Effect of Asymmetry on a Trap Model for Solar Hard X-ray Bursts

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Introduction

The basic model for the precipitation of trapped energetic particles from a magnetic flux tube is Kennel and Petschek's (1966) model. Their model is symmetric, implying equal precipitation rates at the two feet of the flux tube. We have developed a model for precipitation in an asymmetric flux tube (Melrose and White 1979). Here we explore some of the consequences for the precipitation model of Melrose and Brown (1976) for solar hard X-ray bursts. In Melrose and Brown's model roughly half the X-rays arise from precipitating electrons. With present instruments it is not possible to resolve the two feet of the flux tube. However, if the feet can be resolved, either directly by future X-ray telescopes, or indirectly through secondary optical, UV or radio observations, then, as we shall show, the additional information obtained could be used to derive information on processes in the magnetic trap.

The Trap Model

The magnetic trap in our model is taken to be a loop protruding into the corona. The 'feet' are defined by the intersection of the loop with the photosphere, or more specifically, with the layer of the solar atmosphere where a typical precipitating electron would radiate a hard X-ray due to thick-target bremsstrahlung. The precipitation is caused by pitch-angle scattering, feeding electrons into the loss-cones. As illustrated in Figure 1, we assume the scattering region to be near the top of the loop since Coulomb scattering should be more efficient in the large volume at the top of the loop than in the more dense regions at lower height. In any case we do not believe this assumption to be restrictive.

There are three possible sources of asymmetry. First, and most important, the magnetic induction **B** at the two feet can be different. The area A at each foot is inversely proportional to the value of B there. Let us write

$$A_1B_1 = A_2B_2 = A_0B_0, \qquad (1)$$

where the subscripts 1 and 2 refer to the two feet, and where the subscript 0 refers to the scattering region. We assume that the scattering region is at the point where B is a minimum. Let a particle with pitch angle $\alpha > \pi/2$ be travelling to the left (toward foot 1). Then the loss-cones are at $\alpha < \alpha_1$ and $\alpha > \pi - \alpha_2$

$$\alpha_i = \arc\sin\left(\frac{B_0}{B_i}\right)^{1/2}, \; i = 1 \text{ or } 2. \qquad (2)$$

Without loss of generality we assume $B_1 \geq B_2$, and hence $A_1 \leq A_2$ and $\alpha_1 \leq \alpha_2$.

Precipitation in the weaker field region (foot 2) is favoured. Consider a particle with $\alpha > \alpha_2$ moving to the left. As it passes through the scattering region its pitch angle will change. If the new pitch angle is in the range $\alpha < \alpha_1$ it will precipitate at foot 1, and if it is in the range $\alpha_1 < \alpha < \alpha_2$ the particle mirrors and passes back through the scattering region, and would precipitate at foot 2 if it were not scattered again. (We find as a result of our detailed investigation (Melrose and White 1979) that in a statistical sense the particles with $\alpha_1 < \alpha < \alpha_2$ are not scattered again, i.e. to every scattering of a particle into the range $\alpha_1 < \alpha < \alpha_2$ there corresponds a precipitation of a particle at foot 2.) On the other hand, particles moving to the right and scattered into the range $\alpha > \pi - \alpha_2$ precipitate at foot 2.

A second source of asymmetry is that the scattering rate for particles moving to the left and to the right may be different. If the scattering is due to Coulomb interactions, as we assume here, then this source of asymmetry is absent, and the foregoing discussion implies that the precipitation rate at foot 2 exceeds that at foot 1.

A third source of asymmetry is that the energetic particles may be injected into the trap (or accelerated within the trap) in an asymmetric way. We have found that an asymmetry in the injection rate has no important consequences.

Strong and Weak Diffusion

The nature of our solution depends strongly on whether the diffusion in pitch angle is strong or weak. Let $T(v)/\cos\alpha$ be the time taken for a particle of speed v and pitch angle α to

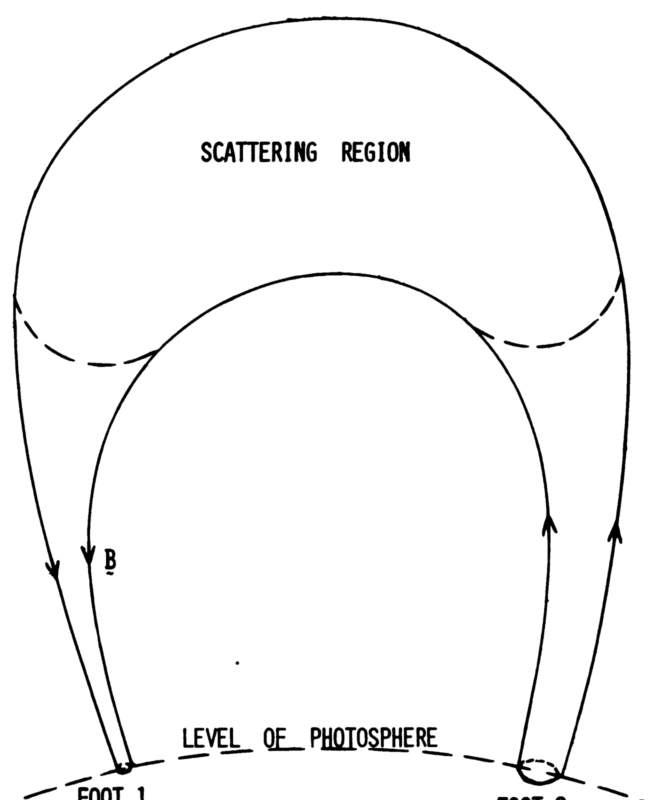

Figure 1.

pass through the scattering region, and let $D(v)$ be the pitch-angle diffusion rate. Strong diffusion corresponds to

$$D(v)T(v) \gg \alpha_2^2 \ , \tag{3a}$$

and weak diffusion corresponds to

$$D(v)T(v) \ll \alpha_2^2 \tag{3b}$$

In strong diffusion the loss-cones remain almost full because the time for scattering to fill the loss-cones is much shorter than the time for precipitation to empty them. In weak diffusion the loss-cones remain almost empty.

