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THE GYROSYNCHROTRON EMISSION FROM QUASI-THERMAL ELECTRONS AND APPLICATIONS TO SOLAR FLARES

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ABSTRACT

We present theoretical results on the gyrosynchrotron radiation from electrons with a Maxwellian energy distribution. We review the analytical expressions for the gyromagnetic absorption coefficient and find two which cover the range of interest for microwave emission from solar flares, i.e., frequencies ω to $\sim 100\Omega_e$ and temperatures T_e to $\sim 10^9$ K. Numerical calculations are used to check the analytic expressions and to derive simplified empirical formulae which relate the observable characteristics of the radiation to the temperature and magnetic field in the source.

We apply the results to the sources of impulsive microwave and hard X-ray bursts from solar flares. For an isothermal source the theory predicts a microwave spectrum where the flux density rises as f^2 at low frequencies, maximizes as some frequency $f_{\rm peak}$, and falls very rapidly thereafter; this shape fits the observed spectra qualitatively. The optical depth τ of the source varies rapidly with f, with $\tau = 1$ at $f \approx f_{\rm peak}$. For $T_e \gtrsim 10^8$ K we derive the relation $f_{\rm peak} \propto T_e^{0.7} B$, which allows a direct estimate of the magnetic field B in the impulsive burst source if the temperature is known—for instance, from hard X-ray observations. For the impulsive burst of 1972 May 18, reported by Hoyng and Stevens, we find that the microwave and hard X-ray data are well fitted by a model source with $T_e \approx 2.3 \times 10^8$ K, $B \approx 370$ gauss, $n_e \approx 2 \times 10^9$ cm^{-3}, and scale length $L \approx 8600$ km.

Subject headings: Sun: flares — Sun: radio radiation — Sun: X-rays — synchrotron radiation

I. INTRODUCTION

Our purpose in this paper is to present theoretical results on the gyrosynchrotron emission from electrons with a Maxwellian energy distribution. It is well known that gyromagnetic emission from nonrelativistic electrons is concentrated at the electron gyrofrequency and its first few harmonics (e.g., Bekefi 1966, p. 203), and that gyromagnetic emission from ultrarelativistic particles ($\gamma \sin \alpha \gg 1$, γ = Lorentz factor, α = pitch angle) is concentrated at very high harmonics [$s \approx (\gamma \sin \alpha)^3$] (e.g., Ginzburg and Syrovatskii 1965). In this paper we are concerned with an application in which emission occurs at harmonics between about 10 and 100, and in which we assume the electron distribution to be a high-temperature Maxwellian (10^8–10^9 K). A general analytic expression for gyromagnetic emission and absorption by a relativistic Maxwellian distribution was derived by Trubnikov (1958) (see also Drummond and Rosenbluth 1963 and Shkarovsky 1966). Trubnikov's general result is too cumbersome to be of direct use in most applications. Here, in § II, we derive an approximation to the absorption coefficient in the nonrelativistic limit and explore two approximations due to Trubnikov. Although these approximations are useful for checking numerical calculations, they remain too cumbersome for many purposes. Consequently, we have used our numerical results to derive simple empirical relations; these are presented in § III.

Our work is motivated by the problem of explaining impulsive microwave bursts from the Sun. Because of the detailed correspondence in the time variations of impulsive microwave and hard X-ray bursts, it is believed that both result from the same electron distribution, with the microwaves and X-rays emitted mainly by electrons with $E \gtrsim 100$ keV and $E \lesssim 100$ keV, respectively (Holt and Ramaty 1969; Takakura 1972; Ramaty and Petrosian 1972). In existing treatments the common assumptions are that the electrons are nonthermal, with a power-law energy distribution, and that the microwaves are due to gyrosynchrotron emission by mildly relativistic electrons (e.g., Takakura 1960*a*, *b*, 1967); the emission and absorption coefficients are calculated numerically (e.g., Ramaty 1969). The hard X-rays are commonly thought to be due to bremsstrahlung when the electrons penetrate a thick target. Characteristically, the models require a total of 10^{34}–10^{39} electrons with $E > 10$ keV with a power-law index $\alpha \approx 3$–5.

Our interest in a thermal interpretation of impulsive microwave bursts follows from the renewed interest in the idea (Chubb, Kreplin, and Friedman 1966) that the hard X-rays may result from electrons which are heated in bulk rather than being accelerated to form a nonthermal tail. Crannell *et al.* (1978) have provided recent evidence for this interpretation, and Mätzler *et al.* (1978) have argued for it (see also Ramaty 1979). Such "bulk-energization" of electrons to form a quasi-thermal distribution with $T \gtrsim 10^8$ K has several attractive features from the point of view of both theory and observations:

1) The amount of energy associated with the impulsive bursts need not be a substantial fraction of the total flare energy, contrary to the implications of the usual interpretation. In thermal models the efficiency of X-ray production can be increased by a factor $\gtrsim 10$ compared with nonthermal models (Smith and Lilliequist 1979) because the major mechanism of energy loss in nonthermal, thick-target models—namely, heating of cold, ambient electrons by fast electrons—is absent.

2) The acceleration of large numbers of electrons to form a strongly nonthermal distribution is not required. Few or no electrons need be accelerated into a high-energy tail; at most, a small number ($< 1\%$) may be needed as "seeds" for second-stage acceleration and to provide interplanetary electron bursts (Lin 1974). Heating seems to be more favorable than systematic acceleration; Smith and Lilliequist (1979) estimate that under coronal conditions only $\sim 10\%$ of energy released goes into systematic acceleration.

3) The sources can easily be small ($\lesssim 5''$) and dense ($\sim 10^{10}$ cm^{-3}), consistent with the inferences from some X-ray data (e.g., Kahler, Petrasso, and Kane 1976), radio data (Crannell *et al.* 1978), and the few radio observations of very small sources (Kundu, Velusamy, and Becker 1974; Alissandrakis and Kundu 1978; Marsh, Zirin, and Hurford 1979).

4) The observed time scales of impulsive bursts, $\sim$1–10 s (cf. Hoyng, Brown, and van Beek 1976), can be reasonably explained by postulating that the hot regions are bounded by ion-acoustic conduction fronts which move apart at about the ion sound speed (Brown, Melrose, and Spicer 1979). Thus the most serious difficulty of thermal models, viz., the excessively rapid heat conduction—and excessively small time scales—is avoided. (The implications of the fine structure ($\sim$ 10 ms) in decimeter bursts [e.g., Slottje 1978] remain to be investigated; they may give information on the basic energy release processes.)

