FREQUENCY SPLITTING IN STRIA BURSTS: POSSIBLE ROLES OF LOW-FREQUENCY WAVES

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Abstract. The kinematics of the process $L \pm F \rightarrow L'$ are explored where L represents a parallel Langmuir wave, F represents a low frequency fluctuation and L' represents a secondary Langmuir wave, and the results are used to discuss (a) a possible interpretation of the frequency splitting in stria bursts in terms of the processes $L \pm F \rightarrow L'$, $L' \pm F' \rightarrow t$, where t represents a transverse wave, and (b) second harmonic emission due to the processes $L \pm s \rightarrow L'$, $L + L' \rightarrow t$, where s represents an ion sound wave. The following results are obtained:

(1) The processes $L \pm s \rightarrow L'$ are allowed only for $k_s < 2k_L \pm k_0$, respectively, with $k_0 = \omega_p/65\,V_e$.

(2) The inclusion of a magnetic field does not alter the result (1) and adds further kinematic restrictions related to angles of propagation; the kinematic restriction $T_e > 5 \times 10^5$ K for second harmonic emission through process (b) above is also unchanged by inclusion of the magnetic field. The effect of a spread in the wavevectors of the Langmuir waves on this restriction is discussed in the Appendix.

(3) For parallel Langmuir waves the process $L - F \rightarrow L'$ is forbidden for lower hybrid waves and for nearly perpendicular resonant whistlers, and the process $L + F \rightarrow L'$ is allowed only for resonant whistlers at $\omega_F \gtrsim \frac{1}{2}\omega_p(\Omega_e/\omega_p)^2$.

(4) The sequential three wave processes $L \pm s \rightarrow L'$, $L' \pm s \rightarrow t$ and $L + F \rightarrow L'$, $L' \pm F' \rightarrow t$ encounter difficulties when applied to the interpretation of the splitting in split pair and triple bursts.

(5) The four-wave process $L \pm F \pm F' \rightarrow t$ is kinematically allowed and provides a favourable qualitative interpretation of the splitting when F denotes a resonant whistler near the frequency mentioned in (3) above. The four wave processes should saturate under conditions which are not extreme and produce fundamental plasma emission with brightness temperature T_t equal to the effective temperature T_L of the Langmuir waves.

1. Introduction

The investigation reported here involves an analysis of kinematic restrictions on three waves processes involving a Langmuir (L) wave parallel to the magnetic field and a low-frequency (F) wave, which may be an ion sound (s) wave, a lower hybrid wave or a resonant whistler wave. This investigation has two motivations. First in a recent paper (Melrose, 1982) it was shown that there is a kinematic restriction on second harmonic emission due to the processes $L \pm s \rightarrow L'$ and $L + L' \rightarrow t$, where t describes a transverse wave: this restriction leads to the unexpected conclusion that the process is forbidden for $T_e < 5 \times 10^5$ K. The question arises as to whether there are similar kinematic restrictions on the processes $L \pm s \rightarrow L'$, $L' \pm s' \rightarrow t$, which would lead to fundamental plasma emission. Another question is what effect a magnetic field has on the kinematics. These questions require a careful investigation of the kinematics of the processes $L \pm s \rightarrow L'$. We also explore the kinematics of the alternative processes $L \pm F \rightarrow L'$ involving the other low-frequency waves mentioned above.

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The second motivation concerns the frequency splitting of stria bursts (de la Noë and Boischot, 1972) into split pair and triple bursts (Ellis and McCulloch, 1967). It was proposed recently (Melrose, 1983) that the observed splitting could be explained qualitatively and semi-qualitatively in terms of the processes $L \pm F \pm F' \rightarrow t$. If the low frequency waves have wavevectors $k_F \cong k_{F'} \gg k_L$, then their frequencies should be nearly equal, $\omega_F \cong \omega_{F'}$, and the four processes indicated would lead to transverse waves with frequencies $\omega_t = \omega_L$, $\omega_L \pm 2\omega_F$. This can account naturally for a triple splitting as in a triple burst. Moreover there is plausible reason why the lowest frequency component might sometimes be missing (it may be below ω_p) to leave a double splitting as in a split pair burst. If the low-frequency turbulence is collimated one may show that the highest frequency element would also be absent leaving a single stria burst. However, despite its obvious attraction as a simple explanation for the observed splitting, this theory encounters severe difficulties due to kinematic restrictions (Melrose, 1983). These restrictions are discussed here and a possible way of overcoming them is indicated.

