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Two-Photon Cyclotron Emission in Accretion Columns

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1 Introduction

Cyclotron lines have been observed in the X-ray spectra of two pulsed sources: Her X-1 (Trümper *et al.* 1978) and 4U 0115 + 63 (Wheaton *et al.* 1979). The generally accepted model for these objects involves an accretion flow from a companion star in a close binary system onto small regions close to the magnetic poles of a strongly magnetized neutron star. Immediately above the surface, matter is confined in an accretion column by the magnetic field. On the basis of this model, Basko and Sunyaev (1975) had predicted a cyclotron line in emission, and it is relevant to recall their arguments. Away from the cyclotron frequency, the mean free path for free-free absorption (the inverse of bremsstrahlung emission) of a photon, $\varkappa_{ff}^{-1}$, is much larger than that between scatterings, $\varkappa_{sc}^{-1}$, so that the observed spectrum is reduced from the black-body level by a large factor $(\varkappa_{ff}/\varkappa_{sc})^{-1/2}$ (Felten and Rees 1972); within the line, however, the mean free path for absorption of cyclotron photons, $\varkappa_{cyc}^{-1}$, is much smaller than $\varkappa_{sc}^{-1}$, and hence the observed spectrum should rise to the black-body level near the cyclotron frequency Ω_e. This argument is no longer accepted, basically because cyclotron absorption is viewed as a resonance in the Thomson (or Compton) scattering cross-section (e.g. Ventura 1979). The essential point is that purely collisional excitations do not control the populations of the Landau levels, because the rate of radiative decay of an excited state is much more rapid than the collision rate. Another point relating to Basko and Sunyaev's argument is that it involves an implicit LTE assumption, but there is no guarantee that the black-body limit to which the cyclotron photons aspire is represented by the same temperature as the black-body limit from which the non-resonant photons are reduced.

There have been several attempts to model cyclotron line spectra using a non-LTE approach. Diffusion in frequency space (Bonazzola *et al.* 1979, Wassermann and Salpeter 1980), scattering from the x-mode into the o-mode (Nagel 1980) and vacuum polarization and thermal effects (Kirk and Mészáros 1980, Kaminker *et al.* 1983) have all been considered as important effects.

In this paper we propose alternative mechanisms for the production of the cyclotron and the non-cyclotron photons; neither mechanism involves bremsstrahlung. A central feature of our approach involves the separation into cyclotron and non-cyclotron photons, coupled with a separation of processes according to 'fast' and 'slow' timescales. On the fast timescale the only processes of relevance are those affecting cyclotron photons. In section 2 we extend the ideas of Melrose (1981) and of Langer and Rappaport (1982) on the production and transport of the cyclotron photons. In section 3 we introduce another effect, that of the indirect production of photons in other parts of the spectrum from cyclotron photons, which hitherto has been neglected, but which may provide a more effective source of non-cyclotron photons than does bremsstrahlung. Section 4 contains our conclusions.

2 Resonant Photons

The resonance at $\omega = \Omega_e$ in the Thomson cross-section may be attributed to cyclotron absorption and re-emission; the collision rate between electrons in a typical accretion column is very slow and collisional de-excitation is extremely improbable. Melrose (1981) formalized this concept by introducing a double time-scale approach. Cyclotron absorption followed immediately by emission defines a fast time-scale. On this time-scale a steady state is maintained between the occupation numbers of cyclotron photons, N_c, and of electrons in the ground and first excited states, n_0 and n_1, respectively:

$$\frac{n_1}{n_0} = \frac{N_c}{1 + N_c} . \tag{1}$$

Implicit in this equation is a classification of photons into cyclotron (i.e. belonging to N_c) and non-cyclotron. The boundary between the two distinguishes the core of the line from the wings and should be drawn where the probability of radiative decay is of the same order as that of collisional de-excitation. Melrose (1981) derived a transport equation for cyclotron photons closely similar to the transport equation in the case of pure scattering.

The creation of cyclotron photons is attributed here to collisional excitation of a ground state electron to the first excited state followed by radiative decay back to the ground state. This

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process is just 'resonant contribution' to bremsstrahlung in the same way in which cyclotron absorption is the 'resonant contribution' to Thomson scattering (e.g. Kirk and Mészáros 1980). The cross-section for collisional excitation has been calculated by Ventura (1973) and Langer (1981). This process adds a source term to Melrose's equation so that, at least in the case of small occupation numbers, cyclotron photons are transported in the same way as are non-cyclotron ones, e.g. as described by Felten and Rees (1972), the only difference being the replacement of $\varkappa_{ff}$ by the collisional rate, and of $\varkappa_{sc}$ by the cyclotron absorption rate. A process similar to this appears to have been considered by Langer and Rappaport (1982) in their calculation of the rate at which plasma cools by cyclotron emission.