An important feature of the spectra of some impulsive microwave bursts is the steep, sometimes exponential, decrease in flux density at high frequencies, i.e., at $f > f_{\text{peak}}$. It is difficult to assess the prevalence of such cutoffs because the number of published spectra of *impulsive* bursts is very limited (most published spectra seem to be for great outbursts, i.e., microwave type IV's). Examples of impulsive burst spectra have been given by Hachenberg and Wallis (1961), Takakura and Kai (1966), Holt and Ramaty (1969), and Hoyng and Stevens (1973) (see also Hoyng 1975). Commonly the flux density increases with frequency from $\sim$1 GHz to a maximum at $f_{\text{peak}} \sim 10$ GHz and then decreases. Early in the burst, when the X-ray spectrum is hardest, the microwave peak is at its highest frequency, i.e., $f_{\text{peak}} \gtrsim 10$ GHz, and the high-frequency cutoff is steepest (e.g., Fig. 1*b*).

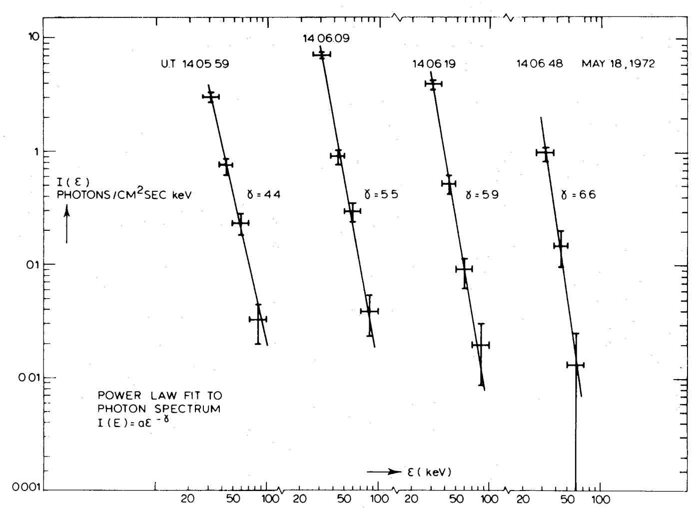

FIG. 1*a*

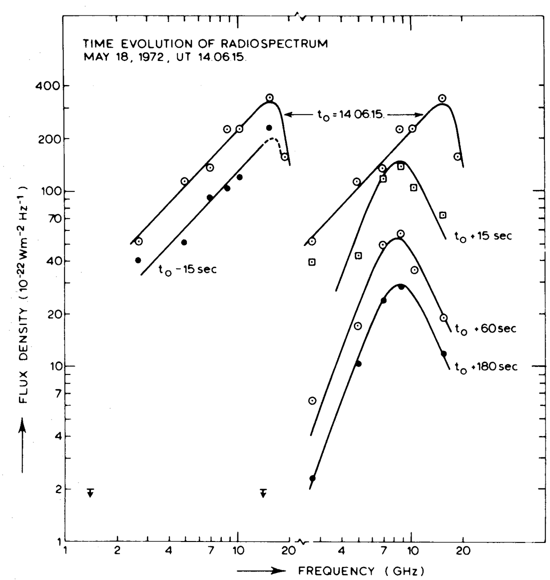

FIG. 1*b*

FIG. 1.—(*a*) X-ray spectra at four different times during the impulsive burst of 1972 May 18 obtained with the ESRO TD1A spacecraft (reproduced from Hoyng and Stevens 1973). (*b*) Microwave spectra at different times during the event. The data are mostly from the Sagamore Hill Radio Observatory (reproduced from Hoyng and Stevens 1973).

This steep decrease in flux is explained in the usual (nonthermal) models only by invoking a sharp high-energy cutoff in the electron energy spectrum which mimics a Maxwellian. Electron bursts in interplanetary space show such an exponential cutoff at high energies (Lin 1970).

As noted originally by Hachenberg and Wallis (1961), impulsive microwave spectra with their positive slopes (often of index $\alpha \approx 2$) at $f < f_{\rm peak}$ and steep negative slopes at $f > f_{\rm peak}$ are suggestive of sources where the electrons have a quasi-thermal distribution, and which are optically thick at $f \lesssim f_{\rm peak}$ and optically thin at $f > f_{\rm peak}$. Hachenberg and Wallis investigated whether bremsstrahlung from a hot plasma could produce the microwaves and found it insufficient to produce the observed flux densities S of ~ 10 to ~ 1000 solar flux units (SFU) (1 SFU $= 10^{-22}$ W m^{-2} Hz^{-1}).

In this paper we investigate the possibility that impulsive bursts result from gyrosynchrotron emission from quasi-thermal electrons. We assume that the flare energy is manifested mainly in very rapid, bulk heating of all electrons rather than in systematic acceleration of only some of them. We consider the radio emission from a source which is small (a few thousand kilometers in extent), dense ($n_e \gtrsim 10^9$ cm^{-3}), and hot ($T_e \gtrsim 10^8$ K). We emphasize that our "thermal" interpretation of impulsive bursts does not imply thermal equilibrium. We do not believe that the electron energy distribution will be strictly Maxwellian or that the electron and ion temperatures will be equal. The important characteristic of our thermal model is that there is a single component of electrons with a high mean energy and an exponential high-energy tail.

The reason that bremsstrahlung from a small thermal source cannot produce the observed flux densities and the implied brightness temperatures $T_b \gtrsim 10^8$ K is that the opacity varies as $T_e^{-3/2}$, resulting in optical depths $\tau \ll 1$ and $T_b \ll T_e$ unless the density is extremely high ($\sim 10^{13}$ cm^{-3}). However, in the presence of a magnetic field, magnetobremsstrahlung (gyrosynchrotron emission) can be much more efficient than field-free bremsstrahlung, and the required opacity can be attained. At the high harmonics ($\omega/\Omega_e \sim 10$ to ~ 100) of interest here, gyromagnetic emission and absorption are dominated by higher-energy particles, and any distribution which varies exponentially at high energies, i.e., $N(E) \propto \exp(-E/E_0)$, should be equivalent to a Maxwellian at temperature $T_e = E_0/k$ ($E_0 = 8.6$ keV corresponds to $T_e = 10^8$ K).

