terms of typical flare fields and plasma densities and much exceed the requirements for both radio emission (type III) and interplanetary particles. Nevertheless the role of collective interactions in the electron streams and the background plasma of these models has so far been largely ignored though such interactions may arise even with the low stream densities typical of type III radio emission (cf. Kaplan *et al.*, 1974; Smith, 1974).

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Three proposals have been made to overcome this problem of large electron numbers. First that the hard X-ray emission might be largely thermal (Chubb, 1971; Brown, 1974). Second that the emission mechanism might be inverse Compton scattering rather than bremsstrahlung, the former requiring far fewer electrons but of very high individual energy (Korchak, 1971; Brown, 1976). Third that a relatively small number of electrons might emit bremsstrahlung while confined in a dense plasma region with continuous reacceleration *in situ* to offset the high Coulomb energy losses (Hoyng, 1975; Brown, 1975). No detailed study has yet been made of collective interactions in any of these situations, nor in the more conventional bremsstrahlung models – thick-target (Brown, 1971; Hudson, 1972), thin-target (Datlowe and Lin, 1973) and coronal magnetic trap (Takakura and Kai, 1966; Brown and Hoyng, 1975).

In this paper we attempt to evaluate three of these possible configurations – thick- and thin-target and electron confinement (i.e., those involving continuous primary acceleration of electrons) in a manner more consistent with constraints from plasma collective processes. We deal with the coronal magnetic trap model elsewhere (Melrose and Brown, 1976; cf. Wentzel, 1976) since this requires a different approach. The thermal and inverse Compton hypotheses are postponed for future consideration.

2. Observational Requirements

It is usually accepted that hard X-ray bursts are 'impulsive' or 'non-thermal' in nature at photon energies $\varepsilon \gtrsim 10$ keV since the time profile is rapidly varying and the energy spectrum is a quasi power-law $\varepsilon^{-\gamma}$ in contrast to the gradual behaviour and near exponential (isothermal) spectrum at lower energies. The actual transition energy from thermal emission is highly uncertain (Brown, 1974) but the consensus is that photons of $\varepsilon \gtrsim 25$ keV are fairly certain to be non-thermal. In what follows, however, it is important to remember that electron numbers and total energies are much greater than quoted here if their steep spectrum ($E^{-\delta}$, $\delta = 2$–6) extends much below 25 keV.

The typical range of photon fluxes I, at the Earth, above 25 keV is (Kane, 1974; Hoyng *et al.*, 1976)

$$I \simeq \begin{cases} 10^1 \\ 10^3 \end{cases} \mathrm{cm}^{-2}\,\mathrm{s}^{-1} \tag{1}$$

with burst durations τ_x ranging over

$$\tau_x \simeq \begin{cases} 10^1 \\ 10^3 \end{cases} \mathrm{s}, \tag{2}$$

where the *upper and lower values in* { *correspond to small and large events respectively, throughout this paper.*

For the collisional bremsstrahlung mechanism source electrons have energies E close to the emitted photon energies ε and the burst intensity I is determined by $n_0 N$ where n_0 is the mean source proton density and N is the number of electrons of $E \geqslant 25$ keV.

The 'non-thermal emission measures' $n_0 N$ corresponding to (1) are found to be

$$n_0 N \simeq \begin{cases} 10^{45} \\ 10^{47} \end{cases} \text{cm}^{-3}. \tag{3}$$

No direct information is available on n_0 so that models are based on postulating high or low n_0 values. The main effect of n_0, which is the density of ambient electrons as well as protons, is to determine the Coulomb collision lifetime τ_{coll} of the fast electrons viz (e.g., Brown, 1971)

$$\tau_{\text{coll}} \simeq \frac{2 \times 10^{10}}{n_0} \left[\frac{E(\text{keV})}{25}\right]^{3/2} \text{s}. \tag{4}$$

An essential feature of *all* bremsstrahlung models is that the collisional energy deposition rate P_{coll} is fixed by the bramsstrahlung emission rate (i.e., by I) essentially independently of the source density n_0 and its distribution. (This is because $P_{\text{coll}} \simeq N Q_{\text{coll}} \bar{E} n_0 v$ and $I \simeq N Q_B n_0 v$ where $Q_{\text{coll}}(E)$ and $Q_B(\varepsilon, E)$ are the Coulomb and bremsstrahlung loss cross sections. Thus, apart from factors around unity due to spectral distribution and inhomogeneous plasma ionisation, P_{coll}/I is independent of n_0 and N everywhere – cf. Brown, 1971.) This fact leads to typical collisional deposition rates (corresponding to (1)) of

$$P_{\text{coll}} \simeq \begin{cases} 2 \times 10^{27} \\ 2 \times 10^{29} \end{cases} \text{erg s}^{-1} \tag{5}$$

and so to total energy depositions $\mathscr{E}_{\text{coll}} \simeq P_{\text{coll}} \tau_x$ of

$$\mathscr{E}_{\text{coll}} \simeq \begin{cases} 2 \times 10^{28} \\ 2 \times 10^{32} \end{cases} \text{erg}. \tag{6}$$

