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ELECTRON-CYCLOTRON MASER EMISSION: RELATIVE GROWTH AND DAMPING RATES FOR DIFFERENT MODES AND HARMONICS

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Abstract. We calculate and compare the temporal growth rate and the number of e-folding growths for the following wave modes due to a loss-cone-driven cyclotron maser: fundamental x, o, and z modes and second harmonic x and o modes. The dominant mode of the maser should be the fastest growing mode for a saturated maser and should be the mode with the greatest number of e-folding growths for an unsaturated maser; this mode is the fundamental x mode for $\omega_p/\Omega_e \lesssim 0.3$, the z mode (or perhaps the fundamental o mode) for $0.3 \lesssim \omega_p/\Omega_e < 1.0$, and the z mode (or perhaps the second harmonic x mode) for $1.0 < \omega_p/\Omega_e < 1.3$. We discuss the effect of cyclotron damping by thermal electrons on the growth. Numerical calculations show that the effect is important only when the ratio of the mean energies of the thermal and maser emitting electrons exceeds 0.1-0.2. An analytic expression for the damping rate is derived and is used to show that some earlier treatments of cyclotron damping greatly overestimate the effect for loss-cone-driven maser emission. These results, when applied to AKR, imply that only either the fundamental x mode (for $\omega_p/\Omega_e < 0.3$) or the z mode (for $\omega_p/\Omega_e \gtrsim 0.3$) is produced directly by maser emission. We suggest (1) that an o mode component in AKR might be due to partial reflection of x mode radiation incident onto sharp overdense plasma intrusions of the kind observed in the auroral cavity and (2) that a second harmonic component can be produced by coalescence of two z mode waves

1. Introduction

There is now considerable support for the suggestion by Wu and Lee [1979] that the auroral kilometric radiation (AKR) is due to electron-cyclotron maser emission driven by electrons with a loss-cone anisotropy (Lee and Wu, 1980; Omidi and Gurnett, 1982; Melrose et al., 1982; Dusenbery and Lyons, 1982, Wu et al., 1982]. This theory also seems favorable for the interpretation of Jupiter's decametric radio emission (DAM) [Hewitt et al., 1981] and similar emission from Saturn, for solar microwave spike bursts [Holman et al., 1980; Melrose and Dulk, 1982; Sharma et al., 19821 and for microwave emission from some flare stars [Melrose and Dulk, 1982; Dulk et al., 19831.

In its simplest form the theory involves a maser which is driven by electrons with a loss-cone anisotropy in a magnetic flux tube. Downgoing (pitch angles $0 < a < \pi/2$) electrons with small a are lost by precipitation into the atmosphere while others reflect and produce an upgoing distribution with a loss-cone anisotropy. At least for small values of the ratio of the plasma frequency ω_p to the cyclotron frequency Ω_e, the fastest growing mode is the fundamental ($s = 1$) x mode in a narrow frequency range just above Ω_e and in a narrow range of wave normal angles θ nearly perpendicular to the magnetic field lines but directed slightly upward ($\theta > \pi/2$). Our purpose in this paper is to discuss some details of the maser emission process, specifically the possible importance of (1) growth in the o mode and the z mode, (2) growth at higher harmonics, and (3) damping by thermal electrons.

AKR is usually x mode radiation but there is sometimes a weak o mode component and there is evidence for higher harmonics [Calvert, 1981a, b; Benson, 1982, 1984; Benson and Akasofu, 19841. There is also an auroral z mode component observed below Ω_e [Gurnett et al., 1983], and this may be generated in a way analogous to AKR [Omidi et al., 1984; Hewitt et al., 19831. Two types of o mode component are claimed: one is a weak widespread component $\approx$ 20-30 dB weaker than the x mode component [Shawhan and Gurnett, 1982], and the other "o mode" component is observed at relatively high values of ω_p/Ω_e [Benson, 1984; Benson and Akasofu, 1984]. We argue here that the latter component, which is restricted to between Ω_e and the upper hybrid frequency $(\omega_p^2 + \Omega_e^2)^{1/2}$, is z mode radiation generated by the process discussed by Hewitt et al. [1983].