In § II we give two analytic expressions for the gyroabsorption coefficient for electrons with a Maxwellian distribution and compare them with numerical computations. In § III we state some useful formulae, compare them with observations of microwave bursts, and derive the parameters of a quasi-thermal source from the combined radio and X-ray data of the burst of 1972 May 18.

II. GYROSYNCHROTRON RADIATION FROM QUASI-THERMAL ELECTRONS

The general characteristics of the electron distribution are as follows:

1. For a Maxwellian distribution with $T_e \gtrsim 10^8$ K, the average electron energy is $\gtrsim 10$ keV and there are large numbers of electrons with energies up to about 100 keV.
2. Above about 100 keV, the distribution $N(E)$ decreases exponentially.
3. For $B \gtrsim 100$ gauss the gyromagnetic frequency is $f_B = \Omega_e/2\pi \gtrsim 300$ MHz. Thus the range of interest for microwave emission is $\omega \sim 10\Omega_e$ to $\sim 100\Omega_e$.

Because most of the electrons are not highly relativistic, the Airy integral approximation, used to obtain the synchrotron formulae, is not valid. Thus the formulae for the emission and absorption coefficients involve sums over Bessel functions and their derivatives. However, as will now be shown, for a nonrelativistic Maxwellian distribution the power series expansion of the Bessel functions converges rapidly and only the leading terms need be retained. The sums may then be performed analytically.

a) *Analytic Expressions for the Gyromagnetic Absorption Coefficient*

Useful analytic expressions for the gyromagnetic absorption are available for (i) a nonrelativistic thermal plasma in which the wave properties are assumed to be given by the magnetoionic theory, and (ii) a mildly relativistic plasma for emission perpendicular to the field lines and with the wave properties those of transverse waves *in vacuo*.

i) *Nonrelativistic Electrons in a Plasma*

The gyromagnetic absorption coefficient, γ, at the sth harmonic for nonrelativistic Maxwellian electrons at temperature T_e is (Sitenko and Stepanov 1956; Ginzburg and Zheleznyakov 1959; Melrose 1979*b*, p. 274)

$$\gamma^\sigma(s, \omega, \theta) = \left(\frac{\pi}{2}\right)^{1/2} \frac{\omega_p{}^2 A^\sigma(s, \omega, \theta)}{\omega n_\sigma{}^2 \beta_0 |\cos\theta| \partial(\omega n_\sigma)/\partial\omega} \exp\left[-\frac{(\omega - s\Omega_e)^2}{2\omega^2 n_\sigma{}^2 \beta_0{}^2 \cos^2\theta}\right], \quad (1)$$

where σ denotes either of the two magnetoionic modes ($\sigma = +1$ for the o-mode, $\sigma = -1$ for the x-mode), $n_\sigma(\omega)$ is the refractive index of that mode, θ is the angle between the wave-normal direction and the magnetic field, Ω_e and ω_p are respectively the electron gyrofrequency and plasma frequency, and $\beta_0 = (kT_e/mc^2)^{1/2}$ with $k =$ Boltzmann's constant. The polarization of the modes appears in A^σ as described by Melrose and Sy (1972) and Melrose (1979*a*, p. 43).

The absorption coefficient (1) is sharply peaked around integral values of ω/Ω_e, a characteristic feature of the nonrelativistic approximation. It is convenient to define an averaged absorption coefficient at the sth harmonic by writing

$$\gamma^\sigma(s, \theta) = \int_{-\infty}^{\infty} \frac{d\omega}{\Omega_e} \gamma^\sigma(s, \omega, \theta) . \tag{2}$$

The simplest useful approximation to $\gamma^\sigma(s, \theta)$ is one given by Zheleznyakov (1970, p. 454). It may be derived by inserting (1) in (2), retaining only the leading term in the power series expansion of the modified Bessel functions which appear in A^σ, making the quasi-circular approximation to the magnetoionic wave properties and evaluating the ω-integral by the method of steepest descents. We find

$$\gamma^\sigma(s, \theta) = \frac{\pi\omega_p^2}{\Omega_e}\left[1 - 2\beta_0^2 \cos^2\theta\left(1 - \frac{9}{4}\frac{\omega_p^2}{s^2\Omega_e^2}\right)\right]A^\sigma(s, \omega_0, \theta) , \tag{3}$$

$$A^\sigma(s, \omega_0, \theta) = \frac{e^{-\lambda}}{4}\frac{s^2}{s!}\left(\frac{\lambda}{2}\right)^{s-1}\left(1 - \frac{\sigma}{2s}\frac{\sin^2\theta}{\cos\theta}\right) \times \left\{(1 - \sigma\cos\theta)^2 - (1 - \sigma\cos\theta)\sin^2\theta\left[\frac{1}{s} + 2\sigma\beta_0^2 n^4|\cos\theta|\right]\right\}, \tag{4}$$

with

$$\lambda = s^2\beta_0^2 n^2 \sin^2\theta(1 - 2\beta_0^2 n^4 \cos^2\theta) \qquad \text{and} \qquad n = \left(1 - \frac{\omega_p^2}{s^2\Omega_e^2}\right)^{1/2} .$$

To lowest order in β_0^2 and in $1/s$, equation (3) with (4) reduces to

$$\gamma^\sigma(s, \theta) = \frac{\pi}{4}\frac{\omega_p^2}{\Omega_e}\frac{s^2}{s!}\left(\frac{n^2 s^2 \beta_0^2 \sin^2\theta}{2}\right)^{s-1}(1 - \sigma|\cos\theta|)^2 , \tag{5}$$

which is essentially Zheleznyakov's result. For θ sufficiently close to $\pi/2$ the quasi-circular approximation breaks down and (5) does not apply. For $\theta = \pi/2$ the counterpart to (5) is

$$\gamma^x\left(s, \frac{\pi}{2}\right) = \frac{\pi}{2}\frac{\omega_p^2}{\Omega_e}\frac{s^2}{s!}\left(\frac{s^2\beta_0^2}{2}\right)^{s-1} , \tag{6a}$$

$$\gamma^0\left(s, \frac{\pi}{2}\right) = 0 . \tag{6b}$$

The major limitations on the use of (5) and (6a, b) arise from the neglect of relativistic effects and from the range of validity of the expansion of the Bessel functions. The power series expansion, and hence (5), is expected to break down for $s^2\beta_0^2 \gtrsim 2$. Comparison of analytic and numerical results below confirms that (5) breaks down for $s^2\beta_0^2$ between one and two.

