

transitions from $n = 1$ to $n = 0$. There are two allowable initial states, labelled 1 and 1', and two allowed final states labelled 0 and 0'. The values of p_{\parallel} ($= p_1, p_1', p_0$ and p_0') may be found from the values of v_{\parallel} at the intersections of the initial and final ellipses with the 1-ellipse and the 0-ellipse (the v_{\parallel} -axis) respectively by using (10a). The condition for maser action is

$$N_1(p_1) + N_1(p_1') > N_0(p_0) + N_0(p_0'). \quad (16)$$

We propose to discuss the possibility of cyclotron maser emission from neutron stars in a future publication.

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Coherent Gyromagnetic Emission

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Introduction

Observations of extremely high brightness temperatures in astrophysical objects imply that coherent emission processes must be occurring. An emission process may be coherent due to an inverted energy distribution leading to maser action, or due to bunching of particles in a region less than or of order of the wavelength of the radiation. Calculations of emission by bunches have generally used either a fluid model, e.g. Buschauer and Benford (1976), or a single-particle approach which requires all particles to have the same velocity, e.g. Saggion (1975) uses elements of both. In this paper we investigate gyromagnetic emission by bunches using a single-particle approach which includes the effects of differing particle velocities on the radiation.

Emission by a bunch may be regarded as consisting of two components. One is the sum of the incoherent powers that each particle would emit in the absence of the others. The other is the radiation due to interference between the particles, taken in pairs. Each pair emits a spike of radiation as the

particles come together. Pairs which do not approach closely do not produce significant radiation.

We explore the properties of the interference radiation. We start by explaining the idea of interference more formally and then identify the kinematic requirements for it to occur. We neglect radiation reaction, and hence exclude the possibility of absorption or maser action. We show the dependence of the interference energy on the particle velocities, and then present the current associated with each particle. These describe the incoherent radiation of each particle. The remaining term is a cross-product between the two currents which describes the effect of interference between the two particles on the radiation.

For a single particle gyromagnetic emission is continuous. The energy radiated is then identified as the power radiated times the duration of emission and this power is the quantity of interest. If the two particles have identical velocities then their interference radiation is also continuous, and is of the same form as the incoherent power, times a "coherence factor" which depends on the orbital phase difference and geometrical separation of the particles. For N particles the maximum value which this coherence factor can attain is $N(N-1)$. If the velocities of the particles differ then the interference radiation is significant only when they are close. Consequently their interaction time is effectively finite, and their interference provides a finite energy contribution even over an infinite time. This energy contribution may be positive or negative. If positive, the coherent power during the interaction is greater than the incoherent power; if negative the coherent power is less than the incoherent power.

Frequency and Angle of Emission

The current density of a single particle in a magnetic field can be resonant with only those waves whose wavevector \mathbf{k} and frequency ω satisfy the condition (Melrose 1980, p. 101) numerical calculations of emission by bunches. Finally we discuss the relation of this work to curvature emission by bunches.

Interference Radiation

Two charges q_1 and q_2 moving along orbits $\mathbf{r} = \mathbf{r}_1(t)$ and $\mathbf{r} = \mathbf{r}_2(t)$ with instantaneous velocities $\mathbf{v}_1(t)$ and $\mathbf{v}_2(t)$ give a current density

$$\mathbf{j}(\mathbf{r}, t) = q_1 \mathbf{v}_1(t) \delta(\mathbf{r} - \mathbf{r}_1(t)) + q_2 \mathbf{v}_2(t) \delta(\mathbf{r} - \mathbf{r}_2(t)) \quad (1)$$