In section 2 we discuss the criterion for the dominant mode of a loss-cone-driven maser, and suggest that this is the x mode for $\omega_p/\Omega_e < 0.3$ and the z mode for

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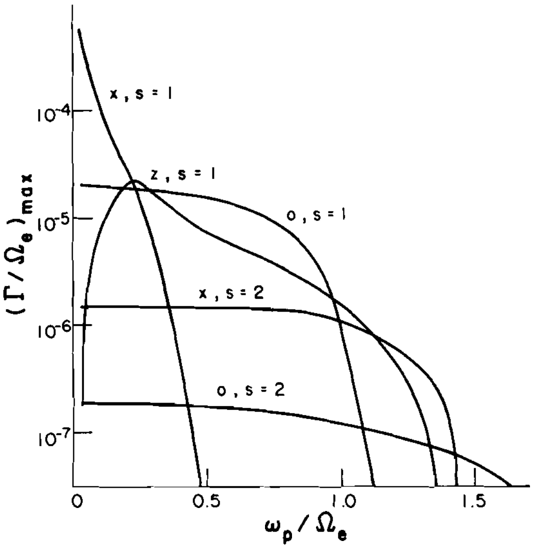

Fig. 1. The normalized maximum (over all θ and ω) temporal growth rate $(\Gamma/\Omega_e)_{max}$ plotted as a function of ω_p/Ω_e for the fundamental $(s = 1)$ x, o, and z modes and for the second harmonic $(s = 2)$ x and o modes. Cyclotron damping by the cold background electrons has been neglected.

$0.3 \lesssim \tilde{\omega}_p/\Omega_e \lesssim 1.3$. We also suggest that growth at higher harmonics may never be important. In section 3 we present a detailed discussion of the effects of cyclotron damping by thermal electrons on the maser emission. In section 4 we discuss implications of the results derived in section 2, suggesting how o mode and second-harmonic components in AKR might be produced other than by direct maser emission.

2. Relative Growth Rates

In this section we address the following question: As ω_p/Ω_e is increased from $\lesssim 0.3$ to > 0.3 and the growth of the fundamental x mode becomes suppressed, which mode becomes the dominant mode of the maser? A subsidiary question concerns the criterion for the dominant mode. We propose two criteria, namely the fastest growing mode and the mode with the largest value of the product of the spatial growth rate and the bandwidth of the growing waves, i.e., the largest number of e-folding growths. We discuss which of these is likely to be the more relevant in particular circumstances.

Detailed Calculations

All our detailed numerical calculations are for the idealized distribution function adopted by Hewitt et al. [1982] and Hewitt and Melrose [1983]. The hot (H) electron distribution has a one-sided loss cone with loss-cone pitch angle α_0 (>90°); it has distribution function

$$f_H(v,\alpha) = \frac{n_H g_N(\alpha)}{(2\pi\, V_H^2)^{3/2}} \exp[-\frac{v^2}{2V_H^2}] \qquad (1)$$

$$g_N(\alpha) = \begin{cases} a_N & \alpha \leq \alpha_0 \\ a_N[\sin\{(\pi/2)(\pi-\alpha)/(\pi-\alpha_0)\}]^N & \alpha > \alpha_0 \end{cases} \qquad (2)$$

with a_N close to unity. We choose N = 6, α_0 = 150°, and $T_H (= m_e V_H^2/\kappa) = 10^8$ K. The "cold" (C) distribution function is taken to be Maxwellian. In the calculations the number density of the hot electrons n_H and Ω_e are held fixed and the number density of the cold electrons n_C (and hence ω_p/Ω_e) is varied. The growth rate Γ for the hot electrons scales as Ω_e so that Γ/Ω_e is a function of the ratios n_H/n_C and ω_p/Ω_e and not of ω_p and Ω_e separately. The parameters are chosen such that $n_H/n_C = 10^{-2}$ when $\omega_p/\Omega_e = 0.1$.

We have determined the maximum dimensionless temporal growth rates $(\Gamma/\Omega_e)_{max}$, for each value of ω_p/Ω_e, by maximizing in θ and ω for each mode and harmonic. The results for the fundamental x, o, and z modes and for the second harmonic x and o modes are plotted in Figure 1. (With the above hot distribution function the z mode can be generated at s = 2 only for $\omega_p/\Omega_e > 3^{1/2}$ and we have not considered this case.) A plot similar to Figure 1 was presented by Dulk and Melrose [1983] who did not consider the z mode and speculated on the relative growth rate for Langmuir waves.

Sharma et al. [1982] also presented similar plots without the z mode. Their detailed results differ from ours, notably in the relative growth rates for the x mode at s = 2 and s = 1 and in the relative growth rates of the o mode and the x mode. (Note that they include an additional factor n_C/n_H in their plots, and this causes their curves to approach zero as $\omega_p/\Omega_e \propto (n_C/n_H)^{1/2}$ approaches zero.) In part the differences between our results and theirs may be attributed to their manner of including thermal damping, which leads to an overestimate of the effect as shown in section 3 below. Wu and Qiu [1983] also compared the growth rates for the x mode at s = 1 and s = 2 and found that the latter becomes the larger for $\omega_p/\Omega_e \gtrsim 0.4$.