ii) *Mildly Relativistic Electrons in Vacuo Emitting at $\theta = \pi/2$*

Relativistic effects were included for $\theta = \pi/2$ by Trubnikov (1958); subsequent discussions (e.g., Bekefi 1966, pp. 200–208) have been largely based on Trubnikov's results. For $\theta = \pi/2$, equation (1) implies zero line width in the nonrelativistic approximation. One of the important relativistic effects is the spread in gyrofrequencies Ω_e/γ due to the range of the Lorentz factor γ. Another is the fact that the peak in the absorption at the sth harmonic occurs at $\omega/\Omega_e < s$, with the value of $\omega/s\Omega_e$ at the peak decreasing with increasing s (e.g., Bekefi 1966, p. 207). Trubnikov (1958, eqs. [3.14] and [3.16]) gives the following two approximate expressions for a mildly relativistic plasma and for $\theta = \pi/2$:

For

$$\rho = \frac{9}{2}\frac{\omega}{\Omega_e}\beta_0^2 \gg 1 \tag{7}$$

he finds

$$\gamma^x(\omega, \pi/2) = \frac{\omega_p^2}{\Omega_e}\frac{3\pi^{1/2}}{e\beta_0\rho}\exp\left\{-\frac{1}{\beta_0^2}\left[\rho^{1/3} - 1 + \frac{9}{20\rho^{1/3}}\right]\right\}, \tag{8a}$$

$$\gamma^0(\omega, \pi/2) = \frac{\beta_0^2}{\rho^{1/3}}\gamma^x(\omega, \pi/2); \tag{8b}$$

and for $(\omega/\Omega_e)^2\beta_0^2 \ll 1$ he finds

$$\gamma^x(\omega, \pi/2) = \frac{\omega_p^2}{\Omega_e}\left(\frac{\pi\Omega_e}{2\omega}\right)^{1/2}\frac{1}{\beta_0^2}\left(\frac{e}{2}\beta_0^2\frac{\omega}{\Omega_e}\right)^{\omega/\Omega_e}, \tag{9a}$$

$$\gamma^0(\omega, \pi/2) = \beta_0^2\gamma^x(\omega, \pi/2). \tag{9b}$$

iii) *Comparison and Generalization*

On making Stirling's approximation to $s!$, (6a) becomes identical to (9) provided one rewrites s as ω/Ω_e. Although s is equal to ω/Ω_e in the nonrelativistic approximation, this equality does not hold in the relativistic case because of the relativistic frequency shift. The equivalence of (6a) and (9) leads to the important conclusion that relativistic effects may be included in (6a) simply by writing ω/Ω_e in place of s and disregarding the nonrelativistic equality $s = \omega/\Omega_e$. It is reasonable to apply the same procedure for $\theta \neq \pi/2$ and to generalize (5) to

$$\gamma^\sigma(s, \theta) = \frac{\omega_p^2}{\Omega_e}\left(\frac{\pi}{2}\frac{\Omega_e}{\omega}\right)^{1/2}\frac{1}{\beta_0^2\sin^2\theta}\left(\frac{e\beta_0^2\omega\sin^2\theta}{2\Omega_e}\right)^{\omega/\Omega_e}(1 - \sigma|\cos\theta|)^2, \tag{10}$$

which should be valid for $(\omega/\Omega_e)^2\beta_0^2\sin^2\theta \ll 2$.

As far as we are aware, the approximations (8) and (10) have not been compared with numerical results; we do so in the next section and find them to be accurate approximations in their stated ranges of validity. It would be desirable to generalize (8a, b) to arbitrary angles θ, as (9a, b) are generalized by (10), but we have no theoretical basis on which to make such a generalization. Empirically, replacing β_0^2 by $\beta_0^2 \sin\theta$ in (8a) seems to give a reasonably good fit over most of the range of validity.

b) Comparison with Numerical Calculations

In this section we compare the foregoing analytical results with numerical computations. The numerical work is based on the equations for the gyrosynchrotron absorption coefficient of Eidman (1958) as given by Ramaty (1969) (but including the correction pointed out by Trulsen and Fejer 1970). The sums over Bessel functions are simplified by use of the formulae of Wild and Hill (1971). Relativistic effects and the influence of the plasma (medium suppression, i.e., the Razin effect) are taken into account. The electron energy distribution $u(\gamma)$, where γ is the Lorentz factor, is taken to be the Maxwellian

$$u(\gamma)d\gamma = \frac{\pi}{2}\left(\frac{2mc^2}{\pi kT_e}\right)^{3/2}\gamma(\gamma^2 - 1)^{1/2}\exp\left[-\frac{(\gamma - 1)mc^2}{kT_e}\right]d\gamma.$$

Except for the normalizing factor, this equation is valid for all T_e; the form of the normalization used here is accurate for $kT_e \ll mc^2$, or $T_e \ll 6 \times 10^9$ K.

In Figure 2 we compare values of the absorption coefficient, κ, from the analytical results ([8] and [10]) with the numerical computations. (Note that $\kappa = \gamma/c$.) For (10) (Fig. 2*a*), the agreement is very good for $10^7 \lesssim T_e \lesssim 10^8$ K and $\omega/\Omega_e \lesssim 15$, i.e., for $(\omega/\Omega_e)^2\beta_0^2\sin^2\theta \lesssim 2$. For (8) (Fig. 2*b*), the agreement is very good for $10^8 \lesssim T_e \lesssim 10^9$ K and $\omega/\Omega_e \gtrsim 10$, i.e., for $(9/2)\omega/\Omega_e\beta_0^2 \gtrsim 1$. Note that in drawing our numerical curves we have averaged the absorption coefficient at low harmonics by eye whenever individual harmonics appeared. A definite averaging procedure, defined by (3), has been adopted for the analytic results. Therefore, some disagreement at low harmonics may be attributed to the different averaging procedures.