The most direct mechanism for fundamental plasma emission involving ion sound waves is due to the processes $L \pm s \rightarrow t$ (e.g. Dawson and Oberman, 1963; Tsytovich, 1966; Melrose, 1980a). The processes considered here are more complicated than this process due to their involving either an additional intermediate step or a higher order nonlinear process. There are two reasons why it is relevant to consider such processes. First, note that the ion sound waves required for the processes $L \pm s \rightarrow t$ have $\mathbf{k}_s = \mp \mathbf{k}_L$; this is due to the wave number of the transverse waves being negligibly small. Because the processes $L \pm s \rightarrow t$ are restricted to a narrow part of the spectrum of ion sound waves, it is quite possible that the required ion sound waves are weak or absent, and that processes which can involve a broader spectrum of ion sound waves are more favourable. The ion sound waves observed in the interplanetary medium (e.g. Gurnett *et al.*, 1979) peak around $k_s \lambda_{De} \cong 0.5$, which corresponds to $k_s \gtrsim 10 k_L$ for the k_L values expected in type III bursts. These observed ion sound waves could not contribute to the processes $L \pm s \rightarrow t$, and it is relevant to ask whether they might cause fundamental plasma due to the two stage processes $L \pm s \rightarrow L'$, $L' \pm s \rightarrow t$. Second, even if both the single stage and two stage processes are allowed it is not necessarily the case that the two stage process is unimportant. Indeed quite the contrary: under quite mild conditions the rate at which three wave processes proceed is so fast that they saturate. The saturation level for all processes involving a net conversion $L \rightarrow t$ corresponds to the effective temperature of the transverse waves being equal to that of the Langmuir waves. Thus when processes involving low frequency turbulence are invoked, all processes which saturate should be taken into accound. The surface brightness at the source is independent of the details of each such process provided that it saturates, i.e. that it corresponds to an optical depth greater than unity.

In Section 2 the kinematics of the processes $L \pm s \rightarrow L'$ are explored with the main purpose being to determine whether they are allowed for $k_s \gg k_L$. It is found that the processes are allowed only for $k_s \leq 2k_L \pm k_0$, respectively. Thus ion sound turbulence with $k_s \lambda_{De}$ close to unity, as observed in the interplanetary medium, cannot lead to fundamental plasma emission due to these processes. In Sections 3 and 4 the effect of the magnetic field is considered for three wave processes involving an initially parallel Langmuir wave and a low frequency wave. In Section 3 the case of ion sound waves is considered. It is shown that the restriction $k_s \leq 2k_L \pm k_0$ obtained in Section 2 still applies, together with further restrictions on certain wave angles. It is also shown that the condition $T_e > 5 \times 10^5$ K (Melrose, 1982) for second harmonic emission to be allowed due to the processes $L \pm s \rightarrow L'$, $L' + L \rightarrow t$ continues to apply in the presence of a magnetic field. (A referee has suggested that the restriction $T_e > 5 \times 10^5$ K may not be relevant to type III bursts because the spectrum of $\mathbf{k}_L$ is expected to be relatively broad (Takakura, 1979, 1982; Magelssen and Smith, 1977); it is shown in the Appendix that over a broad spectrum a similar restriction may apply.) In Section 4 lower hybrid and resonant whistler waves are considered in place of ion sound waves. (Very recently Marsch and Chang (1982) have argued that the observed electrostatic turbulence, interpreted as ion sound turbulence above, is actually in lower hybrid or resonant whistler waves.) The waves propagate nearly perpendicularly to the field lines. It is found that the process $L - F \rightarrow L'$ is not allowed and $L + F \rightarrow L'$ is also not allowed for lower hybrid waves. Recall that the initial Langmuir wave is assumed parallel to the magnetic field and this result does not invalidate the type I theories (Melrose, 1980b; Benz and Wentzel, 1981) involving lower hybrid waves and Langmuir waves generated by trapped electrons.

In Section 5 we discuss the proposed interpretation of the frequency splitting in stria bursts in terms of the two stage processes $L \pm F \rightarrow L'$, $L' \pm F' \rightarrow t$. As the foregoing outline suggests there are serious kinematic difficulties with this interpretation. An alternative is explored briefly in Section 6: the processes $L \pm F \pm F' \rightarrow t$ are allowed due to a four-wave interaction. A rough estimate of the optical depth for this process is made and it is concluded that it can be greater than unity under conditions which are not extreme.

2. The Processes $L \pm s \rightarrow L'$

In this section we discuss the implications of the equations

$$\omega_L \pm \omega_s = \omega_{L'}, \qquad \mathbf{k}_L \pm \mathbf{k}_s = \mathbf{k}_{L'} \tag{1a, b}$$

with

$$\omega_L = \omega_p + \frac{3}{2}\frac{k_L^2 V_e^2}{\omega_p}, \tag{2}$$

$$\omega_s = k_s v_s . \tag{3}$$

We denote unit vectors along $\mathbf{k}_L$, $\mathbf{k}_s$, and $\boldsymbol{\kappa}_L$, $\boldsymbol{\kappa}_s$, and $\boldsymbol{\kappa}_{L'}$, respectively.