The energy radiated by this current over an infinite time is found by integrating $-\mathbf{j}(\mathbf{r}, t) \cdot \mathbf{E}(\mathbf{r}, t)$ over all time and space, where $\mathbf{E}(\mathbf{r}, t)$ is the electric field generated by $\mathbf{j}(\mathbf{r}, t)$. Using the power theorem for Fourier transforms the energy radiated is

$$U = - \int \frac{d^3k}{(2\pi)^4} \frac{d\omega}{4} \mathbf{j}(\mathbf{k}, \omega) \cdot \mathbf{E}^*(\mathbf{k}, \omega) \quad (2)$$

where $\mathbf{j}(\mathbf{k}, \omega)$ and $\mathbf{E}(\mathbf{k}, \omega)$ denote the Fourier transforms of the current and electric field ("*" denotes complex conjugation). Solving Maxwell's equations (after Fourier transforming) for $\mathbf{E}(\mathbf{k}, \omega)$ in terms of $\mathbf{j}(\mathbf{k}, \omega)$, we may evaluate U using (2) (e.g. Melrose, 1980, p. 67). We find

$$U = \int \frac{d^3k}{(2\pi)^3} 2\pi |\mathbf{e}^* \cdot \mathbf{j}(\mathbf{k}, \omega)|^2 \quad (3)$$

for emission in vacuo of transverse waves with polarization vector \mathbf{e} ($\mathbf{e} \cdot \mathbf{k} = 0$ and $\mathbf{e} \cdot \mathbf{e}^* = 1$, and the dispersion relation $\omega = c\mathbf{k}$ has been used). On identifying $\mathbf{j}(\mathbf{k}, \omega)$ with the Fourier transform of (1), the energy radiated, U , includes two terms from the squares of

$$\omega - c k_z \beta_z - s\Omega/\gamma = 0 \quad (4)$$

for some integer s , where the z -direction is chosen along the magnetic field, $c\beta$ is the particle's velocity and $\gamma = (1 - \beta^2)^{-1/2}$. $\Omega = eB/mc$ is the gyrofrequency of a particle of mass m in a magnetic field B . Physically we interpret (4) to require that in the frame moving with the particle along the field, the wave frequency matches one of the harmonics of the particle gyrofrequency, permitting resonant interaction between particle and wave. In vacuo we write

$$k_z = \frac{\omega}{c} \cos\theta \quad (5)$$

where θ is the angle between \mathbf{k} and the magnetic field. Equation (2) then relates the frequency of emission to the angle θ .

However if the particles have different velocities two conditions must be met simultaneously for them to produce interference radiation. These conditions are

$$\omega - \omega \beta_{z1} \cos\theta - s\Omega/\gamma_1 = 0 \quad (6)$$

and

$$\omega - \omega \beta_{z2} \cos\theta - s\Omega/\gamma_2 = 0 \quad (7)$$

Given \mathbf{B}_1 and \mathbf{B}_2 these two equations uniquely determine ω and θ . Hence the interference emission by such dispersing particles occurs only at one angle and harmonics of one frequency, i.e. at

$$\cos\theta = \frac{\gamma_1 - \gamma_2}{\gamma_1 \beta_{z1} - \gamma_2 \beta_{z2}} \quad (8)$$

and

$$\omega = s\omega_0 \quad (9)$$

where

$$\omega_0 = \Omega \frac{\gamma_1 \beta_{z1} - \gamma_2 \beta_{z2}}{\gamma_1 \gamma_2 (\beta_{z1} - \beta_{z2})} \quad (10)$$

when $\phi = 0$ and with

$$\omega_0 \equiv \Omega \quad (11)$$

Similarly (8) simplifies to

$$\cos\theta = \frac{1}{2} \frac{\beta_{z1}^2 - \beta_{z2}^2}{\beta_{z1} - \beta_{z2}} \quad (12)$$

In the ultrarelativistic limit for dispersing particles, (10) becomes

$$\omega_0 \equiv \Omega \frac{2\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \quad (13)$$

and (8) becomes

$$\theta \equiv \frac{1}{(\gamma_1 \gamma_2)^{1/2}} \quad (14)$$

For a single particle in this limit the emission is just synchrotron radiation which peaks at $\omega \equiv 3/2 \gamma^3 \Omega$ and is strongly concentrated on the surface cone of half angle $\theta \equiv \arccos(\beta_z/\beta)$ and of thickness $\Delta\theta \equiv 1/\gamma$.

It is interesting to note that for certain mildly relativistic pairs β_{z1}, β_{z2} such that $\gamma_1 \beta_{z1} \equiv \gamma_2 \beta_{z2}$ but β_{z1} and β_{z2} differ, the fundamental frequency of coherent emission, ω_0 , may be much less than the gyrofrequency.