In Figure 3 we show the results of numerical computations of the absorption coefficient for temperatures up to 10^9 K and frequencies ω up to $100\Omega_e$. The reason for multiplying κ by B/n_e is to make the results very nearly independent of B or n_e; only for calculating the index of refraction were specific values of B and n_e required (we used $B = 300$ gauss and $n_e = 10^{10}$ cm^{-3}). Additional computations have shown that for fixed n_e, there are negligible changes in the quantity $\kappa B/n_e$ when B is in the range <100 to >300 gauss; however, when B is fixed and n_e is lowered, say to 10^8 cm^{-3}, $\kappa B/n_e$ changes little for $\omega/\Omega_e \gtrsim 15$ but rises by up to a factor of 10 at $\omega/\Omega_e < 10$. This is because medium suppression, which mainly affects low frequencies, has little effect when the density is low.

Figure 3 shows curves for both $\theta = 45°$ and $\theta = 75°$. At low ω/Ω_e, the analytical result (eq. [10]) predicts that changing θ from 45° to 75° (an 87% increase in $\sin^2\theta$) will be equivalent to an 87% increase of temperature. The numerical calculations are in qualitative but not quantitative agreement with this prediction, with the increase being less than predicted, going approximately as $\sin\theta$ (i.e., similar to what we find for eq. [8]).

III. APPLICATIONS TO IMPULSIVE BURSTS

We now apply the results just derived to the sources of impulsive hard X-ray and microwave bursts from solar flares. These are taken to be small regions, possibly portions of magnetic loops. We assume that the sources have characteristics similar to those postulated at the Skylab Workshop on Solar Flares (Ramaty 1979): a scale size

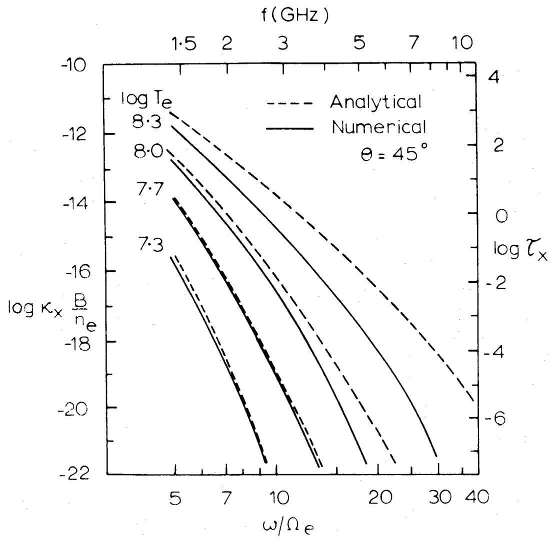

FIG. 2a

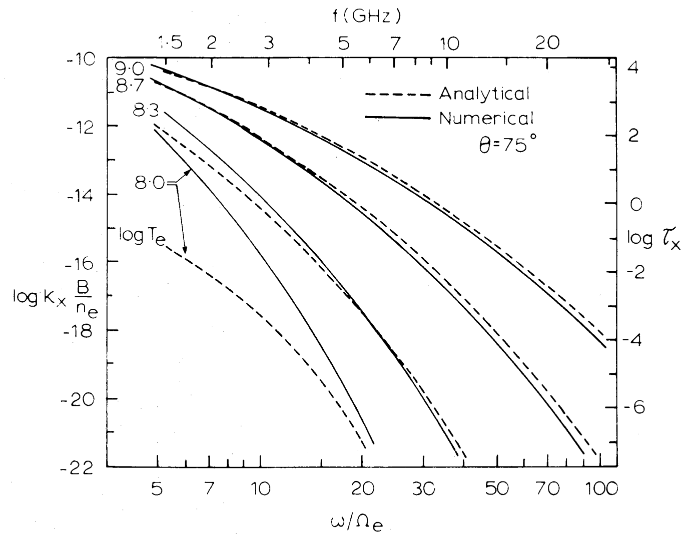

FIG. 2b

FIG. 2.—The gyromagnetic absorption coefficient for the x-mode κ_x (units: cm^{-1}) derived from the analytical results, eq. (10) (Fig. 2a) and eq. (8) (Fig. 2b). compared with numerical computations. The magnetic field was assumed to be inclined at $\theta = 45°$ from the line of sight for Fig. 2a and 75° for Fig. 2b. In the numerical work, the refractive index was calculated for a source with a plasma frequency $f_p = 90$ MHz ($n_e = 10^8\ cm^{-3}$) and a gyromagnetic frequency $f_B = 280$ MHz ($B = 100$ gauss). The right and top scales show optical depth τ_x and microwave frequency respectively for such a source with a scale length $L = 2000$ km.

L of a few thousand kilometers, a density $n_e \sim 10^{10}\ cm^{-3}$, a magnetic field $B \sim 100$ gauss and an electron temperature $T_e \gtrsim 10^8$ K. In this paper our purpose is to test whether this model, in its simplest form, can fit the microwave spectra. We take a single, isothermal source with the same scale length in depth as in projected area. We consider only x-mode radiation, reserving a discussion of polarization to the end. We consider only the early part of an impulsive burst, when the X-ray spectrum is hardest and the microwave peak is at its highest frequency; we do not consider the subsequent (presumed) expansion of the sources. We assume that the electrons have an isotropic velocity distribution.

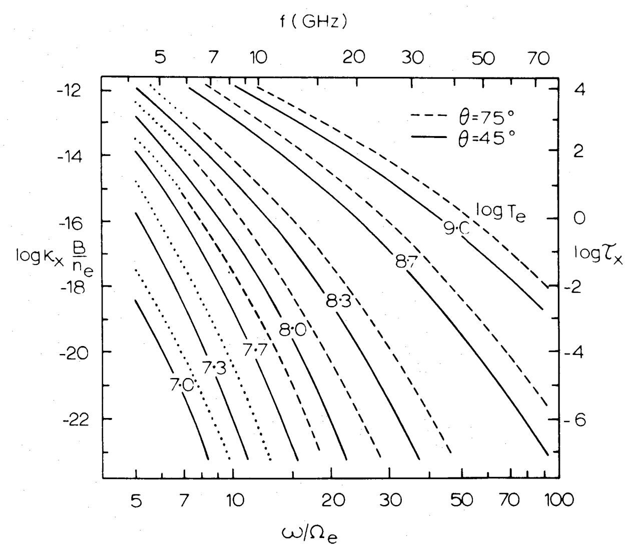

FIG. 3.—The gyromagnetic absorption coefficient for the x-mode from numerical computations for two values of θ and seven values of T_e. The shortened dashes indicate the general trend in regions where there is fine structure because the influence of in dividual harmonics is not entirely wiped out by Doppler shifts. When calculating the refractive index, $f_p = 900$ MHz ($n_e = 10^{10}\ cm^{-3}$) and $f_B = 840$ MHz ($B = 300$ gauss) were assumed. The right and top scales show the optical depth τ_x and microwave frequency, respectively, for such a source with scale length $L = 2000$ km.