In Figure 2 we plot, in place of $(\Gamma/\Omega_e)_{max}$, the maximum of the product in dimensionless form of the spatial growth rate and the bandwidth of the growing waves. The spatial growth rate is Γ/v_g, where both Γ and the group speed v_g are calculated numerically for each mode and for each value of ω and θ. The bandwidth $\Delta\omega$ of the growing waves is defined as the separation (at fixed θ) between the frequencies at which the spatial growth rate is half its maximum value.

Comparison of Figures 1 and 2 shows that as the x mode is suppressed (for $\omega_p/\Omega_e > 0.3$) either the fundamental o mode or the z mode becomes the dominant mode. While in Figure 1 the o mode is marginally the faster growing mode, the z mode seems much the more favorable in Figure 2, i.e., in terms of the alternative criterion discussed in detail below. The second-harmonic x mode is the fastest growing mode only under very special conditions, and even then it is less favorable than the z mode in terms of the criterion based on Figure 2. A possible exception is when the growth is due to higher energy electrons, as suggested by Melrose and Dulk [1982]: the growth rate at the sth harmonic involves a factor $(v/c)^{2s}$ in the integrand and hence the relative importance of higher harmonics increases with increasing energy. Our choice (1) of distribution function is unfavorable for discussion of this effect; for other distributions the number of relevant (fast) electrons may be much larger than for a Maxwellian.

Effective Growth

In order to determine whether the o mode or the z mode is the dominant mode (when the fundamental x mode is suppressed) we need to decide which is the relevant criterion for the dominant mode. We discuss two cases.

Case 1: Saturation of the fastest growing mode. If the maser saturates then the fastest growing mode should be the dominant mode. It extracts free energy from the unstable distribution the fastest, and when it saturates there is no free energy left for the other modes to continue their growth. (An obvious proviso is that the modes under comparison be driven by the same source of free energy, i.e., by the same particles in the distribution function.)

From Figure 1 we see that the fundamental o and z modes have nearly equal growth rates for $\omega_p/\Omega_e \lesssim 1.0$, and the z and s = 2, x modes have nearly equal growth rates for $1.0 \lesssim \omega_p/\Omega_e \lesssim 1.3$. A detailed consideration of the parameters in particular circumstances is needed in order to decide which mode would dominate.

Case 2: Nonsaturation of the fastest growing mode. If saturation does not occur the relevant criterion for the dominant mode is that it experience the largest number of e-folding growths. This number depends on the group speed v_g and the bandwidth Δω of the growing waves as well as on Γ. Over a path length L the number of e-folding growths is (Γ/v_g) L, and L is restricted to $\lesssim (\Delta\omega/\Omega_e)(L_B/|\cos\chi|)$, where L_B is the characteristic distance over which Ω_e changes and χ is the angle between v_g and grad B. For given L_B and χ the number of e-folding growths is proportional to the quantity (Γ/v_g) Δω plotted in dimensionless form in Figure 2.

In most applications the growth of the maser is likely to be limited by the spatial structure of the source regions. Assuming the maser does not saturate, the z mode should dominate over the o mode when the fundamental x mode is suppressed. (We remark that it is possible in principle for the fastest mode not to saturate and for a slower growing mode to saturate due to a larger value of $(\Gamma/v_g)\Delta\omega/|\cos\chi|$, but this could occur only under very special circumstances.) In the discussion in section 4 we assume that the z mode dominates for $\omega_p/\Omega_e \gtrsim 0.3$ and discuss the implications of this.

3. Damping by Thermal Electrons

Cyclotron damping by thermal electrons can suppress growth of the maser. Shkarofsky [1966] gave a general treatment of damping by thermal electrons in both the nonrelativistic and relativistic limits. His results however are not cast in a form where their implications for cyclotron masers are readily apparent. In this section we present both numerical and analytic results for the relevant cyclotron damping and then we comment on the treatment of this by Sharma et al. [1982].