3. Thick and Thin-Target Models

In the thick-target model (Brown, 1971; Hudson, 1972) the bremsstrahlung source lies in the dense chromosphere ($n_0 \simeq 10^{12}$–10^{13} cm^{-3}), electrons being injected and penetrating to there from a coronal supply region. This hypothesis allows rather a small instantaneous number of electrons in the source, viz. by (3)

$$N \simeq \frac{10^{45}\text{–}10^{47}}{n_0} \lesssim \begin{cases} 10^{33} \\ 10^{35}. \end{cases} \tag{7}$$

However, the high density implies a short collisional lifetime (4) and new electrons have to be continuously injected throughout the burst duration τ_x(2). If collisions are the only loss then the injection rate required is independent of n_0, viz.

$$F = \frac{N}{\tau_{\text{coll}}} = \frac{n_0 N}{n_0 \tau_{\text{coll}}} \simeq \begin{cases} 5 \times 10^{34} \\ 5 \times 10^{36} \end{cases} \text{s}^{-1} \tag{8}$$

using (3) and (4). Then the total number of electrons injected is

$$\mathcal{N} = F\tau_x \simeq \begin{cases} 5 \times 10^{35} \\ 5 \times 10^{39} \end{cases} \tag{9}$$

using (2). The area over which this injection takes place is unknown but a plausible value (and upper limit) is the observed area A of the optical flare, viz.

$$A \simeq \begin{cases} 10^{18} \\ 10^{19} \end{cases} \text{cm}^{-2}, \tag{10}$$

where we have very loosely associated intense X-ray bursts with large flare areas.

In the thin-target model the electrons are supposed to be injected continuously upward through the corona (Datlowe and Lin, 1973). Here collisional losses are low and the bremsstrahlung emission lifetime of an electron is governed by escape, i.e., by the density scale height of the source region. The model is somewhat less efficient than the thick target ($\frac{1}{2}$ to 1 order of magnitude down in bremsstrahlung yield per electron (Brown and McClymont, 1975)) so that required electron injection rates would be even higher than (8) though the injection could possibly be spread over larger areas A.

The severity of the above numbers may be illustrated in several ways. Firstly the rates F in (8) correspond to currents of 10^{17}–10^{21} A or current densities of 10^3–10^6 A m^{-2}. Secondly the total number $\mathcal{N}$ of electrons accelerated in the corona is *all* the ambient electrons at a density $n_0 = 10^8$ cm^{-3} in a volume of $(0.02\ R_\odot)^3$–$(0.5\ R_\odot)^3$ or *all* the ambient electrons overlying the flare area A down to the depth where $n_0 \simeq 2 \times 10^8$ cm^{-3} in small flares but down to $n_0 \simeq 3 \times 10^{13}$ cm^{-3} in large flares (i.e., every electron within the flare volume). Thirdly the total electron energy corresponding to (9) is 2×10^{28}–2×10^{32} ergs. This is comparable to the entire magnetic energy available from an active region.

Even if such intense streams can be *produced* (cf. Section 4) it is essential to examine whether they can survive in the solar atmosphere without being destroyed by plasma collective processes (cf. Hoyng *et al.*, 1976). The first candidate is the two stream instability. Smith (1975) has emphasised that the growth time for Langmuir waves would be very much less than the beam duration even though the beam is dilute (density $n \simeq F/Av \simeq 5 \times 10^6$–$5 \times 10^7$ cm$^{-3} \ll n_0$). Thus for a beam with a positive slope in velocity space ($\partial f/\partial v > 0$) the beam energy would be rapidly converted into Langmuir waves in layers of the atmosphere near the site of electron production. This situation is worsened by the fact that such drastic beam losses reduce the electron lifetime for bremsstrahlung emission to $\ll \tau_{\text{coll}}$ (Brown, 1975) so that much higher fluxes F than (8) would be needed to produce a certain X-ray flux, the instability then proceeding still faster. Matters are even worse in the thin target model with its higher beam fluxes and lower background densities. However, Smith (1975) further points out that the beam may never develop a regime with $\partial f/\partial v > 0$ necessary for the instability since the distribution may have a strong negative slope at injection. Then propagation effects can lead to a positive slope only if the injection is sufficiently impulsive ($\lesssim$propagation time from injection to source region) (Harvey, 1975; Smith, 1975) which observations show to be unlikely (Hoyng *et al.*, 1966). Consequently the beam should be longitudinally stable.