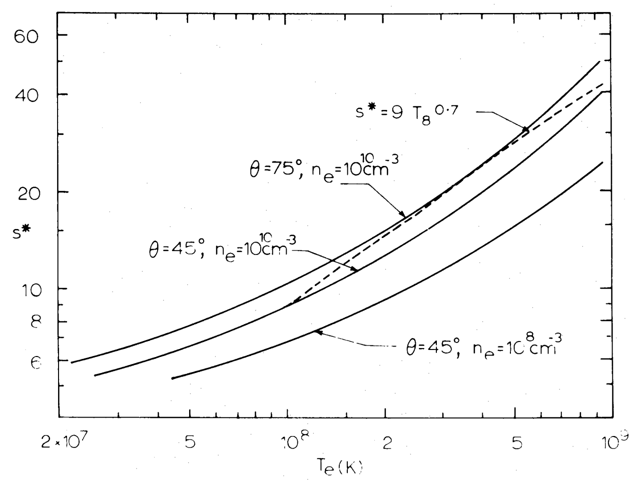

FIG. 4.—The harmonic number s^*, for which $\tau_x = 1$, for the indicated values of θ and n_e; a scale length $L = 2000$ km was assumed. The dashed line between $10^8 \lesssim T_e \lesssim 10^9$ K shows the approximation used in the text.

The right-hand scales on Figures 2 and 3 show the optical depth $\tau_x = \kappa_x L$ of such sources; it is important to note the small range of ω/Ω_e over which sources of given temperature change from optically thick to optically thin. The scales at the top of the figures show the microwave frequencies corresponding to the various values of ω/Ω_e. For a source at a given temperature, the frequency of largest flux density, f_{peak}, is that for which $\tau \approx 1$; at lower frequencies $\tau \gg 1$, $T_b \approx T_e$, and the flux density $S \propto f^2$, while at higher frequencies $\tau \ll 1$, $T_b \ll T_e$, and S decreases very rapidly with f.

In Figure 4 we plot the value of ω/Ω_e for which $\tau_x = 1$, denoted s^*, as a function of temperature. Because s^* varies with θ, the observed value of f_{peak} depends on the orientation of the magnetic field relative to the observer; thus if magnetic fields in impulsive burst regions have a preferred orientation (say horizontal or vertical), a center-to-limb effect is to be expected. (If this is the cause of the slight [20–50%] decrease in burst intensity near the limb found by Kakinuma, Yamashita, and Enome (1969), it would imply that the magnetic fields tend to be horizontal so that, on average, θ is larger near disk center.)

Figure 4 also shows that s^* is insensitive to a change in source density (or equivalently scale length, since $\tau \propto n_e L$); a change of two orders of magnitude in n_e produces less than a 50% change in s^*.

We are now able to write down some relationships which are of use in interpreting the data. For the temperature range of about 10^8–10^9 K, the results of Figures 3 and 4 can be approximated by the simplest useful relations

$$\tau_x \approx 3 \times 10^9 T_8{}^7 \sin^6 \theta (\omega/\Omega_e)^{-10} , \tag{11a}$$

$$\tau_0 \approx \beta_0{}^2 \tau_x \qquad (\theta \approx \pi/2) ,$$

$$\approx [(1 - |\cos\theta|)/(1 + |\cos\theta|)]^2 \tau_x \quad (|\pi/2 - \theta| \gg \Omega_e/2\omega) , \tag{11b}$$

$$s^* = 9 T_8{}^{0.7} , \tag{11c}$$

where T_8 is the electron temperature in units of 10^8 K. On taking the (small) density, angle, and scale-length dependences into account and changing ω to f, Ω_e to B, equations (11a, c) are replaced by slightly more general expressions:

$$\tau_x \approx 5 \times 10^3 T_8{}^7 \sin^6 \theta (n_{10} L_8/B_2) B_2{}^{10} f_9{}^{-10} , \tag{12a}$$

$$s^* \approx 10 (n_{10} L_8/B_2)^{0.1} \sin^{0.6} \theta T_8{}^{0.7} , \tag{12b}$$

where n_{10} is the electron density in units of 10^{10} cm^{-3}, B_2 is the magnetic field strength in units of 10^2 gauss, L_8 is the scale length in units of 10^8 cm = 1000 km, and f_9 is the frequency in GHz. Changing from s^* to f_{peak}, which occurs at $\tau \approx 2.5$ and $\omega/\Omega_e \approx 0.9 s^*$, we have

$$f_{\text{peak}} \approx 2.3 T_8{}^{0.7} B_2 , \tag{13a}$$

or, using (12b) in place of (11c),

$$f_{\rm peak} \approx 2.4(n_{10}L_8/B_2)^{0.1} \sin^{0.6}\theta T_8^{0.7} B_2 , \tag{13b}$$

where $f_{\rm peak}$ is in GHz.[1]

The peak frequency $f_{\rm peak}$ is an observed quantity in a microwave burst, and T_8 can be derived from an X-ray spectrum. Consequently (13a) can be used to estimate the magnetic field in the source region.

Another useful relation comes from the microwave flux density S, which, for $f \lesssim f_{\rm peak}$, is given by the Rayleigh-Jeans law for an optically thick source. In terms of the quantities introduced above, S (in units of SFU) can be written

$$S \approx 1.36 \times 10^{-2} T_8 f_9^2 L_8^2 . \tag{14}$$

Thus given T_8 and the flux density at any $f \lesssim f_{\rm peak}$, this equation can be used to estimate the scale size of the source region.

The two other parameters of most interest, the temperature and density of the source, can be derived with the help of a hard X-ray spectrum. The photon spectrum $I(E)$ can be written as (e.g., Crannell *et al.* 1978)

$$I(E) = 1.3 \times 10^3\, {\rm EM}_{45}\, E^{-1.4} T^{-0.1} \exp(-E/T) , \tag{15}$$

where $I(E)$ is in units of photons cm^{-2} s^{-1} keV^{-1}, the photon energy E and the temperature T are in keV, and the emission measure EM$_{45}$ (in units of 10^{45} cm^{-3}) is given by

$${\rm EM}_{45} = 0.1 n_{10}^2 L_8^3 . \tag{16}$$

Thus both the temperature and emission measure of the source of an impulsive burst can be estimated if the X-ray intensity is measured at two or more photon energies, E_1 and E_2 say.