In the limit of strong diffusion our model implies rates of precipitation R_1 and R_2 at the two feet in the ratio (for $\alpha_1^2 \ll 1$, $\alpha_2^2 \ll 1$)

$$\frac{R_1}{R_2} = \frac{\alpha_1^2}{2\alpha_2^2 - \alpha_1^2} \ , \tag{4}$$

whereas on the basis of intuition we expected $R_1/R_2 \propto \alpha_1^2/\alpha_2^2$ in this case. A simple interpretation of (4) is as follows. In the strong diffusion limit the pitch-angle distribution is isotropic. The numbers of particles in the pitch-angle ranges $\alpha < \alpha_1$, $\alpha_1 < \alpha < \alpha_2$, $\alpha > \pi - \alpha_2$ are in the ratio $\alpha_1^2 : \alpha_2^2 - \alpha_1^2 : \alpha_2^2$. Those with $\alpha < \alpha_1$ precipitate at foot 1, and as mentioned above, at least in a statistical sense those with $\alpha_1 < \alpha < \alpha_2$ must eventually precipitate at foot 2, as do those in the range $\alpha > \pi - \alpha_2$.

In the limit of weak diffusion the number of particles with $\alpha < \alpha_2$ decreases exponentially, with a half-width of order

$$\triangle\alpha \approx \frac{1}{[D(v)\ T(v)]^{1/2}} \tag{5}$$

For $\triangle \ll \alpha_2 - \alpha_1$, a negligible fraction of the particles precipitate at foot 1.

An unexpected feature of the solution in this case is the appearance of a bump in the pitch-angle distribution for $\pi - \alpha_2 > \alpha \gtrsim \pi - \alpha_2 - \triangle\alpha$, in the case when $\alpha_2 > \alpha_1$, and a corresponding dip for $\alpha_2 < \alpha \lesssim \alpha_2 + \triangle\alpha$. When the loss-cones are equal, i.e. $\alpha_1 = \alpha_2$, we again find a bump and conjugate dip allocated according as scattering is greater or weaker respectively at each loss cone. Although interesting, this result does not seem important in the present application.

Lifetime of Trapped Particles

Given sufficient spatial resolution in hard X-rays one could measure the areas A_1 and A_2 of the two feet, and estimate the area A_0 and length L, and hence the volume $V = A_0L$, of the trap. We now show that the lifetime of the trapped particles is related to these parameters, but in different ways in the strong and weak diffusion limits. The lifetime can be estimated from the time evolution of the source.

The lifetime of a trapped particle in the strong diffusion limit in our model turns out to be (for $\alpha_2^2 \gg \alpha_1^2$)

$$\tau(v) = \frac{2T(v)}{\alpha_2^2} \tag{6}$$

where

$$T(v) = \frac{L}{v} \tag{7}$$

may be regarded as a defnition of L. Using (1) and (2), (6) implies

$$\tau(v) = \frac{2V}{vA_2} \tag{8}$$

In this case, the lifetime is independent of the scattering rate, but depends on the structure of the source. However, it is not strongly dependent on the asymmetry. In any trap model, the lifetime should be roughly equal to V/vA, provided the diffusion is strong. We wish to emphasize that (8) provides a relation between four parameters which can be observed in principle, although the spatial resolution of present X-ray telescopes is not adequate to estimate V and A_2 directly.

In the limit of weak diffusion, (8) is replaced by

$$\tau(v) \approx \frac{1}{2D(v)} \ \ell n\left(\frac{A_0}{A_2}\right) \tag{9}$$

In this case the lifetime is proportional to the scattering scale time, $1/D(v)$, and only weakly dependent on the structure of the source.

Coulomb Scattering and X-ray Sources

We now consider likely conditions in solar X-ray emission regions assuming only Coulomb scattering. The diffusion coefficient in Coulomb scattering is, e.g. by Sivukhin (1966),

$$D(v,\alpha) \propto v^{-3} \tag{10}$$

which implies, by (9),

$$\tau(v) \propto v^3 \tag{11}$$

for weak diffusion. Thus in weak diffusion the lifetime increases with velocity: slower particles will precipitate more rapidly than faster particles, and the particle distribution should harden, conversely in strong diffusion the particle distribution should soften in time since fast particles precipitate more rapidly, cf. (8).

Vorpahl *et al.* (1977) analyzed a series of long-lived, high, X-ray loops on the Sun, observed from Skylab on August 13-14, 1973. They deduced a peak temperature of 6.8 x 10^6 K and peak plasma density of 3.5 x 10^9 c $^{-3}$ a the top of the loop. The separation of the feet of the loop varied from 100,000 - 300,000 km. Using these parameters we calculated the diffusion coefficient D and characteristic diffusion distance v/D as a function of velocity. The figures are presented in Table 1. Over a distance v/D the particle pitch-angle may scatter by an amount of the order of 1 radian. Hence if $v/D < L$ where L is the linear dimension of the trap, strong diffusion applies.

If we choose $L = 10^5$ km, then by Table I strong diffusion applies for particles with energy

$$E < 5 \text{ keV} \tag{11}$$

The particles thought to be responsible for hard X-ray bremsstrahlung have $E \gtrsim 20$ keV and should experience weak