As shown previously (Melrose, 1982), (1a, b) imply

$$k_{L'}^2 = k_L^2 \pm k_0 k_s , \tag{4}$$

$$\boldsymbol{\kappa}_L \cdot \boldsymbol{\kappa}_s = \frac{k_0 \mp k_s}{2k_L}, \tag{5}$$

with
$$k_0 := \frac{2\omega_p v_s}{3V_e} = \frac{\omega_p}{65V_e}\,. \tag{6}$$

where ': =' denotes a definition.

(i) *u-case*

The up (u) conversion corresponds to the upper signs in (4) and (5). Inspection of (5) shows that for k_s arbitrarily small we have $\boldsymbol{\kappa}_L \cdot \boldsymbol{\kappa}_s = k_0/2k_L$, which is physically acceptable for $k_L > k_0/2$, and that $\boldsymbol{\kappa}_L \cdot \boldsymbol{\kappa}_s$ decreases with increasing k_s passing through -1 at $k_s = 2k_L + k_0$. Hence k_s is restricted to the range

$$\max[0, k_0 - 2k_L] \le k_s \le 2k_L + k_0\,. \tag{7}$$

According to (4) the value of $k_{L'}^2$ increases with k_s, and corresponding to (7) we have

$$\max[k_L^2, (k_L - k_0)^2] \le k_{L'}^2 \le (k_L + k_0)^2\,. \tag{8}$$

(ii) *d-case*

The down (d) conversion corresponds to the lower signs in (4) and (5). In this case $\boldsymbol{\kappa}_L \cdot \boldsymbol{\kappa}_s$ increases with k_s and reaches unity at $k_s = 2k_L - k_0$. Hence we have

$$0 \le k_s \le 2k_L - k_0\,. \tag{9}$$

Similarly (4) implies

$$(k_L - k_0)^2 \le k_{L'}^2 \le k_L^2\,. \tag{10}$$

The main point to be noted is that the processes $L \pm s \to L'$ are forbidden for sufficiently large k_s. Indeed they are allowed only for $k_s \le 2k_L \pm k_0$. In practice k_0 is less than or of order k_L, and hence these processes are possible only for k_s less than or of order k_L.

3. Magnetized Case $L \pm s \to L'$

In this section the effect of a magnetic field in included: the problem analysed in Section 2 is reanalysed with the dispersion relation (2) replaced by (for $\omega_p^2 \gg \Omega_e^2, k_L^2 V_e^2$)

$$\omega_L = \omega_p + \frac{3}{2}\frac{k_L^2 V_e^2}{\omega_p} + \frac{\Omega_e^2}{2\omega_p}\sin^2\theta_L\,, \tag{11}$$

where Ω_e is the electron gyrofrequency and θ_L is the angle between $\mathbf{k}_L$ and the magnetic field. In addition it is assumed that the initial Langmuir wave is propagating along the magnetic field, i.e. $\sin\theta_L = 0$. This assumption is plausible for Langmuir waves generated by a stream of electrons.

The generalization of (4) when the magnetic correction in (11) is included for the scattered Langmuir wave is

$$k_L^2 \pm k_0 k_s = k_{L'}^2 + k_B^2 \sin^2\theta_{L'} \tag{12}$$

with

$$k_B^2 := \frac{\Omega_e^2}{3V_e^2}\,. \tag{13}$$

Our assumption that $\mathbf{k}_L$ is along the magnetic field implies

$$k_{L'} \cos\theta_{L'} = k_L \pm k_s \cos\theta_s\,, \tag{14a}$$

$$k_{L'} \sin\theta_{L'} = k_s \sin\theta_s\,. \tag{14b}$$

Then (12) reduces to

$$\pm k_0 k_s - k_s^2 \mp 2k_L k_s \cos\theta_s = \frac{k_B^2 k_s^2 \sin^2\theta_s}{k_L^2 + k_s^2 \pm 2k_L k_s \cos\theta_s}\,. \tag{15}$$

The right-hand side of (15) is positive, and hence the left-hand side must be positive. This restricts the range of allowed values of k_s. It is straightforward to show that the allowed ranges are just those given by (7) and (9) for the unmagnetized case. Thus the inclusion of the magnetic field does not weaken the kinematic restrictions on the processes $L \pm s \to L'$.

Equation (15) may be written as a quadratic equation for $\cos\theta_s$ and solved:

$$\cos\theta_s = \frac{1}{k_B^2 - 4k_L^2}\Bigg[\pm\frac{k_L}{k_s}\,\{k_L^2 + k_s^2 \mp k_s(k_0 \mp k_s)\} + \\ + \sigma\Bigg\{k_B^4 + k_B^2\Bigg[-4k_L^2 \mp \frac{1}{k_s}\,(k_0 \mp k_s)(k_L^2 + k_s^2)\Bigg] + \\ + \frac{k_L^2}{k_s^2}\,[k_L^2 \pm k_s k_0]^2\Bigg\}^{1/2}\Bigg]\,, \tag{16}$$

