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RADIO-FREQUENCY HEATING OF THE CORONAL PLASMA DURING FLARES

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ABSTRACT

We develop a model for the radio-frequency (RF) heating of soft X-ray emitting plasma in solar flares due to absorption of amplified cyclotron radiation. The radiation, carrying $\sim 10^{27}$ to $\sim 10^{30}$ ergs s^{-1}, is generated through maser emission following partial precipitation of electrons in one or more flaring loops. The maser operates in a large number of small regions, each producing an "elementary burst" (EB) of short duration. This radiation propagates either directly or after reflection to the second-harmonic absorption layer, where it is absorbed by thermal electrons. The properties of EBs and the heating of the electrons in the absorption layer are discussed in detail. RF heating and evaporation models for the production of soft X-ray emitting plasma are compared. Properties of the RF heating model that explain observed features are energy transport across field lines, rapid heating (in ~ 1 s) of coronal plasma to $\approx 3 \times 10^7$ K, and instigation of turbulent velocities up to the ion sound speed.

Subject headings: radiation mechanisms — Sun: corona — Sun: flares — Sun: radio radiation

I. PRELIMINARY REMARKS

The objective of this investigation is to explore the idea that soft X-rays associated with solar flares may be produced through radio wave heating of the soft X-ray emitting plasma. Specifically, it is envisaged that radiation is emitted copiously near the electron cyclotron frequency Ω_e through maser emission in a source region of plasma frequency $\omega_p \ll \Omega_e$. The free energy is in an anisotropy in the energetic electron distribution where the electrons are assumed to be accelerated near the top of a magnetic flux tube, which is identified as the source of energy for the flare. The radiation is absorbed in a surrounding region at the second harmonic of the cylotron frequency, i.e., in a region where the magnetic field strength B is half its value in the source of the maser emission. The model has been discussed by Melrose and Dulk (1982*a*, *b*).

A strong case can be made that a large fraction of the energy in the flare goes into radio-frequency (RF) emission through the maser process. It is widely accepted that a large fraction of the flare energy goes initially into hot (10–100 keV) electrons near the tops of flux tubes. The spectrum of these electrons is not important in the following discussion, and they may be either nonthermal or thermal (at a temperature 10^8–10^9 K). We assume here that the acceleration mechanism does not lead to electrons which are highly collimated along the field lines. This assumption is essential for the maser mechanism, which relies on free energy in the perpendicular (to $\boldsymbol{B}$) motion of the electrons. We discuss this assumption in § VI. Because of the mirroring in the converging magnetic fields in the legs of the flux tubes, only a fraction of these electrons can precipitate directly. However, data on hard X-rays imply that the precipitating flux fluctuates much as does the trapped flux represented by the microwaves, and in the impulsive phase both the hard X-rays and microwaves are thought to reflect the injection (or acceleration or heating) of the electrons. The point is that the electrons do not remain trapped for many bounce periods as simple theory would suggest, at least for small flares with loops of length $\sim 10^9$ cm (e.g., Marsh and Hurford 1982). The maser emission is the only known process which can lead to the extremely efficient diffusion in pitch angle required to prevent most of the electrons remaining trapped for many bounce periods, i.e., for several to many seconds. The maser emission requires that the electrons radiate away the perpendicular energy which prevents them from precipitating. Efficient diffusion then implies that roughly 50% of the initial energy of the electrons, the perpendicular component, goes into the maser emission.

We now discuss the observations of soft X-ray emission of simple impulsive flares (e.g., Doschek *et al.* 1980; Feldman *et al.* 1980; Culhane *et al.* 1981; Gabriel *et al.* 1981; Antonucci *et al.* 1982; Tanaka *et al.* 1982; Doschek 1983). There are three aspects that are important in the present context: the relative sizes of soft and hard X-ray sources, the great widths of ion lines, and the temperatures and densities achieved in the soft X-ray plasma. The currently favored interpretation involves an evaporative process. The precipitating electrons heat the chromospheric plasma, which boils off to form a hot (few $\times 10^7$ K) region in the corona which emits soft X-rays. The ion lines (e.g., Ca XIX, Fe XXV) are broadened, indicating turbulent motions at several hundred km s^{-1}, and there is an asymmetry indicating upward motions.

Comparing time profiles of (i) excess line width, (ii) integrated line intensity, and (iii) hard X-rays and/or microwaves, three findings are notable: (1) Large excesses of line widths seem to occur only during the impulsive phases of flares at times when intense hard X-ray and microwave emission is occurring, although smaller excesses sometimes occur during decay phases. (2) Large excess widths occur almost entirely while the integrated intensity is increasing. (3) The derivatives of the profiles of intensity (for small, simple flares) correspond well with hard X-ray and microwave burst profiles. The peak hard X-ray and microwave flux generally occurs at the time when the soft X-ray line intensity is increasing most rapidly. Seconds to minutes later, when the hard X-ray flux is back to near-zero, the soft X-ray line intensity reaches its peak.

There are at least two difficulties with the evaporation model that may be overcome or alleviated by an appeal to RF heating. The most serious concerns the relative location and sizes of the hard and soft X-ray sources. Duijveman, Hoyng, and Machado (1982), for example, compared three impulsive flares in soft (3.5–5.5 keV) and hard (16–30 keV) X-rays. Whereas the hard X-rays often show evidence of two footpoints, favoring a precipitation model, in the soft X-ray sources there is little evidence (except perhaps at late times) of two subsources. Furthermore, the soft X-ray sources may well be larger than the hard X-ray sources. Duijveman *et al.* suggested that the soft X-ray sources outline the top of a set of loops. These data imply that the flux tubes in which the soft X-ray plasma is contained do not coincide with those in which the hard X-ray plasma is contained. Similarly, microwave impulsive burst sources are larger at the lower frequencies where soft X-ray plasma emits significantly, e.g., $\gtrsim 10''$ at 5 GHz compared with $\lesssim 3''$ at 15 GHz (e.g., Marsh and Hurford 1982).