Secondly such enormous currents as correspond to (8) are far beyond the Alfvén-Lawson current limit (about 5×10^3 A here) and so are laterally unstable, self-pinching *in vacuo*, as noted by Hoyng *et al.* (1976) in this context. In the presence of a background plasma, however, the effect is immediately to set up a neutralizing reverse current (Benford and Book, 1971; Melrose, 1974a; Hoyng *et al.*, 1976). At any point in the atmosphere (density n_0) the return flow speed v_0 must therefore be such that

$$n_0 v_0 = nv = F/A. \tag{11}$$

This condition can only be met provided that the return current does not require that the ambient electrons flow relative to the ions faster than the ion sound speed v_s. Otherwise ion sound turbulence is generated and the beam is impeded (cf. Melrose, 1970a; Hoyng *et al.*, 1976). (Here we have assumed that $T_e > T_i$ due to the heating process, which is generally the case due to the much longer collisional equipartition time for ions than for electrons (e.g., Spitzer, 1963). Thus if the plasma is heated collisionally, either directly by the beam, or indirectly by Ohmic dissipation, a state with $T_e > T_i$ will be set up initially. Typical electron – ion energy exchange times – Spitzer (1963) will then ensure that $T_e > T_i$ continuous to prevail throughout burst durations such as (3) except in the very densest parts of the flare plasma. The essence of our argument here depends only on the lower density regions, as we now show, where conditions for $T_e > T_i$ should be amply met.)

Thus free passage of the beam can only occur in regions satisfying $v_0 < v_s$ which by (8) and (11) requires that

$$A_{18} n_{10} T_7^{1/2} \geqslant \begin{cases} 10^{-1} \\ 10^1 \end{cases} \tag{12}$$

or

$$n_{10} T_7^{1/2} \geqslant \begin{cases} 10^{-1} \\ 1 \end{cases}, \tag{13}$$

where we have again adopted $A = 10^{18}$ cm^2 for small bursts, 10^{19} cm^2 for large and $A_{18} = A/10^{18}$, $n_{10} = n_0/10^{10}$ etc. as in Hoyng *et al.* (1976). Now in the ambient atmosphere we must, by hydrostatic considerations have $n_0 T$ increasing with depth while T decreases so that $n_{10} T_7^{1/2}$ is an increasing function of depth. Consequently reverse current limitation of the beam will occur high in the atmosphere near the electron source where criterion (13) is hardest to satisfy. (This argument is reinforced by the rapid decrease of F with depth due to collisional loss of the numerous low energy electrons in the stream spectrum (Brown, 1973).)

Once the stream is under way, increase of T_7 will permit increasing electron fluxes F to pass stably. An upper limit is set by the observational fact that no very large amount of flare material ever exceeds temperatures of 1–4 $\times 10^7$ K even at flare peak (e.g., Doschek and Meekins, 1973). Thus we can conclude from criterion (13) that the density at the electron acceleration region is

$$n_0{}^* \gtrsim \begin{cases} 10^9 \\ 5 \times 10^9 \end{cases} \mathrm{cm}^{-3} \tag{14}$$

if electron streams with fluxes (8) are to be injected downward. Values (14) are comparable with those suggested on other grounds (e.g., Sweet, 1969; Lin, 1974). This coincidence is at least suggestive that the observed thick-target electron fluxes (8) are indeed determined by self-limitation of the injection rate via the reverse current instability, though the numbers used above are admittedly mostly only known to an order of magnitude at best.

If this configuration is in fact viable it may also provide the answer to the problem of supplying the vast number (9) of electrons to an acceleration site in the corona. It has been suggested that 'chromospheric evaporation' by particle heating (Brown, 1973; Hudson, 1973) might enhance coronal densities in flares (Sturrock, 1974). The time scale for establishing such thermally driven motions is, however, many orders of magnitude more than the time scale for the setting up of a reverse current electrostatically and it is therefore much more likely that the reverse current is the source of supply of electrons to the corona. It remains of course to be worked out how the reverse current electron flow could in practice replenish the acceleration region in a coronal magnetic topology undergoing reconnection.