with $\sigma = \pm 1$ denoting the two solutions. In the limit $k_B \to 0$ (16) reproduces (5) (with $\boldsymbol{\kappa}_L \cdot \boldsymbol{\kappa}_s = \cos\theta_s$) for $\sigma = \mp 1$. Now setting $\sigma = \mp 1$ and expanding, assuming that k_B^2 is either arbitrarily small or arbitrarily large, we find:

small k_B^2:

$$\cos\theta_s \cong \frac{k_0 \mp k_s}{2k_L} \mp \frac{k_B^2 k_s}{8k_L^3}\,\frac{4k_L^2 - (k_0 \mp k_s)^2}{k_L^2 \pm k_s k_0}\,; \tag{17a}$$

large k_B^2:

$$\cos\theta_s \cong \mp 1\left[1 - \frac{(k_L - k_s)^2}{k_B^2}\,\frac{k_L}{k_s}\left(1 \pm \frac{k_0 \mp k_s}{2k_L}\right)\right]. \tag{17b}$$

From (17a, b) one infers that as k_B^2 increases from zero the ranges of angles over which the processes $L \pm s \to L'$ can occur shrink. For the u process the range shrinks from $-1 \leq \cos\theta_s \leq k_0/2k_L$ towards a narrow range at $\cos\theta_s \gtrsim -1$ for large k_B^2, and for the

d process the range shrinks from $k_0/2k_L \leq \cos\theta_s \leq 1$ to a narrow range at $\cos\theta_s \lesssim 1$. In the case (17b) one finds that $\sin^2\theta_s$ is necessarily very small and then using (14b) one finds

$$k_B^2 \sin^2\theta_{L'} \cong 2k_L k_s \left(1 \pm \frac{k_0 \mp k_s}{2k_L}\right). \tag{18}$$

Now it is obvious that for arbitrarily large k_B^2 (12) can be satisfied only if $\sin^2\theta_{L'}$ is small enough that the term $k_B^2 \sin^2\theta_{L'}$ is of the same order as the other terms. The specific form (18) of this additional term is such that it vanishes for $k_s = 2k_L \pm k_0$, when one has $\cos\theta_s = \mp 1$, and also in the limit $k_s = 0$. The maximum value is at $k_s = \frac{1}{2}(2k_L \pm k_0)$ when one has $k_B^2 \sin^2\theta_{L'} = k_s^2$.

Qualitatively the effect of the magnetic correction is to impede the processes $L \pm s \rightarrow L'$ by restricting them to ranges where all the waves are nearly along the magnetic field. Although we have considered only the case $\sin^2\theta_L = 0$, it is not difficult to see that these restrictions apply to processes in which $\sin^2\theta_{L'}$ exceeds $\sin^2\theta_L$ whenever $\sin^2\theta_L$ is non zero. On the other hand the kinematic restrictions on processes which reduce the angle ($\sin^2\theta_{L'} < \sin^2\theta_L$) are less severe than in the absence of a magnetic field.

3.1. Effect on Second Harmonic Emission

As discussed by Melrose (1982), the simplest sequences of processes involving ion sound waves and leading to second harmonic type III emission consists of $L \pm s \rightarrow L'$, $L' + L \rightarrow t$. The second step requires

$$(\mathbf{k}_{L'} + \mathbf{k}_L)^2 = k_{SH}^2 = \frac{3\omega_p^2}{c^2}. \tag{19}$$

Using (12) and (14a, b) one finds

$$k_{SH}^2 - k_0^2 = 4k_L^2 - (k_0 \mp k_s)^2 - 2k_B^2 \sin^2\theta_{L'}. \tag{20}$$

For $k_B^2 = 0$ the restrictions (7) and (9) imply that the right-hand side of (20) is positive, and then $k_{SH}^2 > k_0^2$ implies $T_e > 5 \times 10^5$ K. More generally k_{SH} and k_0 are fixed by the plasma parameters and once k_L is determined (20) determines a unique value of k_s for each of the u and d processes; this value satisfies $k_s < 2k_L \pm k_0$ only for $k_{SH}^2 > k_0^2$.

In the case $k_B^2 \neq 0$ the right-hand side of (20) vanishes for $k_s = 2k_L \pm k_0$. One may show that for $k_s < 2k_L \pm k_0$ the right-hand side of (20) is strictly positive. For example, for the maximum value of $k_B^2 \sin^2\theta_{L'}$ in the limit of large k_B^2, which is $k_s^2 = \frac{1}{4}(2k_L \pm k_0)^2$, one finds that the right-hand side of (20) is equal to $\frac{1}{4}k_s^2$. Hence one concludes that the restriction $k_{SH}^2 > k_0^2$ or $T_e > 5 \times 10^5$ K continues to apply in the magnetized case.