There is, however, another mechanism which effectively creates or destroys cyclotron photons, namely the diffusion of photons in frequency, which arises because of the recoil of the electron upon scattering. In this process photons may cross over the borderline which divides non-cyclotron from cyclotron photons (Wassermann and Salpeter 1980), providing, in the case of an emission line, an additional sink for cyclotron photons.

3 Non-Cyclotron Photons

Assuming that the problem outlined in the previous section has been solved, so that the occupation number of cyclotron photons as a function of distance into the source is known, then slower processes which can generate non-cyclotron photons from the cyclotron ones can be considered. The most important of these is two-photon emission: a transition of an electron from its first excited state into the ground state can occur by emission of two photons of frequencies ω and ω' such that their total energy, $\hbar + \hbar_{\omega}'$, is equal to the cyclotron energy, $\hbar\Omega_e$ (neglecting electron recoil). This process provides source of photons at $\omega < \Omega_e$.

In another process, related to the previous one by a crossing symmetry, a photon of frequency ω impinges on an electron in the excited state $n = 1$. The subsequent scattering event leaves the electron in the ground state, $n = 0$, and the photon emerges with frequency $\omega' = \Omega_e + \omega$. This has the net effect of coalescing a cyclotron photon and a photon with $\omega < \Omega_e$ to produce a photon with frequency between Ω_e and $2\Omega_e$. Thus, these two processes, acting together, can provide a smooth continuum over a range of frequencies from $\omega << \Omega_e$ to $\omega \cong 2\Omega_e$, without the necessity for any contribution from bremsstrahlung. We emphasize that ultimately, the only source of photons is the collisional excitation of the first excited state of an electron.

In order to compare the efficiency of these processes we consider the fate of a photon which enters the plasma: what is the relative probability that it be absorbed by the free-free process and that it be absorbed by the inverse of the two-photon emission process? The first step is to calculate the rate for two-photon decays. Earlier attempts have been incomplete (Melrose and Parle 1981, 1983), so we have reanalyzed the process using the method of Melrose and Parle (1983). For simplicity, we assume that all photon distributions are isotropic. Moreover, we assume that the mean occupation number of the excited states is given by equation (1), which allows us to eliminate any direct reference to the electron distribution. The two-photon decay is then equivalent to a photon splitting $\Omega_e \rightarrow \omega + \omega'$ and the kinetic equation can be written in the known form (e.g. Melrose 1983) viz.

$$\frac{dN(\omega)}{dt} = A(\omega)\, y^{-2}\{N_c[1+N(\omega)+N(\omega')]-N(\omega)\, N(\omega')\} \tag{2}$$

with $\omega' = \Omega_e - \omega$, $y := \omega/\Omega_e$, $y' := \omega'/\Omega_e$ and

$$A(\omega) := \frac{\alpha}{45}\left(\frac{\hbar\omega_p}{mc^2}\right)^2 \frac{\Omega_e}{1+2N_c} F(y,y') \tag{3}$$

where ω_p is the plasma frequency. The function $F(y,y') = F(y',y)$ is given by a cumbersome expression; we plot this function in Figure 1 and find it to be well approximated by

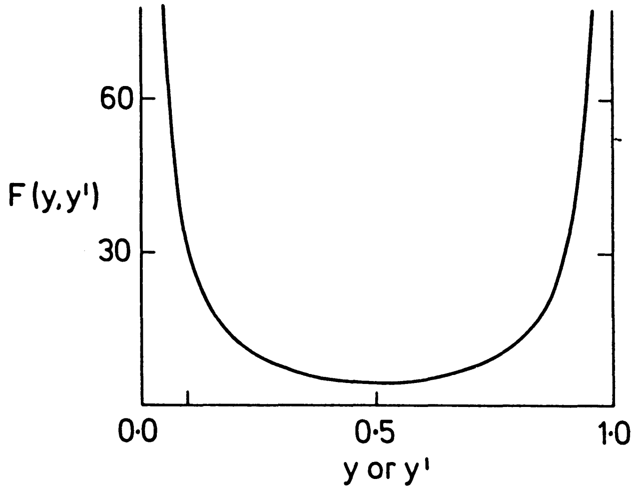

Figure 1. Spectral variation of the rate of two-photon emission, $F(y,y')$ of equation (3). The abscissa is the frequency, normalized to the cyclotron frequency, $y := \omega/\Omega_e$, $y' = 1 - y$.