Thin target models can be considered similarly in terms of the reverse current problem. In this case the electrons are taken to escape upward in order to explain interplanetary electron spectra and the visibility of some bursts at high altitude behind the limb (cf. Datlowe and Lin, 1973; Brown and McClymont, 1975). It is at once obvious that stability criterion (13) severely limits this possibility. Even with the improbably large parameters $T = 10^8$ K, $A = 3 \times 10^{19}$ cm^2 the reverse current instability would impede progress of an upward stream of electrons beyond the height where $n_0 \simeq 10^7$ cm^{-3} in small flares and $n_0 \simeq 10^9$ cm^{-3} in large flares for the fluxes F required by (8). (In fact, as already noted, even greater F are required by the thin-target model.) The only process which can remove this impediment is the lateral and longitudinal spread of the stream and consequent reduction of the stream current density. We can illustrate this effect by means of a rough example without detailing the physics of beam spreading. Suppose that spreading becomes important at about one solar radius from the source, i.e., about $n_0 \simeq 10^6$ cm^{-3}, $T \leqslant 10^7$ K, then the reverse current instability limits streams penetrating up to here to a flux $n_0 v_s \leqslant 5 \times 10^{13}$ cm^{-2} s^{-1}. Supposing further that spreading above this height reduces the beam density sufficiently to overcome the reverse current limitation, we conclude that electrons could not escape through the corona at a rate greater than $5 \times 10^{13} \times A(\mathrm{cm}^2)$ s^{-1} or $\leqslant 5 \times$ $\times 10^{33}$ s^{-1} for an escape area $A \leqslant 10^{20}$ cm^2. This rate is only about 0.1–1 % of fluxes (8) inferred for the X-ray source, a fraction comparable with observations of interplanetary electrons (Lin, 1974). Though this calculation is at best an exploratory illustration it is indicative that reverse current limitation might be as important a factor in determining interplanetary electron escape as trapping on closed field lines.

It seems feasible therefore that thick-target electron streams may be just about stable against collective instabilities, or at least those which we have discussed, and of driving chromospheric electrons upward to replenish the coronal supply. (On these grounds, however, a thin-target hard X-ray burst model is not viable.) On the other hand the configuration is far from rigorously established. Furthermore the model does not offer a convincing explanation of behind-the-limb X-ray bursts (cf. Brown and McClymont, 1975) nor does it specify anything about conditions in the primary (coronal) acceleration region. We therefore now consider this acceleration region and, in particular the possibility that hard X-rays might be emitted *in situ* by comparatively few electrons.

4. Confinement and Reacceleration of Electrons by Plasma Waves

Recently a number of authors (Brown, 1975, 1976; Hoyng, 1975; Smith, 1975) have suggested that, instead of electrons being continuously accelerated in one region and emitting bremsstrahlung after injection to a separate dense source region, acceleration and emission regions might be one and the same. This could enormously reduce the total electron requirements $\mathcal{N}$, by avoiding the replenishment of electrons in a separate emission region, if the following conditions could be attained:

(a) that a high plasma density n_0 ($\simeq 10^{12}$ cm^{-3} say) can exist in the likely region of acceleration processes, viz the corona or upper chromosphere;

(b) that the acceleration mechanism can operate continuously to offset the heavy collisional losses implied by high n_0;

(c) that a very efficient containment mechanism exists to retain the same electrons within a plausible plasma volume, through which acceleration can occur, and to allow multiple (repeated) acceleration;

(d) that this volume of plasma containing the electrons should remain near the acceleration region throughout.

Additionally the volume (c) involved is required to be small by observations of thermal X-ray emission measure combined with requirement (a) (typically $V \simeq n_0^2 V/n_0^2 \simeq 10^{49}$ cm^{-3}/(10^{12} cm^{-3})$^2 \simeq 10^{25}$ cm^3). These conditions are not attainable by simple magnetic confinement in a coronal bottle (i.e. trapping models – cf. Introduction) since collisional scattering alone precipitates electrons out of such a trap on a time scale of order $\tau_{\text{coll}}/2$ (Melrose and Brown, 1976). Secondly the topology of such a trap provides no obvious means of continuously accelerating electrons (b) (cf. Brown and Hoyng, 1975). As suggested by Hoyng (1975) requirements (a)–(c) are more suggestive of acceleration associated with current sheet dissipation. Here we evaluate the extent to which requirements (a)–(d) can be satisfied and for what plasma parameters. In doing so we have developed a number of suggestions first made by Hoyng (1975 – Chapter V).

Requirement (a) is in fact the easiest to realize for although the ambient atmospheric density around a current sheet is presumably low (in order for energy to be stored in the magnetic field there) the density in the immediate vicinity of the sheet must in fact be high when reconnection is occurring (Cowling, personal communication). This is so because magnetic energy $B^2/8\pi$ outside the sheet is converted into thermal energy $2n_0kT$ inside the sheet (Kaplan *et al.*, 1974). Thus, in a steady state, reconnection, pressure balance across the sheet requires that $2n_0kT \simeq B^2/8\pi$ or $n_0 \simeq 1.5 \times \times 10^{11}(B/100)^2/T_7$ cm^{-3}. To the extent therefore that reconnection ever occurs in the corona, and on the assumption that it can be described in steady state terms, high densities around sheets are a natural result. Any attempt to establish this state of affairs more rigorously is well beyond the scope of this paper and we accept it as a working hypothesis to evaluate the implications for the electron number problem.