4. Lower Hybrid and Resonant Whistler Waves

A possible mechanism for fundamental emission is $L \pm F \rightarrow L'$, $L' \pm F' \rightarrow t$ where F represents a low-frequency wave. Besides ion sound waves ($F = s$) other possibilities are lower hybrid waves and resonant whistler waves. The dispersion relation for these waves is independent of wavenumber, k_F say, to a first approximation, with

$$\omega_F = \begin{cases} \omega_{LH}, & |\cos\theta_F| \lesssim \frac{1}{43} \\ \Omega_e |\cos\theta_F|, & |\cos\theta_F| \gtrsim \frac{1}{43} \end{cases} \tag{21}$$

with $\omega_{LH} \cong \Omega_e/43$ here. These waves are restricted to angles of propagation close to perpendicular to the field lines except near the electron cyclotron resonance ($\omega_F \cong \Omega_e$). We assume $\omega_F \ll \Omega_e$, $|\cos\theta_F| \ll 1$.

Beating of a wave with dispersion relation (21) with a parallel propagating Langmuir wave can produce secondary Langmuir waves ($L \pm F \rightarrow L'$) provided the condition

$$k_L^2 \pm k_1^2 = k_{L'}^2 + k_B^2 \sin^2\theta_{L'} \tag{22}$$

is satisfied, where we write

$$k_1^2 := \frac{2}{3} \frac{\omega_F \omega_p}{V_e^2} . \tag{23}$$

Using the fact that $\mathbf{k}_F$ is nearly orthogonal to $\mathbf{k}_L$ we have

$$k_{L'}^2 \cong k_L^2 + k_F^2 , \tag{24}$$

$$\sin^2\theta_{L'} \cong \frac{k_F^2}{k_{L'}^2} \cong \frac{k_F^2}{k_L^2 + k_F^2} . \tag{25}$$

If then follows that only the upper sign (u process) is allowed in (22), and that the value of k_F^2 is given by

$$k_F^2 = \tfrac{1}{2}\{k_1^2 - k_L^2 - k_B^2 + [k_B^4 + 2k_B^2(k_L^2 - k_1^2) + (k_L^2 + k_1^2)^2]^{1/2}\} , \tag{26}$$

where we assume $k_1^2 > k_L^2 + k_B^2$. Allowing k_L^2, k_1^2, and k_B^2 to vary the maximum value of k_F^2 is given by

$$k_F^2 \leq \tfrac{1}{2}(k_1^2 - k_L^2 - k_B^2) . \tag{27}$$

Consider the ratio

$$\frac{k_1^2}{k_B^2} = \begin{cases} \dfrac{2}{43} \dfrac{\omega_p}{\Omega_e} , & |\cos\theta_F| \lesssim \dfrac{1}{43} , \\ \dfrac{2\omega_p}{\Omega_e} |\cos\theta_F| , & |\cos\theta_F| \gtrsim \dfrac{1}{43} . \end{cases} \tag{28}$$

Even for very weak magnetic fields one has $2\omega_p/43\,\Omega_e < 1$ in the solar corona and the interplanetary medium. One then concludes that the process $L + F \rightarrow L'$ is forbidden for lower hybrid waves (recall that we consider only the case where $\mathbf{k}_L$ is along the magnetic field) and is allowed for resonant whistlers only for

$$\omega_F \gtrsim \frac{\omega_p}{2} \left(\frac{\Omega_e}{\omega_p}\right)^2 . \tag{29}$$

When considering the processes $L + F \rightarrow L'$, $L' \pm F' \rightarrow t$ to produce fundamental plasma emission there is an additional restriction on $\cos\theta_F$:

$$k_L + k_F \cos\theta_F \pm k_{F'} \cos\theta_{F'} \cong 0 . \tag{30}$$

For values $|\cos\theta_F| \gtrsim \Omega_e/2\omega_p$ implied by condition (29), Equation (30) requires $k_F \gtrsim k_L\ |\cos\theta_F \cong 3.5k_B(V_e/v_\phi)(\omega_p/\Omega_e)^2$, where we write $k_L = \omega_p/v_\phi$. With $v_\phi \cong 30V_e$ for a type III burst, this condition on k_F is not usually compatible with (27). It may be concluded that there are severe kinematic restrictions on the processes $L + F \rightarrow L'$, $L' \pm F' \rightarrow t$ for initially parallel Langmuir waves.

5. Frequency Splitting in Stria Bursts: Sequential Three-Wave Processes

The most obvious features of the splitting observed in stria bursts include the following:

(1) The splitting is in frequency with the frequency separations exceeding the bandwidths by a factor between one and two.

(2) The bursts may be singlet, doublet or triplet with doublets (split pair bursts) appearing to be like triplets (triple bursts) with the lowest frequency element missing.

(3) The typical magnitude δf of the frequency splitting corresponds to $\delta f/f \cong 1/300$, e.g. $\delta f = 100$ kHz at $f = 30$ MHz. However values of $\delta f/f$ larger than this typical value by a factor of two to three are observed.

(4) In triple bursts the frequency splitting between the upper two elements is roughly the same as that between the lower two elements.