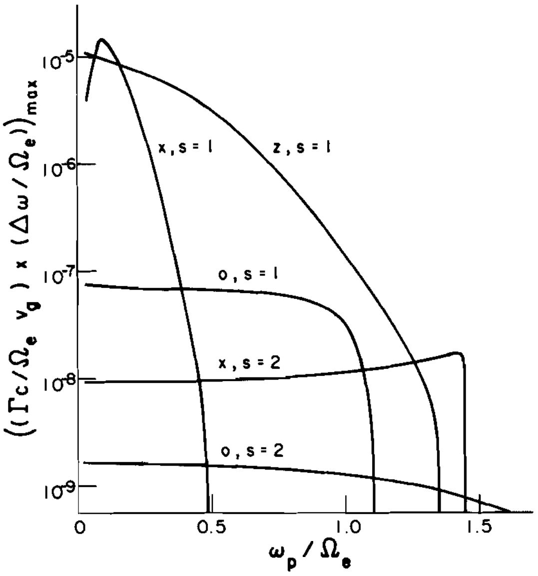

Fig. 2. As in Figure 1 but for the normalized maximum quantity $((\Gamma c/\Omega_e v_g) \times (\Delta\omega/\Omega_e))_{max}$.

Method of Treatment

We include damping by thermal electrons by adding their distribution function

$$f_C(v) = \frac{n_C}{(2\pi V_C^2)^{3/2}} \exp\left[-\frac{v^2}{2V_C^2}\right] \qquad (3)$$

to the distribution (1) and calculating the growth rate for the sum of the two distribution functions. That is, we treat the damping by integrating around the resonance ellipse with contributions from f_H and f_C at every point. As explained by Hewitt et al. [1982], the two contributions add and growth occurs only if the growth rate due to f_H exceeds the damping rate due to f_C. In Figure 3 we show the magnitude of these two quantities for the s = 1, x mode, $n_H/n_C = 10^{-2}$, $\omega_p/\Omega_e = 0.1$ and various values of the ratio T_C/T_H where $T_C = m_e V_C^2/\kappa$. The solid curve is the maximum growth rate plotted as a function of θ, and the dashed curves are the damping rates calculated using the resonance ellipses for the frequencies which give maximum growth for the appropriate values of θ. (This figure is an extension of Table 1 of Hewitt et al. [1982].) For the parameters chosen the damping due to thermal electrons is important only for $T_C/T_H > 0.1$, and growth (at a reduced rate) can exceed damping even for $T_C/T_H > 0.3$.

With the distribution (1) alone, growth is most favorable for resonance ellipses, with

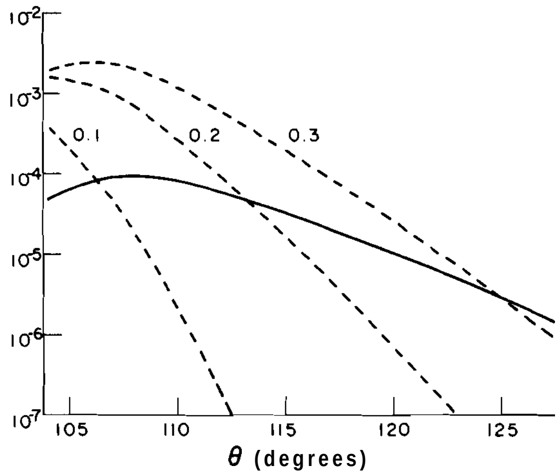

Fig. 3. The normalized maximum (over ω) temporal growth rate (solid) of the s = 1, x mode for the hot distribution and the corresponding normalized damping rates (dashed) for the cold distributions with the indicated values of T_C/T_H, plotted as functions of 8. The ratios n_H/n_C and ω_p/Ω_e are 10^{-2} and 0.1, respectively. Net growth occurs at a given angle only if the solid curve lies above the appropriate dashed curve.

centers at $(v_\parallel, v_\perp) = (v_0, 0)$ and semimajor axes $V \approx |v_0| \sin\alpha_0$, which lie just within the loss cone. The thermal electrons of distribution (3) are concentrated in the region $v \lesssim V_C$ near the origin of $v_\perp - v_\parallel$ space, and they contribute significant damping only if the resonance ellipses pass through this region (i.e., if the ellipses satisfy $|v_0| - V \lesssim V_C$ in the semirelativistic approximation). In treating the maser emission it is important that we use the full relativistic resonance condition

$$\omega - \frac{s\Omega_e}{\gamma} - k_\parallel v_\parallel = 0 \qquad (4)$$

or the semirelativistic approximation to it (i.e., $\gamma^{-1} = 1 - v^2/2c^2$) and not the nonrelativistic approximation ($\gamma = 1$); in all cases the resonance ellipse is nearly circular and hence we must not use the nonrelativistic approximation which corresponds to a straight line in $v_\perp - v_\parallel$ space. It is also important that when calculating the damping we integrate around the same resonance ellipse rather than use the nonrelativistic approximation.