In Hoyng's (1975) analysis it is proposed that electron acceleration and confinement occur in a Langmuir turbulent volume around a current sheet ('shock wave or neutral sheet proper'). On the basis of energy flow arguments alone, Hoyng showed that only a rather small number of fast electrons is needed near the sheet at any instant to produce a typical X-ray burst, stochastic acceleration by resonant scattering on Langmuir waves offsetting the collisional losses. Neither point (c) nor (d) is, however, considered by Hoyng. Thus if scattering were not effective enough to contain the electrons in the turbulent plasma volume, they would leave the volume at once and emit bremsstrahlung elsewhere (i.e., a thick-target situation). Secondly the large scale motion of the flare plasma might carry the electrons away from the source of turbulent acceleration. In either case effective reacceleration would be prevented. We consider point (c) first, deferring (d) to the end of this Section.

Hoyng envisages a source region in the form of a slab of thickness l, side L, cm containing plasma of density n_0 cm^{-3} and filled with energetic electrons ($\geqslant 25$ keV) of density $n = \alpha n_0$. In order to give sufficient non-thermal emission measure (3) these parameters must satisfy (with $L_9 = L/10^9$ etc.).

$$\alpha n_{10}^2 L_9^2 l_7 = \begin{cases} 1 \\ 10^2 \end{cases}. \tag{15}$$

Energy flows in via the slab sides from the surrounding plasma, either by reconnection of inflowing field or by shock dissipation, being released in a current sheet (within the slab) with a suggested high efficiency of conversion into Langmuir wave turbulence. This generally requires a very thin current sheet ($d \leqslant 100$ cm – Hoyng (1975)) and if X-rays were to be emitted in the sheet only ($l = d$ so $l_7 \leqslant 10^{-5}$) then (with $\alpha \leqslant 1$) (15) would require $n_{10}L_9 \geqslant 3 \times 10^2 – 3 \times 10^3$ which is unacceptably large or dense – the more so since in practice $\alpha \ll 1$ (see below). Recognizing this, Hoyng speculates that the Langmuir waves, once generated, may be emitted from the sheet to fill a thicker slab ($l \gg d$) throughout which electrons are accelerated.

We now note that an upper limit to l is set by the damping distance for Langmuir waves in the plasma. For resonant acceleration of electrons of speed v_1 the Langmuir waves must have phase speeds $v_\phi \simeq v_1 \simeq 1\text{–}2 \times 10^{10}$ cm s^{-1} for 20–100 keV electrons. Now the principal damping mechanism for Langmuir waves depends on the ratio v_e/v_ϕ, v_e being the thermal electron speed (Melrose, 1970b; Kaplan *et al.*, 1974). In a plasma at $T_e \simeq 1\text{–}4 \times 10^7$ K (see below) $v_e \simeq 2\text{–}4 \times 10^9$ cm s^{-1} so that $v_e/v_\phi \simeq 2\text{–}10$. For $v_e/v_\phi \geqslant 7$ collisional wave damping is dominant while for $v_e/v_\phi \leqslant 7$ Landau damping is the main mechanism. Clearly here the situation is intermediate and for simplicity we include only collisional damping as a rough estimate of these two (comparable) mechanisms. This will in any case set an upper limit to the slab thickness since we neglect damping of the waves on the non-thermal electrons (cf. Kaplan *et al.*, 1974; Hoyng, 1975). Then the slab can be no thicker than $l \simeq v_G \tau_{\text{coll}}$ ($E \simeq kT$) where v_G is the Langmuir wave group velocity $v_G \simeq 3v_e^2/v_\phi$ corresponding to v_ϕ for, say, 25 keV electrons ($v_1 \simeq c/3$). Thus $l \simeq 9v_e^2/c\tau_{\text{coll}}$ (T_e) must be (using (4))

$$l_7 \simeq 0.6 \frac{T_7^{5/2}}{n_{10}}. \tag{16}$$

Secondly we consider the geometric constraint imposed by containment requirement (c) which, Hoyng suggested, might also be affected by resonant scattering. Clearly, whatever the scattering mechanism, escape of an electron should not occur beyond the acceleration slab dimensions in a time much less than the burst duration, if significant reacceleration is to occur (or in less than one collisional loss time if resonant acceleration is to occur at all). If the electron scattering time for any process is τ_D then during the burst an electron would typically diffuse a distance

$$D \simeq v\sqrt{\tau_x \tau_D}. \tag{17}$$

For scattering on Langmuir waves τ_D is of the same order as the resonant acceleration time which, in equilibrium between acceleration and collisional losses, will equal τ_{coll}. Thus using (2), (4) and (17) we have the diffusion distance under Langmuir scattering