$$F(y,y') \approx 4[1 + y^3/y' + y'^3/y]. \tag{4}$$

It is apparent from Figure 1 that two-photon emission exhibits an infra-red divergence: the rate of two-photon decays tends to infinity as the frequency of one of the photons tends to zero. Such infra-red divergences are well known in QED and the procedure for overcoming them is also well known, e.g. Jauch and Rohrlich (1976, p. 245). In our case, there is a limit which avoids the divergence: ω or ω' cannot be less than the one-photon decay rate:

$$\Gamma = \frac{4}{3}\alpha\Omega_e \frac{B}{B_c} \tag{5}$$

for the excited levels, where B_c is that critical value of the magnetic field at which the energy of a cyclotron photon equals the electron rest energy, $\hbar\Omega_e = mc^2$. There is also a practical limit in that ω cannot be less than ω_P, because if it were, the neglect of the dispersive properties of the waves in the derivation of equation (3) would be unjustified. The infra-red divergence is of no formal consequence, but the fact that the two-photon rate does become large for small ω raises the possibility that it may provide a significant source of low-frequency photons. We do not discuss this possibility further here.

The kinetic equation (2) differs from that for a one-photon process, such as bremsstrahlung, in that the absorption term contains a product of photon occupation numbers. In order to find an absorption coefficient with which to compare the importance of $\varkappa_{ff}$, it is, therefore, necessary to prescribe the occupation number $N(\omega)$ as a function of distance into the plasma. Since our aim here is merely to demonstrate the importance of the two-photon process, we proceed as follows: first $N(\omega)$ is specified under the assumption that free-free absorption is much more important than the two-photon process. Then the two-photon absorption probability is calculated and the assumption is shown to be invalid under a very wide range of parameters.

Following this argument, we assume the photon occupation number to increase linearly with distance x in the source up to a 'thermalization depth' x_T at which point it reaches the black-body level, i.e.

$$N(\omega,x) \approx (\kappa_{sc}\kappa_{ff})^{\frac{1}{2}}\, x/(e^{\hbar\omega/kT} - 1) \qquad (6)$$

for $0 \leqq x \leqq x_T := (\varkappa_{sc}\varkappa_{ff})^{-1/2}$. A simple scattering equation with a source term leads to an equation of the form (6) with x/x_T replaced by $1-\exp(-x/x_T)$, but (6) suffices for our purpose.

In this scattering-dominated medium the path length s traversed by a photon in diffusing a distance x is

$$s \approx N_{sc}\,\kappa_{sc}^{-1} \approx \kappa_{sc}\, x^2 , \qquad (7)$$

where $N_{sc} \cong (\varkappa_{sc}x)^2$ is the typical number of scatterings which the photon undergoes. The probability $P_{2\gamma}$ of absorption by the two-photon process before arriving at $x=x_T$ is then given by integrating the absorption coefficient,

$$\kappa_{2\gamma} := A(\omega)\, N(\omega')/c\, y^2 , \qquad (8)$$

along the photon's path:

$$P_{2\gamma} = \int_{x=0}^{x_T} ds\, \kappa_{2\gamma}$$

$$= \frac{2}{3}\,\frac{A(\omega)}{c\, y^2}\left(\frac{\kappa_{ff}(\omega)}{\kappa^3_{ff}(\omega')}\right)^{\frac{1}{2}} [e^{\hbar\omega'/kT} - 1]^{-1} \qquad (9)$$

Two-photon emission is dominant when $P_{2\gamma} > 1$. At $\omega = \omega' = \Omega_e/2$, $\varkappa_{ff}$ is given to order of magnitude by its non-magnetic value:

$$\kappa_{ff} = \frac{\alpha}{4\sqrt{2\pi}}\left(\frac{m}{kT}\right)^{\frac{1}{2}} \frac{\omega_p^4}{\omega^3}\,(1 - e^{-\hbar\omega/kT}). \qquad (10)$$

For parameters appropriate to accretion columns—$\hbar\Omega_e = 50$ keV, $kT = 10$ keV, $n_e = 10^{22}$ cm^{-3}—one finds that $P_{2\gamma}$ exceeds unity by some three orders of magnitude. Hence bremsstrahlung should be a negligible source of non-cyclotron photons at least above several keV.

4 Discussion and Conclusions

We have argued in this paper that two-photon emission (and absorption) may play an important role in forming the spectrum near the cyclotron line in the emission from an accretion column above a magnetized neutron star. Qualitatively, processes are available which allow the entire spectrum to be produced from one source of photons: (non-radiative) collisional excitation of the Landau levels followed by a radiative decay. Semi-quantitative estimates suggest that two-photon decay of the first excited level is a more important source of photons at $\omega < \Omega_e$ than is bremsstrahlung.

No attempt has been made to describe the line structure because of the complexity of incorporating the processes described in section 2 into a model. However, our main conclusion on the importance of a hitherto neglected source of photons is unaffected by these difficulties.

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