These and other detailed properties of the bursts have been reviewed by McCulloch and Ellis (1977), Fomichev and Chertok (1977), Elgarøy (1977), and Melrose (1983). In this section we discuss the possibility that the observed splitting is due to the underlying emission process involving two sequential three-wave processes $L \pm F \rightarrow L'$, $L' \pm F' \rightarrow t$.

5.1. Ion sound waves

First suppose the low frequency waves are ion sound waves. Then one may show that the processes *uu*, *ud*, *du*, and *dd* lead to transverse waves with the four frequencies (Melrose, 1983)

$$\omega_{1,2} = \omega_p + (k_L^2/k_0 + k_s \pm \sqrt{k_L^2 + k_0 k_s})v_s , \tag{31a}$$

$$\omega_{3,4} = \omega_p + (k_L^2/k_0 - k_s \pm \sqrt{k_L^2 - k_0 k_s})v_s , \tag{31b}$$

respectively. The frequency splittings are a maximum for $k_s = 2k_L \pm k_0$, giving

$$\omega_1 - \omega_2 \leq 2(k_L + k_0)v_s , \qquad \omega_2 - \omega_3 \leq 2k_L v_s ,$$

$$\omega_3 - \omega_4 \leq 2(k_L - k_0)v_s . \tag{32}$$

With $k_0 = \omega_p/65V_e$ and $v_s = V_e/43$, for the largest possible frequency splitting

$(\omega_1 - \omega_2)/\omega_p$ to correspond to 1/300 would require $k_L = \omega_p/v_\phi$ with $v_\phi \cong 20 V_e$. Such a value is not unreasonable for Langmuir waves generated by a type III electron stream.

However, there are several difficulties with this interpretation. One is that it requires implausible values to account for frequency splitting twice or three times the average value, and such splittings are observed. A splitting twice the average would require $v_\phi \lesssim 10 V_e$ which is much slower than inferred for type IIIb streams. Another difficulty is that the maximum splitting invoked above requires parallel ion sound waves with $k_s = 2k_L \pm k_0$, and one would then expect parellel ion sound waves with $k_s = k_L$ also to be present. These latter waves should lead to emission due to the processes $L \pm s \to t$, with resulting frequencies

$$\omega_{5,6} = \omega_p + (k_L^2/k_0 \pm k_L) v_s . \tag{33}$$

There seems no good reason why these frequencies should not also be observed. The processes leading to (33) were invoked for the splitting by Melrose and Sy (1971) and Yip (1973), with a third line at $\omega_7 = \omega_p + (k_L^2/k_0) v_s$ due to induced scattering by ions; however the resulting splitting is then much smaller than that observed for plausible values of k_L. Another apparent difficulty with (31a, b) is the presence of four rather than three frequencies. However one can plausibly argue away ω_4 as being too close to the plasma frequency to escape (it is actually below the plasma frequency for $k_s = 2k_L - k_0$ and $k_L < 2k_0$).

These difficulties are compounded by the fact that one has no good reason to expect waves with $k_s \lesssim 2k_L$ to dominate the ion sound spectrum. Consequently although this interpretation cannot be ruled out, it does not seem favourable.

5.2. Lower hybrid waves and resonant whistler waves

As pointed out by Ellis and McCulloch (1967) the observed frequency splitting can be plausibly explained in terms of a splitting of the order of the lower hybrid frequency. Specifically setting $\omega_{LH}/\omega_p = 1/300$ implies $\Omega_e/\omega_p = 1/7$ and setting $2\omega_{LH}/\omega_p = 1/300$ implies $\Omega_e/\omega_p = 1/14$. At heights where stria bursts are observed a value $\Omega_e/\omega_p \cong 1/10$ is reasonable, e.g. Dulk and McLean (1978).

The discussion in Section 4 above implies that the processes $L \pm F \to L'$, $L' \pm F' \to L$ are not favourable for lower hybrid waves (in the case considered where the initial Langmuir wave is parallel to **B**). Specifically the process $L - F \to L'$ is forbidden, and then $L' \pm F' \to L$ cannot produce a triple splitting. The process $L + F \to L'$ is also forbidden for lower hybrid waves, but it is possible for resonant whistlers at a frequency only slightly higher than ω_{LH}. According to (29) one could account for a single splitting with $\delta f/f \gtrsim (\Omega_e/\omega_p)^2$ in this case, and $\Omega_e/\omega_p \cong 1/17$ would then be required to give $\delta f/f \cong 1/300$. Besides the absence of a third element this process is subject to the restriction (30) which is not easily satisfied.

Thus while the processes involving waves near the lower hybrid frequency seem to offer an attractive interpretation of the observed splitting, the kinematic restrictions on the three wave processes involved turn out to be so severe that the proposed mechanism seems untenable.