$$D^l \geqslant \frac{1}{n_{10}} \times \begin{cases} 5 \times 10^{10} \\ 5 \times 10^{11} \end{cases} \text{cm} \tag{18}$$

which enormously exceeds the slab thickness l (16) and exceeds even the probable maximum slab dimension L. Therefore a much more efficient scattering process than Langmuir waves is necessary for electron confinement, the best candidate being whistlers for which an extremely lower limit to τ_D (Melrose, 1974b) is

$$\tau_D^W \simeq \frac{1}{2\pi\alpha\nu_B} \simeq \frac{5 \times 10^{-8}}{\alpha B} \text{ s}, \tag{19}$$

where B (gauss) is the field strength and ν_B the corresponding electron gyro frequency. From (17) the resulting diffusion distance is

$$D^W \simeq \frac{1}{(10^2\alpha)^{1/2}} \frac{1}{B^{1/2}} \times \begin{cases} 5 \times 10^7 \\ 5 \times 10^8 \end{cases} \text{cm}. \tag{20}$$

(Note that whistler scattering is only effective for electrons with $v \geqslant 43 v_A \simeq 2 \times 10^6 B/n_{10}^{1/2}$ cm s^{-1} which will be amply satisfied by $\geqslant 25$ keV electrons for the fields and densities invoked here.)

An upper limit to α can be set by assuming the volume of hard X-ray emission to equal that of soft X-ray emission. Then $\alpha = n/n_0 = nn_0 V/n_0^2 V = n_0 N/n_0^2 V =$ ratio of non-thermal to thermal emission measure, typically 10^{-2}–10^{-3} which is reasonable for the fraction of electrons accelerated (Hoyng, 1975). Thus the scaled factor $10^2\alpha$ is of order unity.

Consequently the slab must have a length

$$L_9 \geqslant D^W \simeq \frac{1}{(10^2\alpha)^{1/2}} \frac{1}{B^{1/2}} \times \begin{cases} 5 \times 10^{-2} \\ 5 \times 10^{-1} \end{cases} \tag{21a}$$

in order that electrons should not escape *along* the slab during the burst. The equivalent constraint on l due to transverse diffusion is given by noting that the diffusion time across the field is of order $(2\pi v_B \tau_D^W)^2 \simeq 1/\alpha^2$ times longer than (19) so that

$$l_7 \geqslant \frac{(10^2\alpha)^{3/2}}{B^{1/2}} \times \begin{cases} 5 \times 10^{-4} \\ 5 \times 10^{-3} \end{cases}. \tag{21b}$$

We now utilize (15), (16) and (21) to derive the required model parameters by combining them with the two energy equations given by Hoyng (1975). The first states that the bulk of the energy flowing into the current sheet is channelled, via Langmuir turbulence, into the fast electrons (as deduced, in many flares, from the hard X-ray bursts (e.g., Brown, 1975)). The second determines the Langmuir power generated by the energy inflow for a given set of plasma conditions. Clearly the form of these equations will differ according to whether the current sheet is a neutral sheet proper or a shock wave (cf. Hoyng). We follow Hoyng and consider only a neutral sheet situation since reasonable estimates of the power produced have been made in this case.

Firstly if we take magnetic energy inflow to occur at about the Alfvén speed v_A then the total inflow rate across the slab $\simeq B^2/8\pi \times v_A \times L^2$ erg s^{-1} must balance the collisional dissipation rate of the electrons given by (5) (Hoyng, 1975). Thus

$$\frac{B^3 L_9^2}{n_{10}^{1/2}} \simeq \begin{cases} 2 \times 10^4 \\ 2 \times 10^6 \end{cases}. \tag{22}$$

Secondly the power generated in the form of Langmuir waves depends on the specific situation prevailing in the sheet (cf. Kaplan *et al.*, 1974). An estimate of the power generated has been made for the extreme case of onset of the Buneman instability (Buneman, 1959; Hamberger and Jancarik, 1972; Hoyng, 1975) which we take (following Hoyng) as an illustrative example, viz.

$$P_B = 10^9 w (n_{10} T_7)^{1/2} B L_9^2, \tag{23}$$

where $w = W^l/n_0 kT$, W^l being the energy density of Langmuir waves and the field B in the sheet neighbourhood has been taken to be comparable to that in the inflowing plasma. Equating P_B to the electron collisional dissipation (Hoyng, 1975) P_c then

$$wn_{10}^{1/2}T_7^{1/2}BL_9^2 \simeq \begin{cases} 2 \\ 2 \times 10^2 \end{cases}. \tag{24}$$