Energy of Interference Radiation

As is well known (Jackson 1962) the power from gyromagnetic emission in vacuo by a particle of velocity $c\beta$ is

$$P = \frac{2}{3} \frac{q^2 \Omega^2}{c} \gamma^2 \beta_{z1}^2 \quad (15)$$

We compare this with the form of the energy due to the interference radiation. In the non-relativistic limit the energy emitted due to the interference of two particles is

$$U = 4\pi \frac{q^2 \Omega}{c} \frac{\beta_{z1} \beta_{z2}}{|\beta_{z1} - \beta_{z2}|} \cos\phi \quad (16)$$

when ϕ denotes the difference in orbital phase between the two particles when one passes the other. The reason why this particular phase factor is important is as follows. Conditions (6) and (7) imply that in the frame of the wave both particles are (rotationally) oscillating at the same rate, such that wave oscillations occur at a harmonic of the particle oscillations. Consequently as far as the radiation field is concerned the

relative phase between the oscillations of the two particles is invariant during their interaction although this is clearly not true in the laboratory frame. If the phase is zero, the particles interfere constructively. If the phase is π , they interfere destructively at odd harmonics. If we average (16) over a random distribution of the phase ϕ , to first order we find that the net interference energy is zero. For this emission process, therefore, we may have a bunch of particles coherent on a scale much smaller than a wavelength but for whom the net coherent emission is no different from the incoherent emission.

At ultra relativistic energies the formula for the interference energy is a complicated function of ϕ . The maximum value it can attain is

$$U_{\text{tot}} = 4\pi \frac{q^2 \Omega}{c} \bar{\gamma}^{-4} \frac{\beta_{11} \beta_{12}}{|\gamma_1 - \gamma_2|} \quad (17)$$

when $\phi = 0$ and with

$$\bar{\gamma} = \frac{2\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \quad (18)$$

Numerical Calculations of Radiation by Bunches

We have calculated the radiation from a bunch of 200 particles in a magnetic field. The bunch is chosen (using a pseudo-random-number generator) to have a Maxwellian velocity

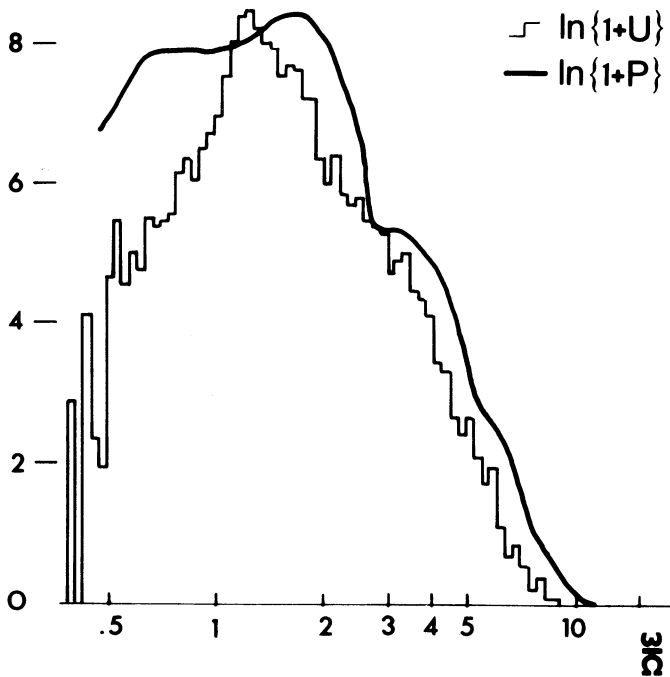


Figure 1. Plot of $\ln(1+U)$ (narrow line histogram) and $\ln(1+P)$ (heavy line) versus frequency, ω/Ω , for emission by a bunch of 200 particles, all with the same phase (see text). The total interference energy emitted is $42879 q^2 \Omega/c$. The total incoherent power emitted is $1.9733 q^2 \Omega^2/c$. U is in units of $q^2 \Omega/c$, P is in units of $q^2 \Omega^2/c$, and the values of power, P , have been multiplied by 3360 before plotting to allow direct comparison of the shapes of the spectra.