6. The Four-Wave Processes $L \pm F \pm F' \to t$

The processes $L \pm F \pm F' \to t$ can occur in a single step due to a four-wave interaction. The kinematic restrictions include

$$\omega_L \pm \omega_F \pm \omega_{F'} = \omega_t\,, \tag{33a}$$

$$\mathbf{k}_L \pm \mathbf{k}_F \pm \mathbf{k}_{F'} = \mathbf{k}_t\,, \tag{33b}$$

and the dispersion relations, with the dispersion relation for transverse waves requiring k_t to be negligibly small here. The only relevant restriction following from (33a) is for the *dd* process, for which one must have

$$\omega_F + \omega_{F'} < \omega_L - \omega_p\,. \tag{34}$$

When the condition (34) is not satisfied the *dd* process is forbidden and for $\omega_{F'} \cong \omega_F$ this provides a possible explanation for the absence of the lowest frequency element in a split pair burst compared with a triple burst. The condition (33b) now imposes no restriction on the magnitude of k_F. We may have $k_F \gg k_L$ and satisfy (33b) with $k_{F'} \cong k_F$. Thus in principle the four-wave process is allowed for the large k low frequency waves observed in the interplanetary medium. However for lower hybrid waves and parallel Langmuir waves the condition (30) still proves to be a severe restriction. With $k_{F'} \cong k_F$ (30) requires $|\cos\theta_F| \cong 2k_L/k_F$ and with $|\cos\theta_F| \lesssim 1/43$ for lower hybrid waves this reduces to $k_F \gtrsim 86k_L$. However for resonant whistlers at a frequency two to three times the lower hybrid frequency the condition $|\cos\theta_F| \cong 2k_L/k_F$ can be satisfied for $k_F\lambda_{\mathrm{De}} < 1$. That is one may satisfy (30) for waves with frequency satisfying (29) and then the four wave process allows a triple splitting with

$$\frac{\delta f}{f} \cong \left(\frac{\Omega_e}{\omega_p}\right)^2. \tag{35}$$

Only a double splitting results when (34) is not satisfied, and this occurs for $k_L = \omega_p/v_\phi$ with

$$\frac{v_\phi}{V_e} \lesssim 1.2\,\frac{\omega_p}{\Omega_e}\,. \tag{36}$$

6.1. Saturation of the four-wave process

A four-wave process can be due to a cubic nonlinear response of the plasma or to two quadratic nonlinear responses. In the latter case two waves beat to form a virtual wave which then beats with the third wave and produces the fourth wave. A rough estimate of the possible contributions shows that none exceeds the sequential quadratic nonlinear responses in which the virtual wave is obtained by the beating of the initial Langmuir wave and one of the low frequency waves. The matrix element for this process involves the product of the matrix elements for the processes $L \pm F \to (L')$, $(L') \pm F' \to t$, where now (L') denotes a virtual Langmuir wave. These matrix elements are well known.

Using them the following rough estimate of the optical depth for the four-wave process has been obtained:

$$\tau \cong \left(\frac{m_e}{m_i}\right)^2 \frac{r_0^2 L \omega_p^8}{c^3 \omega_F^5} \left(\frac{c}{V_e}\right)^{10} \frac{1}{(k_F \lambda_{\mathrm{DE}})^2} \left(\frac{T_F}{m_e c^2}\right)^2 . \tag{37}$$

In (37) r_0 is the classical radius of the electron, L is the distance through the source and T_F is the effective temperature of the low frequency waves, given roughly by

$$\frac{k_F^3}{2\pi^2} T_F = W_F , \tag{38}$$

where W_F is the energy density in the waves in a range $\Delta k_F \cong k_F$ about k_F. The most important factor in (37) is $(T_F/m_e c^2)^2$; it is only if this factor is sufficiently large that one can have $\tau > 1$. The value of T_F is quite uncertain. The results of Gurnett *et al.* (1979) for the peak level of ion sound waves in the interplanetary medium lead to $T_F/m_e c^2$ of order 10^8. A level several orders of magnitude less than $T_F/m_e c^2 = 10^8$ seems reasonable. Let us adopt $T_F/m_e c^2 = 10^8$ and set $\omega_p = 10^8 \text{ s}^{-1}$, $c/V_e = 10^2$, $k_F \lambda_{\mathrm{De}} = \frac{1}{2}$, and use $\delta f/f = 2\omega_F/\omega_p$ to write $\omega_F/\omega_p \cong 1/600$. Then (37) implies $\tau \cong 10^{11} L$ with L in centimetres. The maximum possible value of L is that in which the plasma frequency changes by an amount equal to the bandwidth of the radiation, say $L \cong 10^8$ cm implying $\tau \cong 10^{19}$. Although there is considerable uncertainty in these parameters, it is clear that one expects $\tau \gg 1$ for any plausible values.

Although the foregoing rough estimates are based on favourable choices for some of the parameters, it does seem reasonable to conclude that the condition for the four-wave process to saturate is relatively mild. Indeed one would expect it to saturate whenever Langmuir waves and low frequency waves coexist in any volume large enough to be of relevance to solar radio emission.