In summary, given the hard X-ray intensity $I(E_1)$ and $I(E_2)$ and the microwave $f_{\rm peak}$ and $S_{\rm peak}$, then the temperature and emission measure can be derived from equation (15), the magnetic field from equation (17), the scale length from equation (14), and the source density from equation (16).

a) *Impulsive Burst of 1972 May 18*

In this section we illustrate the application of the foregoing results by considering the impulsive burst of 1972 May 18, 1406 UT, reported by Hoyng and Stevens (1973), Hoyng (1975), and Hoyng, Brown, and van Beek (1976). We choose this event because both the X-ray and microwave observations are of high quality. It is, perhaps, not an ideal event for illustration because it was relatively strong and complex, with several peaks; thus multiple source regions or particles produced by second-phase acceleration may play some role. However, it is of particular interest because of its high value of $f_{\rm peak} \approx 15$ GHz.

Figure 1 shows the X-ray and microwave spectra at different times during the burst. We will concentrate on the spectra taken near the X-ray and microwave peaks, 1406:09 to 1406:15 UT. From the X-ray spectrum and equation (15), we derive

$$T_e = 2.3 \times 10^8 \text{ K} \quad \text{and} \quad {\rm EM} = 2.0 \times 10^{45} \text{ cm}^{-3} .$$

Using these values, the radio spectrum, and equations (13), (14), and (16), we derive

$$L = 8600 \text{ km} , \quad B = 370 \text{ gauss} , \quad \text{and} \quad n_e = 2 \times 10^9 \text{ cm}^{-3} .$$

As seen on Figure 5, these values give good agreement with the X-ray intensity at the three higher energies but predict a lower intensity than observed at the lowest energy, 30 keV. Figure 5 also shows that values of T_8 in the range of about 1.5–3.0 give poorer but still acceptable fits. On Figure 6 we compare the observed radio spectrum with one calculated from the foregoing parameters and equation (12a). The agreement is good at frequencies near $f_{\rm peak}$, but the observed points at several of the lower frequencies are higher than the calculated ones —i.e., the observed spectrum rises more slowly than f^2.

In sum, both the radio and X-ray data indicate that a single isothermal source is a surprisingly good representation, especially with respect to the high radio frequencies and high X-ray energies which come from the hottest, densest part of the flare. However, they both indicate the need for additional material at a lower temperature to

[1] We should remark on the exponent of nL/B in (12b): under somewhat similar conditions Bekefi (1966, p. 207) found an exponent $\frac{1}{6}$ rather than our 0.1. Bekefi's estimate was for $T = 50$ keV, and for $n_e = 8 \times 10^{14}$ cm^{-3}, $L = 10^2$ cm, $B = 5 \times 10^4$ gauss, i.e., a value of $n_e L/B$ some three orders of magnitude less than ours. On using our curve for $T = 5 \times 10^8$ K in Figure 3, noting that Bekefi's exponent is for fixed T, and using Bekefi's value of nL/B, we find that $s^* \propto (n_e L/B)^{1/6}$ fits quite well. This comparison emphasizes that the exponents in (11), (12), and (13) apply around the chosen values $n_{10} = 1$, $T_8 = 3$, $L_8 = 2$, $B_2 = 3$, and that somewhat different exponents would be obtained for substantially different values of any of these parameters. For example, changing T_8 from 8 to 9 and leaving the other parameters the same, the exponent 10 in (11a) changes from about 14 to about 8.

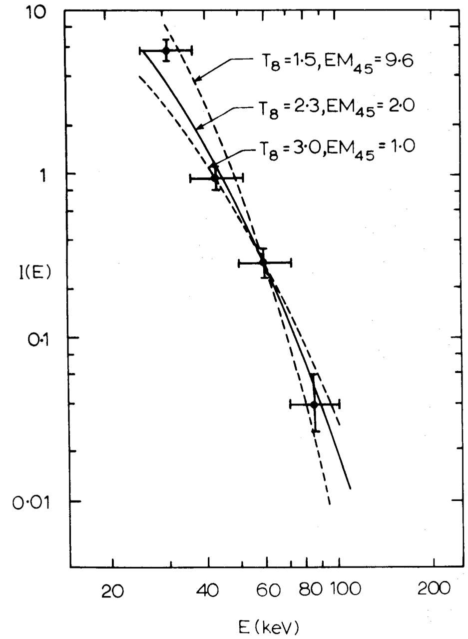

FIG. 5

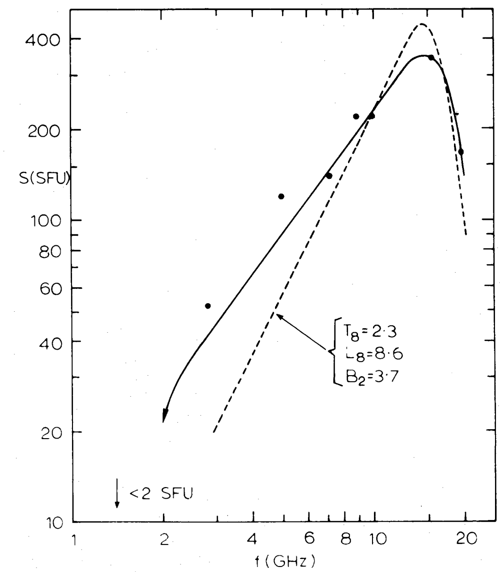

FIG. 6

FIG. 5.—X-ray spectrum of the impulsive burst of 1972 May 18, 1406:09 UT. The data points are from Hoyng and Stevens (1973). *Solid curve*, spectrum for a source with the parameters derived in the text. *Dashed curves*, for other values of temperature and emission measure as indicated.

FIG. 6.—Microwave spectrum of the event at 1406:15 UT. The data points are from Hoyng and Stevens (1973). *Solid curve*, a hand-drawn fit to the data points. Dashed line is computed using the parameters derived in the text and the curves of Fig. 3.

give the extra low-frequency/low-energy flux. As demonstrated by Milkey (1971) and Brown (1974), most X-ray spectra can be fitted by the radiation from a thermal plasma of varying temperature; a similar situation may apply to the radio spectrum (Piddington 1950; see also Pawsey and Smerd 1953).