Analytic Approximation for the Damping Rate

In order to discuss the foregoing point quantitatively we evaluate the damping rate for s = 1 analytically in the appendix under the assumption that the semirelativistic approximation applies. This approximation is valid for $k_\parallel^2 c^2 \ll \omega^2$, which corresponds to $\cos^2 8 \ll 1$ in the present case. The nonrelativistic approximation is valid in the opposite limit $k_\parallel^2 c^2 \gg \omega^2$. The analytic expression derived in the appendix reproduces the nonrelativistic expression in one instance. Mathematically there are three relevant cases: $|v_0| \ll V$ (a semicircle centered near the origin), $|v_0| \gg V$ (a semicircle nowhere approaching the origin), and $|v_0| \approx V$ (a semicircle with radius nearly equal to the distance of its center from the origin). As we now demonstrate in the third case nonrelativistic and semirelativistic approximations lead to the same expression for the damping rate since the section of the resonance ellipse in the region $v \lesssim V_C$ near the origin is reasonably approximated by a straight line when $|v_0| \approx V \gg V_C$.

The simplest useful approximate expression which covers the second and third cases is

$$\frac{\Gamma}{\Omega_e} \approx \frac{1}{2}\left(\frac{\pi}{2}\right)^{1/2} \frac{\omega_p^2}{\Omega_e^2} \frac{Vc^2}{v_0^2 V_C} \frac{\left(1 + \frac{V}{c}\frac{\cos\theta}{|\cos\theta|} T\right)^2}{1 + T^2} \left[\exp - \frac{(|v_0| - V)^2}{2V_C^2}\right] \qquad (5)$$

with

$$\frac{v_0}{c} \approx \cos\theta \qquad (6a)$$

$$\frac{V}{c} \approx \left[\frac{v_0^2}{c^2} \frac{2(\omega - \Omega_e)}{\Omega_e}\right]^{1/2} \qquad (6b)$$

where T is the axial ratio of the polarization ellipse and where the refractive index is approximated by unity. (More general expressions are given in the appendix.) The simplest relevant approximation to T is $T_x = \cos\theta$ and $T_o = -\sec\theta$ for the x and o modes respectively; these apply for $\omega \approx \Omega_e \gg \omega_p$.

When $|v_0| \approx V$ the exponential in (5) becomes $\exp[-(\omega - \Omega_e)^2 c^2/2\Omega_e^2 V_C^2 \cos^2\theta]$ and (5) reproduces a well-known result obtained using the nonrelativistic approximation [e.g., Ginzburg, 1964, p. 131; Zheleznyakov, 1920, p. 449; Melrose, 1980a, p. 2761. In the case of relevance to a loss-cone instability, however, we have $|v_0| \gg V \approx |v_0| \sin\alpha_0$. In this case, by using $|v_0| - V = (v_0^2 - V^2)/(|v_0| + V) \approx (v_0^2 - V^2)/|v_0|$, the exponential in (5) becomes $\exp[-2(\omega - \Omega_e)^2 c^2/\Omega_e^2 V_C^2 \cos^2\theta]$. This is of the same form as the nonrelativistic expression but with an extra factor of 4 in the exponent. For example, for a loss-cone-driven maser, typical values of this exponential factor are e^{-2} to e^{-5} or less in the nonrelativistic case; these become e^{-8} to e^{-20} or less in the semirelativistic case for $|v_0| \gg V$.

We conclude that incorrect use of the nonrelativistic approximation can lead to a very large overestimate of the importance of cyclotron damping by thermal electrons in loss-cone-driven masers. This point is relevant to all modes. The results of Sharma et al. [1982] are dependent on the nonrelativistic approximation to the resonance condition, due to their use of a dielectric tensor derived by making the nonrelativistic approximation. In view of the present results and those of Hewitt et al. [1982], we conclude that there is no justification for Sharma et al.'s criticism of the neglect of cyclotron damping by Melrose and Dulk [1982].

A more general implication of the foregoing discussion is that the familiar formulas for the dielectric tensor of a thermal plasma [e.g., Stix, 1962, p. 189; Bekefi, 1966, p. 229] cannot be used to treat thermal damping in cases where the nonrelativistic approximation is invalid. Intrinsically relativistic effects become important near perpendicular propagation [e.g., Shkarofsky, 1966] and must be taken into account correctly.

4. Generation of Other Modes and Harmonics

The most plausible conclusion which may be drawn from the discussion in section 2 is that for $\omega_p/\Omega_e < 0.3$ the fundamental x mode is the dominant mode of the maser, and for $0.3 < \omega_p/\Omega_e < 1.3$ the z mode becomes the dominant mode. Here we assume this to be the case and address the question of how an o mode component and a second-harmonic x mode component might arise.