7. Conclusions

There are two important qualitative conclusions which follow from the results presented above. The first is that the kinematic restrictions on three-wave processes involving a Langmuir wave and a low-frequency wave are more stringent that has been realized previously. Although it is well known that the processes $L \pm s \to t$ require $\mathbf{k}_s = \mp \mathbf{k}_L$, it has not been recognized that the processes $L \pm s \to L'$ require $k_s \lesssim 2k_L \pm k_0$, with $k_0 = \omega_p/65 V_e$. This implies that three-wave processes cannot occur for the ion sound turbulence with $k_s \gg k_L$ observed in the interplanetary medium. A similar restriction forbids the processes $L \pm F \to L'$ for lower hybrid and resonant whistler waves with large k_F. The other important qualitative conclusion is that the four-wave processes $L \pm F \pm F' \to t$ appear to be fast enough to saturate under far from extreme conditions in the solar corona. However the discussion of the four-wave processes here has been at best semiquantitative; a detailed analysis is required to confirm the tentative conclusions reached.

The application which partly motivated the current investigation is the interpretation of the frequency splitting in split pair and triple bursts, i.e. in the stria bursts whose envelopes defined type IIIb bursts. Existing theories involve either the processes $L \pm s \rightarrow t$ (Melrose and Sy, 1971; Yip, 1973) or the development of sidebands on the Langmuir spectrum (Smith and de la Noë, 1976); the alternative theory outlined in the Introduction above overcomes difficulties with these theories (Melrose, 1983). This alternative theory encounters kinematic difficulties when the net processes $L \pm F \pm F' \rightarrow t$ are attributed to sequential three wave processes $L \pm F \rightarrow L'$, $L' \pm F' \rightarrow t$. Based on kinematic arguments alone the most favourable such theory involves the four wave processes $L \pm F \pm F' \rightarrow t$ with $k_F \cong k_{F'} \gg k_F$, $\omega_F \cong \omega_{F'} \cong \frac{1}{2}\omega_p(\Omega_e/\omega_p)^2$, leading to a frequency splitting $\delta f/f \cong (\Omega_e/\omega_p)^2$. This mechanism would involve resonant whistler waves at a frequency somewhat above the lower hybrid frequency. Such waves are observed in the terrestrial magnetosphere as VLF hiss. The suggestion that hiss is generated through Čerenkov emission (Ellis, 1957) is supported by both observational data (e.g. Gurnett and Frank, 1972; Laaspere and Hoffman, 1976) and by theory (Swift and Kan, 1975; Maggs, 1976; Yamamoto, 1979). It is at least plausible that these waves might be generated by streams of electrons in the solar corona.

The ideas put forward here on the interpretation of the splitting of stria bursts has not been combined with a model for propagation of type IIIb streams. However, the ideas themselves seem attractive and it is clearly desirable that a detailed model incorporating them should be developed.

Appendix: Effect of a Spread in k_L on the Condition $k_{SH}^2 > k_0^2$ for Second Harmonic Generation

The requirement $k_{SH}^2 > k_0^2$, which is equivalent to $T_e > 5 \times 10^5$ K (Melrose, 1983), for second harmonic generation follows from (20) in the case where the initial Langmuir waves all have the same $\mathbf{k}_L$. Suppose the initial waves have a spread $\Delta\mathbf{k}_L$ in $\mathbf{k}_L$. Neglecting the magnetic field, coalescence of a scattered Langmuir wave $\mathbf{k}_{L'} = \mathbf{k}L \pm \mathbf{k}_s$ with another wave $\mathbf{k}_L + \Delta\mathbf{k}_L$ in the initial distribution leads to, in place of (20),

$$k_{SH}^2 - k_0^2 = 4k_L^2 - (k_0 \mp k_s)^2 + \Delta k_L^2 + 2\mathbf{k}_L \cdot \Delta\mathbf{k}_L . \qquad \text{(A1)}$$

The condition $k_{SH}^2 > k_0^2$ is relaxed for $\Delta k_L^2 + 2\mathbf{k}_L \cdot \Delta\mathbf{k}_L < 0$, i.e. when $|k_L + \Delta k_L|$ is smaller than k_L. For $\Delta k_L \ll k_L$, (A1) can be satisfied for any value of k_{SH}^2/k_0^2 provided one has

$$\frac{\Delta k_L}{k_L} \gtrsim \frac{1}{2}\left(\frac{k_0}{k_L}\right)^2 \cong \frac{1}{2}\left(\frac{v_\phi}{65\,V_e}\right)^2 , \qquad \text{(A2)}$$

with $v_\phi = \omega_p/k_L$. Even for $\Delta k_L \cong \frac{1}{2}k_L$ one requires $T_e \gtrsim 10^8\,(v_\phi/c)^2$ K, which is similar to the requirement $k_{SH}^2 > k_0^2$ for $v_\phi/c \cong 0.1$.

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