If we now again take $T_7 \simeq 1$–4 then collecting conditions (15), (16), (21), (22) and (24) together we obtain, after some simple manipulation, n_{10}, B and l_7 in terms of L_9 (with $(10^2 w)$ of order unity as we show shortly) viz. –

$$B \simeq \frac{1}{(10^2 w)^{1/4}} \times \frac{1}{L_9} \times \begin{cases} 45 \\ 375 \end{cases}, \tag{25}$$

$$n_{10} \simeq \frac{1}{(10^2 w)^{3/2}} \times \frac{1}{L_9^2} \times \begin{cases} 20 \\ 700 \end{cases}, \tag{26}$$

$$l_7 \simeq \frac{(10^2 w)^3}{(10^2 \alpha)} L_9^2 \times \begin{cases} 2 \times 10^{-1} \\ 2 \times 10^{-2} \end{cases}, \tag{27}$$

$$n_{10} l_7 \simeq \begin{cases} 0.6 \\ 18 \end{cases}. \tag{28}$$

We see from (26) and (27) that (28) can be satisfied by any L_9 but defines w in relation to α viz.

$$w = 10^{-2}(10^2\alpha)^{2/3} \times \begin{cases} 0.2 \\ 1 \end{cases} \tag{29}$$

and we can re-express (25)–(27) finally as

$$B \simeq \frac{1}{(10^2\alpha)^{1/6} L_9} \times \begin{cases} 70 \\ 350 \end{cases} \text{G}, \tag{30}$$

$$n_0 \simeq \frac{1}{(10^2\alpha) L_9^2} \times \begin{cases} 2 \times 10^{12} \\ 5 \times 10^{12} \end{cases} \text{cm}^{-3}, \tag{31}$$

$$l \simeq (10^2\alpha) L_9^2 \times \begin{cases} 3 \times 10^4 \\ 3 \times 10^5 \end{cases} \text{cm}. \tag{32}$$

(According to Kaplan *et al.* (1974) the solution for the particle spectrum when wave input balances collisional losses $\sim E^{-\delta}$ with $\delta \sim 1/w$. Thus by (29) we would expect, for fixed α, a variation in δ of about 5:1 from small to large events, which is comparable to observations.)

Finally we use (21), (30) and (32) to test whether electrons are in fact effectively contained by whistler scattering. This requires

$$(10^2\alpha)^{5/6} L_9 \geqslant \begin{cases} 3 \times 10^{-5} \\ 3 \times 10^{-4} \end{cases} \quad \text{and} \quad \frac{L_9}{(10^2\alpha)^{7/18}} \geqslant \begin{cases} 6 \times 10^{-2} \\ 4 \times 10^{-2} \end{cases} \tag{33}$$

which are certainly satisfied for the range of α and L_9 we expect.

Equations (29)–(33) let us check the feasibility of the parameters needed by the confinement model. That is, does the physics we suggest is required to satisfy conditions (a)–(c) lead to parameters n_0, L, l, B and w which are consistent and reasonable? In fact our equations cannot prescribe the slab length L so we take this to have the plausible value of 10 000 kms ($L_9 = 1$). Then (30) requires coronal fields of 70–350 G (almost independent of α) which are in the usually accepted range for flare active regions. Only a detailed treatment of the acceleration process would allow α to be predicted but observations suggest values around 10^{-2}. This leads (32) to an emitting slab of thickness $l \simeq 0.3$–3 km while the relative Langmuir energy density in the slab would be 0.2–1 %. This latter value is consistent with the theoretical considerations of Kaplan *et al.* (1974) but is an order of magnitude higher than suggested by Hoyng (1975).

The high density invoked is readily shown to be consistent with the gas/magnetic pressure balance required across the slab for reconnection to occur. Thus by (30) and (31) again with $T \simeq 1$–4×10^7 K we find

$$\frac{n_0 k T}{B^2/8\pi} \simeq (10^2\alpha)^2 \times \begin{cases} 1.4 \\ 0.7 \end{cases}. \tag{34}$$

Though flare density values around 10^{10} cm^{-3} are often quoted from soft X-ray line data (e.g., Doschek and Meekins, 1973) these results are based on the improbable assumption of steady ionization equilibrium. The most recent analyses show that densities up to 10^{13} cm^{-3} may in fact be involved (Mason – personal communication). We must also check that high n_0 would not radiatively cool the hot plasma in times $\ll \tau_x$. The free-free cooling time is of order $10^4 T_7^{1/2}/n_{10}$, i.e., around 50 s which seems satisfactory. (The turbulent state of the plasma will greatly inhibit cooling by thermal conduction.)

In summary therefore requirements (a)–(c) for a confinement model can be realized with reasonable values of the parameters involved. However, for the model to effectively reduce the total electron requirements it is necessary to satisfy (d) – i.e., that the electrons should not only be confined within the turbulent plasma slab but also that this *same* plasma should remain, throughout the burst, around the region (sheet) of magnetic energy dissipation. This in fact is very unlikely to be feasible, as we now show.