4.1. The o Mode Component in AKR

Polarization measurements by Gurnett and Green [1978] and Kaiser et al. [1978] showed that AKR is predominantly in the x mode; this has been challenged by Oya and Morioka [1983] and defended further by Shawhan and Gurnett [1982]. However, it does seem that there is a weak o mode component with an intensity about 20 dB below that of the dominant x mode component [Shawhan and Gurnett, 1982; W. Calvert, private communication, 1983]. It seems unlikely that this component is generated directly by the maser. The growth of the o mode is much slower than that of the x mode for $\omega_p/\Omega_e < 0.3$ and seems less effective than that of the z mode for $\omega_p/\Omega_e > 0.3$.

The o mode component could be generated as a propagation effect, as mentioned by Calvert [1982]. In the auroral cavity there are localized higher density regions in the otherwise underdense region where AKR is generated [Benson and Calvert, 1979; Benson et al., 1980; Calvert 1981a, b]. An x mode wave incident on an interface with a sharp density jump leads, in general, to reflected and transmitted components in both magnetoionic modes. A detailed investigation is currently in progress to determine whether this mechanism can account for the observed o mode component in AKR. One case which is relatively simple to treat is the limit $\omega \gg \omega_p, \Omega_e$ for a geometry such that B lies in the plane of the interface: then the transmitted component is almost totally in the x mode and the relatively weak reflected component is almost totally in the o mode.

The density gradients associated with the density spikes in the auroral cavity have been invoked to account for generation of the o mode by mode conversion from the z mode [Oya, 1974; Jones, 1977; Melrose, 1980b]. These same density gradients could produce a reflected o mode component from AKR incident in the x mode provided that the density changes occur over a few kilometers (i.e., less than about a wavelength), as seems to be the case (W. Calvert, private communication, 1983).

4.2. Second Harmonic Components in Solar Spike Bursts and in AKR

In the application to solar microwave spike bursts and similar bursts from other stars, a central problem is how the maser radiation escapes, in view of the very strong absorption at the second harmonic of the cyclotron frequency. Holman et al. [1980] suggested that the radiation escapes through a "window" at $\theta \approx 0$, but this presents severe difficulties for radiation generated at $\theta \approx \pi/2$. Melrose and Dulk [1982] proposed that the escaping radiation is generated at the second harmonic ($s = 2$), i.e., at $\omega > 2\Omega_e$ so that it can escape without being absorbed at the second-harmonic absorption layer. However, in view of the discussion in section 2 above, masering at the second harmonic may be supplanted by masering in the z mode and the second harmonic could be produced directly by the maser only under special conditions.

As an alternative we propose here that the second harmonic is generated by coalescence of two z mode waves which are produced by the maser. It is important that the resulting radiation be above $2\Omega_e$ so that it is not reabsorbed; this requires that the z mode waves be above the cyclotron frequency, as in the mechanism discussed by Hewitt et al. [1983], and not below Ω_e as in the alternative mechanism of Omidi et al. [1984].

Evidence for z mode radiation was reported by Benson [1984] who observed radiation below the x mode cutoff and in the range between Ω_e and the upper hybrid frequency; although Benson suggested that this might be o mode radiation, its polarization could not be determined from the data, and we believe an interpretation in terms of z mode radiation offers a better explanation. From theory, Hewitt et al. [1983] found that z mode waves are generated between Ω_e and the upper hybrid frequency on a cone at $\theta \approx 100°$ (i.e., wave normals directed slightly upward) with refractive indices in a moderately broad range about $n \approx 2$.

The coalescence of two z mode waves (ω', $\mathbf{k}'$, and ω'', $\mathbf{k}''$) can lead to a second-harmonic wave (ω, $\mathbf{k}$) provided the conditions

$$\omega = \omega' + \omega'' \tag{7a}$$

$$\mathbf{k} = \mathbf{k}' + \mathbf{k}'' \tag{7b}$$

are satisfied. For a second-harmonic x or o mode wave with $\omega \approx 2\Omega_e$, we have

$$k \approx 2\frac{\Omega_e}{c}\left[1 + \frac{1}{4}\frac{\omega_p^2}{\Omega_e^2}\right]^{1/2} \tag{8}$$