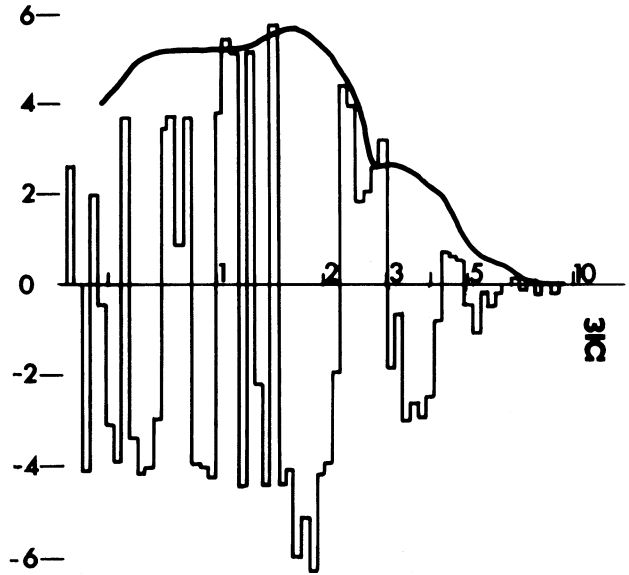


Figure 2. Plot of $\ln(1+U)$ and $\ln(1+P)$ versus frequency for emission of a bunch of particles with the same velocity distribution as in Figure 1, but with random phases, ϕ . The total interference energy in this case is $-716.75 q^2 \Omega/c$. The values of power have been multiplied by 211 before plotting.

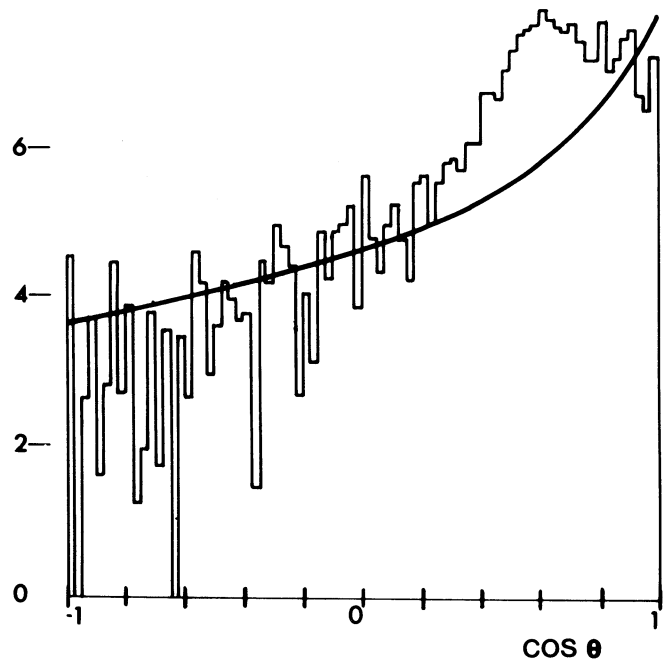


Figure 3. Plot of $\ln(1+U)$ and $\ln(1+P)$ versus $\cos \theta$ for the same bunch, with all having the same phase (cf. Figure 1). The powers have been multiplied by 245 before plotting.

distribution with a temperature of order 5 keV, and is streaming at 0.6c along the field. The particles are all following the same magnetic field line, and at time $t = 0$ all are bunched at the same point on the field line. Figures 1-4 compare the characteristics of the interference radiation of the

