COMMENTS ON 'THE EQUATION OF POLARIZATION TRANSFER IN AN INHOMOGENEOUS MAGNETIZED PLASMA. I. FORMALISM' BY F.T. CHENG AND P.C.W. FUNG

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Abstract. In their discussion of the transfer of polarized radiation in a weakly anisotropic, weakly inhomogeneous plasma, Cheng and Fung (1977) have included terms which describe the effects of 'mode coupling' and of the 'variation of the characteristic polarizations'. The latter effect is non-physical, and only the mode-coupling term should be retained. As the problem was formulated by Cheng and Fung, the two terms would cancel exactly; consequently, when corrected, the work of Cheng and Fung leads to a simple and useful method of including the effects of mode coupling in the weak anisotropy limit.

1. Introduction

Cheng and Fung (1977) have extended Sazonov and Tsytovich's (1968) theory of the transfer of polarized radiation in a weakly anisotropic plasma to include the effects of a weak inhomogeneity. They neglected emission and absorption, and separated into the two orthogonal natural modes, and then considered the effects of an inhomogeneity on the amplitudes, polarization vectors and relative phases of the components in the two modes. In this way they derived their transfer equation (Cheng and Fung 1977; Equation (5.2))

$$\frac{\mathrm{d}I_{pq}}{\mathrm{d}z} = S_{pq} + \sum_{r,s}\left[- K_{pqrs} + R_{pqrs} + C_{pqrs} + N_{pqrs}\right] I_{rs}, \tag{1}$$

where I_{pq} is the polarization tensor for the specific intensity, z denotes distance along the ray path, and where latin subscripts run over the other two coordinates x and y. The terms involving S_{pq} and K_{pqrs} describe emission and absorption, respectively, as in Sazonov and Tsytovich's (1968) transfer equation; they are of no interest here. The term involving R_{pqrs} describes Faraday rotation; it also appears in Sazonov and Tsytovich's (1968) equation and Cheng and Fung reproduced it from the terms involving the derivatives of the relative phase factor. The final two terms in (1) were introduced by Cheng and Fung (1977) to describe the effects of 'mode coupling' and 'variation of the characteristic polarizations', both of which involve the inhomogeneity explicitly.

My purpose here is to make the following points in connection with (1):

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(i) According to Cheng and Fung's prescription for calculating N_{pqrs} in (1), it should be equal to $-C_{pqrs}$. In other words, according to their prescription, the final two terms in (1) should cancel exactly.

(ii) However, the final term in (i) is non-physical and should not appear.

(iii) Nevertheless, in view of (i), Cheng and Fung's prescription provides a useful simple way of calculating C_{pqrs}, i.e., of including the effects of mode coupling.

2. The Non-Existence of the Term N_{pqrs}

The procedure used by Cheng and Fung (1977) is as follows. Let $E_x(z)$ and $E_y(z)$ be the two components of the electric amplitude in the wave, and let γ_{op} and γ_{ep} be the p ($= x$ or y) component of the polarization vector for the o-mode and the e-mode, respectively. Then write

$$E_p(z) = \gamma_{op}A_o(z)\,e^{i\alpha_o(z)} + \gamma_{ep}A_e(z)\,e^{i\alpha_e(z)}, \tag{2}$$

where the A's and α's are the real amplitudes and their phase factors for the two modes. Now construct the outer product

$$E_p(z)E_q^*(z) = \gamma_{op}\gamma_{oq}^*A_o^2(z) + \gamma_{ep}\gamma_{eq}^*A_e^2(z) + A_o(z)A_e(z)[\gamma_{op}\gamma_{eq}^*g(z) + \\ + \gamma_{ep}\gamma_{oq}^*g^*(z)], \tag{3}$$

with

$$g(z) = e^{i\{\alpha_o(z)-\alpha_e(z)\}}. \tag{4}$$

Then on differentiation of (3), the derivatives of g and g^*, of the A's and of the γ's lead to the terms involving R_{pqrs}, C_{pqrs} and N_{pqrs}, respectively, in (1).