IV. DISCUSSION AND CONCLUSION

The results of this paper are relevant to thermal models for impulsive microwave sources. The useful results are the approximate formulae (13) and (14), which relate the peak frequency and peak flux density in a thermal, self-absorbed, gyrosynchrotron source to its temperature, size, and magnetic field strength. The temperature and density may be estimated from the associated hard X-ray burst. Thus, if both the microwave and X-ray spectra of impulsive bursts result from electrons which have been energized in bulk to a near-Maxwellian distribution, it is possible to derive most of the interesting flare parameters using the simple formulae summarized here. The example we have considered is the flare of 1972 May 18, and the derived parameters are $T_e = 2.3 \times 10^8$ K, $B = 370$ gauss, $L = 8600$ km, $n_e = 2 \times 10^9$ cm^{-3}, and EM $= 2.0 \times 10^{45}$ cm^{-3}.

Crannell *et al.* (1978) studied a series of impulsive bursts, most of which were simpler and weaker than the one we considered, and concluded that a thermal interpretation is compatible with the X-ray data. The temperatures they derived ranged from about 1.7 to 7×10^8 K and the emission measures from about 0.1 to 2×10^{45} cm^{-3}, so that our example has a comparatively low temperature and high emission measure. From microwave data they deduced scale lengths of 1700 to 25,000 km for the different bursts.

Mätzler *et al.* (1978) analyzed two of the Crannell *et al.* (1978) events in more detail, in a fashion similar to ours. Using some calculations of gyromagnetic absorption coefficients by Drummond and Rosenbluth (1963), they estimated the magnetic field in the source region to be about 100 gauss. They then suggested that the magnetic energy density was only about 3 times the kinetic energy density, implying that the adiabatic heating postulated by them produced a relatively high beta plasma, i.e., $\beta \approx 1$. In contrast, for our event we find $\beta \approx 0.01$, showing a pronounced dominance of magnetic energy.

Böhme *et al.* (1977) have also studied the burst of 1972 May 18 which we have analyzed. They considered a two-component model, with the microwave radiation at $f \gtrsim 10$ GHz coming from a small core containing electrons with a power-law energy distribution and the radiation at $f \lesssim 5$ GHz coming from a larger halo. They found that the X-ray and microwave observations could be satisfied with such a model if the core diameter was about 15,000 km and, at the center, $n_e \approx 10^{10}$ cm^{-3} and $B \approx 1500$ gauss. These values are larger than ours by factors of 2, 5, and 4, respectively.

As suggested by an anonymous referee, we compare the efficiency of hard X-ray production in thermal and nonthermal models at the same value of electron density, which we take to be 2×10^9 cm^{-3} in accord with our thermal model. We consider the nonthermal case first. From Hoyng (1975, p. 27) we find that near the peak of the burst the hard X-rays can be fitted by a power-law spectrum with index $\gamma \approx 5.5$ and that the number flux F_{25} of electrons with energies $E > 25$ keV into a thick target is $F_{25} \approx 1.5 \times 10^{36}$ s^{-1} (this number would be greater for a thin target). Using these values, we estimate the nonthermal emission measure $n_0 N_{25}$ by the relation (Hoyng 1975, p. 23)

$$n_0 N_{25} = 1.46 \times 10^8 \frac{(25)^{3/2}}{\gamma - 3/2} F_{25} \text{ cm}^{-3} ,$$

and obtain $n_0 N_{25} = 6.8 \times 10^{45}$ cm^{-3}. Taking $n_0 = 2 \times 10^9$ cm^{-3}, we find that $N_{25} = 3.4 \times 10^{36}$ electrons with $E > 25$ keV are required in the nonthermal model. In the thermal model we require a total number of electrons $N = n_e L^3 = 1.3 \times 10^{36}$, of which 8.4×10^{35} have $E > 25$ keV when $T = 2.3 \times 10^8$ K. On this basis, the efficiency of the thermal model is about 4 times greater than the nonthermal model. This is, of course, not a very large factor; it is significantly less than the factor $\gtrsim 10$ estimated by Smith and Lilliequist (1979) for a source of higher density. However, we are not convinced that this comparison is very meaningful because of the thick-target assumption and because the parameters chosen may not be the optimum for a nonthermal model. None of our results prove that a thermal model is the correct one, only that it is feasible.

Finally we make a few remarks about the polarization of the microwave emission. From equation (11b) we see that the gyromagnetic absorption coefficient for the *o*-mode should be smaller than the *x*-mode by a factor of about 0.03 for $\theta = 45°$ and 0.3 for $\theta = 75°$. Thus at low frequencies, where the source is optically thick for both modes, the net polarization is calculated to be zero. At higher frequencies, where $\tau \lesssim 1$, the degree of circular polarization is calculated to be high, up to 90% for $\theta = 45°$. However, we do not expect the observed polarization to agree in detail with these calculations, since they were done for an idealized, homogeneous source with sharp boundaries. In real sources, the density may be nonuniform, the magnetic field both nonuniform and anisotropic, and edge effects may occur (e.g., Melrose 1978). While these factors will have only a minor effect on the microwave spectrum, they could produce major changes in the polarization. Thus the degree of polarization could be nonzero at low frequencies and only moderate at high frequencies. Observations of impulsive bursts at 3.7 and 9.4 GHz by Kakinuma, Yamashita, and Enome (1969) are consistent with these general expectations; there are larger numbers of highly polarized ($\gtrsim 50\%$) bursts at the higher frequency, and bursts with polarization $\gtrsim 20\%$ are not uncommon at either frequency.

After this paper was submitted, a very interesting paper by Mätzler (1978) appeared which addressed similar problems. While Mätzler did not give any analytical results, his numerical calculations were similar to ours; in his Figure 1 the curves (for $\theta = 90°$) are nearly identical to those for $\theta = 75°$ in our Figure 3. In his discussion Mätzler concentrated on the spectrum and polarization expected from model sources, while we have concentrated on the application of the results to a particular hard X-ray and microwave event. In the area of overlap his conclusions agree with ours, especially in regard to the feasibility of a thermal interpretation.

We gratefully acknowledge the inspiration, the stimulating discussions, and the highly perceptive comments given to us by Dr. S. F. Smerd until his untimely death on 1978 December 20. We shall miss him greatly.

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