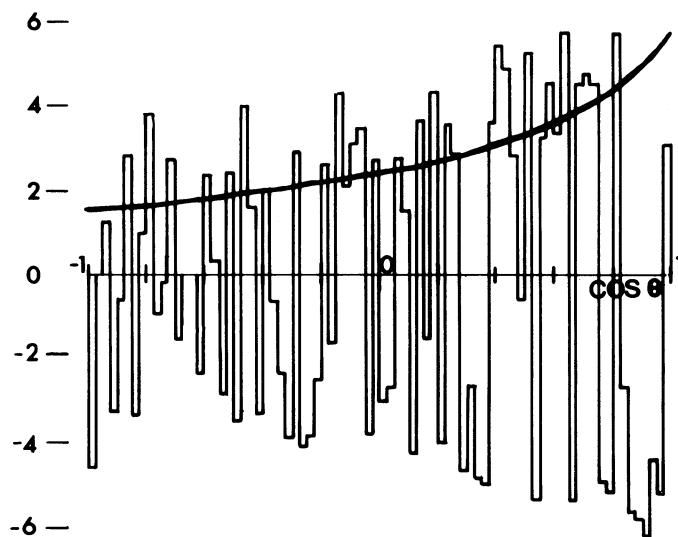


Figure 4. Plot of $\ln(1+U)$ and $\ln(1+P)$ versus $\cos \theta$ for the same bunch, with the distribution of phases being random (cf. Figure 2). The values of power have been multiplied by 31 before plotting.

bunch with those of the incoherent emission. On Figures 1 and 2 we plot the variation of the interference energy U (rendered dimension-less by dividing by $q^2\Omega/c$ and the incoherent power P (divided by $q^2\Omega^2/c$) with frequency. On Figures 3 and 4 we plot power and energy versus $\cos\theta$, to show the angular characteristics of the radiation. In order to show the effects of the phase ϕ , Figures 1 and 3 are calculated on the (unphysical) assumption that $\phi = 0$ for all particles. Figures 2 and 4 assume a random distribution of phases amongst the 200 particles.

For a bunch of 200 particles there are some 20,000 paired interactions in the interference component of the radiation. Ideally this implies over 20,000 zero-width spikes on the spectrum. Rather than show all these spikes we have divided the frequency range into logarithmically equal intervals and added the energies in each interval to form a histogram distribution. To show faint detail we have plotted $\ln(1+U)$ and $\ln(1+P)$ rather than U and P , and the numerical values of P have been scaled up to match those of U .

From Figures 1 and 3 we see that the spectral and angular characteristics of the interference radiation of the zero-phased bunch are very similar to the incoherent radiation of the bunch. However for the randomly phased bunch the energy in a given frequency range is about an order of magnitude smaller than for the zero-phased bunch, and regions of positive and negative interference occur in no particular pattern. These figures show that a non-random distribution of phases may be needed to give significant gyromagnetic interference radiation.

Curvature Emission by Bunches

Coherent emission by bunches of charged particles is a candidate mechanism for pulsar radio emission. The radiation process is thought to be curvature radiation, that is the par-

ticles follow curved field lines and radiate due to their acceleration. Bunching provides the coherence required to explain the high brightness temperatures observed. Coherent curvature emission has not yet been treated to allow for the effects of differing particle velocities on the characteristics of the radiation. When a single-particle approach to incoherent curvature radiation is used (e.g. Saggion 1975), the radiation properties are derived from formulae for gyromagnetic emission by replacing the gyroradius with the field-line curvature. This method may not be used for coherent curvature emission from dispersing bunches for the following reasons. The appropriate analogy in gyromagnetic emission is of two particles moving in a circular rather than a helical orbit, i.e. $\beta_{z1} = \beta_{z2} = 0$, and $\beta_{\perp}c$ is replaced by the speed of the particles along the field line. If the two particles are dispersing then $\beta_{\perp1} \neq \beta_{\perp2}$ in the gyromagnetic formulae. However, in gyromagnetic theory the radius of the orbit depends on β_{\perp} , which is not appropriate for curvature emission. Further the calculations of this paper show that no interference between particles with $\beta_{z1} = \beta_{z2}$ but $\beta_{\perp1} \neq \beta_{\perp2}$ may occur. Hence the gyromagnetic analogy to curvature emission may only give exact results for particles with the same speed along the field line. The results of the calculation in this paper suggest that when we take account of the dispersion of particles in bunches emitting coherent curvature radiation, we may expect that the spectral properties and angular characteristics of the resulting radiation will differ from the predictions of incoherent curvature emission theory. However the details of these differences have yet to be explored.

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Elliptically Polarized Natural Modes in Pulsar Magnetospheres

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Introduction

The most obvious feature of the polarization of the radio emission from most pulsars is the rotation of the plane of linear polarization across pulses. The original interpretation of this in terms of the magnetic pole model (Radhakrishnan 1969, Radhakrishnan *et al.* 1969, Radhakrishnan and Cooke 1969)