and for the z mode waves we write

$$k' \approx n'\frac{\Omega_e}{c} \tag{9a}$$

$$k'' \approx n''\frac{\Omega_e}{c} \tag{9b}$$

If the z mode waves are produced as discussed by Hewitt et al. [1983], then they are confined to a thin cone of angles near perpendicular propagation and to a broad range of refractive indices around n' or $n'' \approx 2$. Hence we expect the z mode waves in the source region to have these properties: $n' \approx n'' \approx 2$, $\omega_p \ll \Omega_e$, an angular distribution confined to the surface of a wide-angle cone, and k, k', and k'' all of comparable magnitudes; then (7a), (7b) are automatically satisfied. With $|\cos\theta'| \approx |\cos\theta''| \approx 0.2$ and $k' \approx k'' = k$ we have $|\cos\theta| \approx 0.4$, i.e., the second harmonic radiation is directed upward at a moderately large angle to B.

We conclude that there is no difficulty in satisfying the kinematic conditions (7a), (7b) for generation of a second-harmonic component from two z mode waves generated by the maser. If indeed most of the electrons' free energy goes into the z mode for $\omega_p/\Omega_e > 0.3$ as indicated above, this seems a more likely source of second harmonic radiation than direct generation by the maser. An implication is that second harmonic radiation should be associated with z mode fundamental radiation. This seems to be the case for the AKR data reported by Benson [1984]: in relatively dense regions ($\omega_p/\Omega_e \gtrsim 0.3$) there is a component just above Ω_e (which is plausibly in the z mode) plus a second harmonic, and sometimes higher harmonics. However, Benson [1984] also reported another case where an x mode component and harmonics were observed in a region of very small ω_p/Ω_e; such a situation cannot be explained by the interpretation offered here.

Another point of interest concerns the fate of energy going into the z mode: it cannot escape directly and must either escape indirectly (e.g., through production of a harmonic component) or be absorbed by the thermal plasma. If, as suggested by Hewitt et al. [1983], the radiation propagates downward to regions with $w < \Omega_e$ it may be partially trapped between the z mode cutoff and the layer $w \simeq \Omega_e$ where the group velocity is roughly horizontal [Omidi et al., 1984]. Absorption by the thermal plasma is weak except near the upper hybrid resonance and at least initially the radiation does not propagate toward this resonance. Hence the radiation intensity may build up to high levels before it is dissipated by some mechanism, as yet unknown.

5. Discussion and Conclusions

Two main points have been made in this paper in connection with loss-cone-driven cyclotron-maser emission: the dominance of the z mode for $0.3 < \omega_p/\Omega_e < 1.3$ and the very weak effect of cyclotron damping.

The dominance of the z mode is shown by the plots of the temporal growth rate (Figure 1) and of the parameter related to the maximum number of e-folding growths (Figure 2). While these results do not totally exclude the possibility of maser emission of the fundamental o mode or of the second harmonic x mode, it seems that they might be produced directly by the maser only under special circumstances. We should remark on the fact that our results have been derived for a specific form of distribution function. Our conclusions cannot be sensitive to this choice provided the resonance ellipses for the different modes are all similar. Put another way, our results should apply provided that the modes under comparison tap the same source of free energy in the loss cone feature of the distribution function. Then the growth rates are sensitive only to the form of the distribution function in a localized region of velocity space (where $\partial f/\partial p_\perp$ is a maximum). We emphasize that our results are specifically for loss-cone distributions; it may be that other sources of free energy are available to one or more modes, and, if so, we have no reason to expect that the relative growth rates for different modes would be similar to the loss cone case discussed here.

In view of the present results it seems that, for $\omega_p/\Omega_e > 0.3$, the z mode becomes the dominant mode of the maser, not the second harmonic x mode as was suggested by Wu and Qiu [1983]. We argue that the second harmonic component in AKR may not be produced directly by the maser, but by coalescence of two z mode waves. Suitable z mode waves are in the range $\Omega_e < w < (\omega_p^2 + \Omega_e^2)^{1/2}$ and propagate at large angles to the magnetic field; for $\omega_p/\Omega_e > 0.3$, such waves are expected to be produced directly by the maser [Hewitt et al., 1983]. With this mechanism both fundamental (z mode) and second harmonic radiation could be observed simultaneously in the AKR source region, and Benson's [1984] data are compatible with such simultaneous observation. Other arguments aside, a difficulty with masering at the second harmonic is that fundamental and second harmonic radiation would be generated in different regions (with $\omega_p/\Omega_e < 0.3$ and $\omega_p/\Omega_e > 0.3$, respectively); there then seems to be no reason why harmonic structure should be observed. Although the coalescence mechanism overcomes this difficulty, there are other data implying the existence of harmonic structure in regions with $\omega_p/\Omega_e \ll 0.3$ and the presence of harmonics higher than the second [Benson, 1982, 19841. These features cannot be explained in terms of the ideas discussed here.