Firstly, it is of the essence of reconnection mechanisms that the plasma outside the neutral sheet is frozen to the field and so is carried through the sheet (leaving at its edges) as magnetic energy is released (cf. Kaplan *et al.*, 1974). Secondly, in the case of current sheet dissipation in a shock, field and plasma must likewise be carried through the shock for energy release to occur. In either case it is impossible to see how the fast electrons, which form part of the plasma, can fail to be continuously swept away through the sheet. (Note that the Langmuir waves in the plasma would damp on a time scale $\simeq \tau_{\text{coll}}$ after the plasma left the sheet so that acceleration would not be sustained in this 'processed' plasma.) Thus the parameters we have derived would apply to a slab of plasma and fast electrons fixed in the frame of the sheet but not in the ion frame.

On this basis we can readily estimate the total number of electrons which must be accelerated in the burst duration. By (3) and (31) the number of electrons *instantaneously* in the slab will be

$$N \simeq (10^2\alpha)L_9^2 \times \begin{cases} 5 \times 10^{32} \\ 5 \times 10^{34} \end{cases} \tag{35}$$

which is of course small, as originally intended. For release of the total energy $\mathscr{E}_c$ (6), however, the slab must be refilled with incoming plasma η times say, viz.

$$\eta \simeq \frac{\mathscr{E}_c}{B^2/8\pi \times V} \simeq \frac{\mathscr{E}_c}{B^2/8\pi \times L^2 l}, \tag{36}$$

where V is the slab volume. By means of (30) and (32) this is

$$\eta \simeq \frac{1}{(10^2\alpha)^{2/3} L_9^2} \times \begin{cases} 3 \times 10^3 \\ 2 \times 10^5 \end{cases} \tag{37}$$

so that the total number of electrons entering (and leaving) the slab during the burst is

$$\mathscr{N} = \eta N \simeq (10^2\alpha)^{1/3} \times \begin{cases} 10^{36} \\ 4 \times 10^{39} \end{cases} \tag{38}$$

which is not significantly different from the thick-target requirement (9).

5. Discussion and Conclusions

(i) In Section 3 we showed that the necessary electron injection rate for chromospheric thick-target production of hard X-ray bursts is of the order of the limit imposed by stability of the reverse current set up in the ambient plasma if acceleration occurs where the *ambient* density $n_0 \geqslant 1\text{–}5 \times 10^9$ cm^{-3}. The total number of electrons injected is $5 \times 10^{35}\text{–}5 \times 10^{39}$ and it is feasible that these are supplied to the acceleration region by the upflowing plasma in the reverse current.

(ii) In Section 4 we demonstrated that resonant acceleration might occur in a Langmuir turbulent dense plasma slab (also permeated by whistlers to prevent rapid electron escape) where the flare enhanced density is sufficient for hard X-ray emission to occur *in situ*. Accelerated electrons are, however, swept out of the slab sides as the inflowing plasma released its energy, the total number of electrons accelerated in this process being again a few times $10^{35}\text{–}10^{39}$ in small–large events.

(iii) The coincidence of these numbers (i) and (ii) makes it tempting to suggest therefore that a turbulent slab surrounding a current sheet might in fact be the acceleration source which feeds electrons into the chromospheric thick target. If this is the case then spatially resolved X-ray observations may reveal both a chromospheric (thick-target) component at the foot points of field lines and a coronal component forming a thin emission region near a current sheet (and visible behind the limb). The ratio of the intensities of the two components could of course only be predicted by a much more sophisticated treatment of the acceleration region than our order of magnitude estimates.

(iv) Finally however, this coincidence should probably not be over-rated for the following reason. Taking a large flare as an example, the release of a total energy $\mathscr{E} \simeq 10^{32}$ erg from magnetic storage in a mean field $\bar{B}$ requires processing of a volume of plasma $V = \mathscr{E}/(\bar{B}^2/8\pi)$ which will contain a total number of electrons $\mathscr{N} \simeq \bar{n}_0 V \simeq \bar{n}_0 \mathscr{E}/(\bar{B}^2/8\pi)$. $\simeq 2 \times 10^{39} \bar{n}_{10}/(\bar{B}/100)^2$. The mean energy acquired per electron is $\bar{E} \simeq \bar{B}^2/8\pi)/\bar{n}_0$, i.e., $\bar{E} \simeq 25\ \text{keV} \times (\bar{B}/100)^2/\bar{n}_{10}$. Thus with fields $\bar{B}$ around 100 G and densities $\bar{n}_0$ around $10^{10}\ \text{cm}^{-3}$ we should *expect* field annihilation to result in some 10^{39} electrons of around 25 keV. To that extent the observed number of electrons involved in hard X-ray emission should not have been regarded as surprising though their acceleration does pose problems whose solution may lie along the lines suggested here.

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