However, the foregoing procedure is not correct because of a misunderstanding of the approach adopted in mode-coupling theory. The 'modes' in an inhomogeneous medium are defined to be the modes of an equivalent homogeneous medium with parameters equal to the local parameters in the inhomogeneous medium. Because of this definition, the γ's in (2) are independent of z by hypothesis, and one is to regard them as constants when differentiating (3). The term involving N_{pqrs} in (1) then does not appear. The 'mode-coupling' term should be present because, with this definition of the 'modes', a disturbance initially in one 'mode' splits partially into components in the two 'modes' as it propagates.

The foregoing point, concerning the constancy of the γ's, is a subtle one and it is appropriate to restate it in different words. The polarization vectors (the γ's) in the mode-coupling approach are to be regarded as constants (independent of z) in (2) in the following sense. When one makes a decomposition into modes at one point, at $z = z_0$ say, one uses the polarization vectors (for an equivalent homogeneous medium) appropriate to that point. This enables one to determine $A_o(z_0)$ and $A_e(z_0)$. In a homogeneous medium the A's would be constant, and the α's would be linear functions of z; the evolution of the α's with z then describes Faraday rotation. However, propagation in an inhomogeneous medium is different, and one takes this into account by allowing the A's to evolve in a coupled way for $z > z_0$. Thus the evolution of $E_p(z)$, in the form (2), is attributed entirely to the evolution of the α's and the A's. One views the propagation as though it were occurring in a homogeneous medium; from this viewpoint one needs to invoke a pseudo-physical effect, called 'mode coupling', to account for the fact that the A's do not remain constant.

Of course, the γ's do vary from point to point, and this must be taken into account at some stage. In mode-coupling theory the variation of the γ's is regarded as a passive one: if one makes the initial separation (2) at any point other than $z = z_0$, then one must choose the γ's appropriate to that point, but once chosen the γ's are constants.

3. The Calculation of C_{pqrs}

The variation of the A's can be determined as follows. Use (2) and the orthogonality of the γ's to find that

$$A_{o,e}(z) = \gamma^*_{o,ep}(z)E_p(z)\,\mathrm{e}^{-i\alpha_{o,e}(z)}. \tag{5}$$

In the absence of the inhomogeneity, the A's would be independent of z, and in the mode-coupling approach the medium is regarded as locally homogeneous. Therefore, one adopts the following viewpoint: the variation attributed to the A's in an inhomogeneous medium must be due entirely to the actual variation of the γ's from one point to another. In other words, one now takes account of the actual change in the γ's to determine how the A's change in (1).

It is straightforward to differentiate (5), keeping E_p and the α's constant, to insert the result in (3) and, hence, to find an explicit expression for C_{pqrs} in (1) in the form

$$C_{pqrs} = \gamma_{op}\gamma^*_{oq}(\gamma^*_{or}\gamma_{os})' + \gamma_{ep}\gamma^*_{eq}(\gamma^*_{er}\gamma_{es})' + \gamma_{op}\gamma^*_{eq}(\gamma^*_{or}\gamma_{es})' + \gamma_{ep}\gamma^*_{oq}(\gamma^*_{er}\gamma_{os})', \tag{6}$$

where the primes denote differentiation with respect to z. A comparison of (6) with Cheng and Fung's Equation (4.3) for N_{pqrs} shows that their N_{pqrs} is equal to $-C_{pqrs}$. (One needs to use only the orthogonality conditions for the γ's to establish this result.) It is in this sense that the final two terms in (1) cancel.

It should be remarked that the result (6) can be derived from a general theory of mode coupling. However, the derivation (which is not given here) is surprisingly tedious in view of the relative simplicity of the result (6). The derivation of (6), which is essentially the same as Cheng and Fung's derivation of their N_{pqrs}, is much simpler than any other existing treatment of mode coupling.

4. Concluding Remarks

In conclusion, I should like to remark on one of the (supposedly) important consequences of mode coupling in a weakly anisotropic plasma: namely, the concept of 'polarization limiting' – i.e. the supposed absence of Faraday rotation in a certain limit. The usefulness of this concept in astrophysical applications (as opposed to the ionospheric application for which it was introduced) has been questioned by Simpson (1976). The corrected form of (1) would be particularly appropriate as the basis of a more detailed discussion of 'polarization limiting'.

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