Regarding the o mode components of AKR, we suggest that they might arise due to propagation in the presence of the dense plasma intrusions observed in the auroral cavity [Calvert, 1981b; W. Calvert, private communication, 1983]. These intrusions may well have sufficiently sharp density gradients to cause partial reflection of an incident x mode wave. The (weak) reflected component is predominantly o mode. A detailed investigation of the reflection and transmission properties is currently in progress.

The other main point made in this paper is an interesting one from a theoretical viewpoint: the standard treatment of cyclotron damping by thermal electrons is not valid for cyclotron masers. We show that one should use the semirelativistic and not the nonrelativistic approximation to the resonance condition. The appropriate approximate formula is similar to the familiar formula except in that an exponential factor $\exp[-(\omega - \Omega_e)^2c^2/2\Omega_e^2V_e^2\cos^2\theta]$ is replaced by $\exp[-2(\omega - \Omega_e)^2c^2/\Omega_e^2V_e^2\cos^2\theta]$. Thus an incorrect use of the nonrelativistic approximation can lead to a serious overestimate of the strength of cyclotron damping and hence of its tendency to suppress maser emission.

Appendix: Absorption by Thermal Electrons

Melrose et al. [1982, equation (19)] wrote down a general expression for (minus) the gyromagnetic coefficient for each harmonic in terms of an integral around the resonance ellipse. On keeping only the fundamental ($s = 1$) term, assuming the distribution function to be given by (3), approximating the resonance ellipse by a circle, keeping only the lowest order terms in the expansion of the Bessel function, ignoring relativistic effects (except in the resonance condition) and setting the refractive index equal to unity, their expression gives

$$\frac{\Gamma_C}{\Omega_e} \simeq \left(\frac{\pi}{2}\right)^{1/2} \frac{\omega_p^2}{\Omega_e^2} \frac{V^3 c^2}{V_C^5} \frac{e^{-(V^2 + v_0^2)/2V_C^2}}{4(1 + T^2)} \int_{-1}^{+1} d\cos\phi$$

$$(1 - \cos^2\phi) \left[1 + K\sin\theta + \left(\cos\theta - \frac{v_0}{c} + \frac{V\cos\phi}{c}\right) T\right]^2 \exp\left[\frac{Vv_0}{V_C^2}\cos\phi\right] \quad \text{(A1)}$$

where we write $v_\perp = V \sin\phi$ and $v_\parallel = v_0 - V\cos\phi$ [Melrose et al., 1982, equations (15a), (15b)]. The integral in (A1) is elementary and on evaluating it we obtain

$$\frac{\Gamma_C}{\Omega_e} = \left(\frac{\pi}{2}\right)^{1/2} \frac{\omega_p^2}{\Omega_e^2} \frac{Vc^2}{V_C v_0^2} \frac{e^{-(V^2 + v_0^2)/2V_C^2}}{1 + T^2}$$

$$\Bigg[(\cosh a - \sinh a)(1 + B)^2 + 2\left(\sinh a - \frac{3\cosh a}{a} + \frac{3\sinh a}{a^2}\right)(1 + B)\frac{V}{c} T$$

$$+ \left(\cosh a - 5\sinh a + \frac{12\cosh a}{a^2} \;\; \frac{12\sinh a}{a^3}\right)\frac{V^2}{c^2} T^2\Bigg] \quad \text{(A2)}$$

with

$$a = \frac{v_0 V}{V_C^2} \quad \text{(A3a)}$$

$$B = K\sin\theta + \left(\cos\theta - \frac{v_0}{c}\right) T \quad \text{(A3b)}$$

Here we have $v_0/c \simeq \cos\theta$ and $|K \sin\theta| \ll 1$, and hence we may approximate 1 + B by unity.

The limiting case of most relevance here is $|a| \gg 1$ when (A2) reduces to (5). In the other limiting case $|a| \ll 1$, which includes perpendicular propagation ($a = 0$), (A2) reduces to

$$\frac{\Gamma_C}{\Omega_e} = \left(\frac{\pi}{2}\right)^{1/2} \frac{\omega_p^2}{\Omega_e^2} \frac{V^3c^2}{V_C^5} \frac{e^{-V^2/2V_C^2}}{3(1 + T^2)} \left(1 + \frac{1}{5}\frac{V^2}{c^2} T^2\right) \quad \text{(A4)}$$

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