How then can heating and evaporation by the precipitating electrons produce the soft X-ray plasma? How is energy transported across field lines into neighboring, or even more distant, flux tubes?

These questions are difficult to answer with the evaporation model, but specific predictions of the RF heating model provide simple answers to them. Another difficulty with the evaporation model is how to produce turbulent motions in the soft X-ray plasma within a few seconds; if the heating is in the high-density chromospheric regions, one would not expect strong, turbulent motions (both up and down) in the plasma that is evaporating into the corona. However, in the RF heating model the turbulence is readily interpreted in terms of the irregular heating caused by the multitudinous maser pulses and minor nonuniformities in the absorbing plasma; the blue-shifted components often observed would arise from the precipitating electrons (their parallel energy) and perhaps from the footpoints of maser-heated loops as described below.

In our heating model the hot plasma is produced at the second-harmonic layer. An idealized geometry would involve a sequence of loops in which the field strength decreases with height such as depicted in Figure 1. In this geometry the second-harmonic layer is an annulus centered on the flaring flux tube and at a distance $L_B = B/|\text{grad } B|$ from it. In a more realistic geometry the annulus would be more complicated. With the maser operating over a range of heights and hence a range of field strengths and frequencies, the heated region is the envelope of the second-harmonic layers corresponding to the range of field strengths.

The idea underlying the present investigation may be summarized as follows. Arguments suggest that up to 50% of the energy in a flare should go into maser radiation at $\approx \Omega_e$. These arguments are reinforced by consideration of how the injected electrons lose their perpendicular energy in order to precipitate into the footpoints and hence heat the chromosphere and generate hard X-rays there. This perpendicular energy is radiated away through amplification of the appropriate (decimetric) radio waves. The radiation is then reabsorbed at the second harmonic of Ω_e, i.e., at points where B is half its value in the source. From energetic considerations there seems to be enough energy in the RF emission to heat the plasma in the

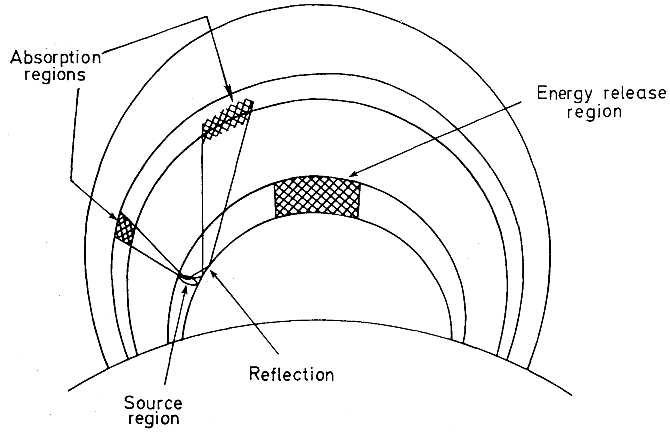

FIG. 1.—Schematic drawing of a sequence of loops showing an energy release region, the maser source region in one leg, where the field strength is B, the cone of maser radiation with a reflection of all radiation directed toward higher field strengths, and the absorption regions, where the field strength is $B/2$.

absorbing region so that it produces the soft X-rays. In addition it is likely that this RF heating process can account for some or most of the other properties of the soft X-rays in impulsive flares, e.g., the large source sizes, excess line widths and their temporal development.

II. ENERGETICS OF THE HARD AND SOFT X-RAY EMITTING PLASMAS

The rate at which energy is released during impulsive flares, derived from observations of hard X-ray bursts and assuming thick-target emission processes, is (e.g., Brown and Smith 1980)

$$\dot{E} \approx 10^{27}\text{–}10^{30} \text{ ergs s}^{-1} .$$

If most of the impulsive hard X-rays come from footpoints of magnetic loops, as implied by the observations reported by Hoyng *et al.* (1981*a*, *b*, 1983), Tsuneta *et al.* (1982), and Ohki *et al.* (1982), we can identify this energy with the parallel component of the energy of the accelerated electrons; then a similar amount is in the perpendicular component provided that the accelerated electrons leaving the energy release site (presumed to be in the upper parts of magnetic flux tubes) are not highly collimated. The perpendicular energy, in conjunction with the loss-cone anisotropy developed by the precipitation of those electrons with small pitch angles into the dense transition region and chromosphere, is the pump for the cylotron maser. The maser converts the perpendicular energy directly into radiation, and the radiative energy is then reabsorbed within the coronal plasma in a volume whose dimensions can be estimated from images made in soft X-ray lines, X-ray continuum at $\lesssim 10$ keV, and microwaves at $f \gtrsim 5$ GHz. Typical observed sizes are $10''$–$60''$, implying volumes of $\sim 10^{27}$ to $\sim 10^{29}$ cm^3. Thus the heating rate in the corona, assuming that the larger values of $\dot{E}$ go with the larger volumes, is

$$\dot{E}/V \approx 1\text{–}10 \text{ ergs cm}^{-3} \text{ s}^{-1} .$$

For comparison, we list some other typical parameters in this volume in Table 1. Thus we see that the internal energy nkT of the preflare plasma can, on average, increase by a factor of 2–20 each second. With the expectedly strongly inhomogeneous heating, some pockets can heat up even more rapidly. With the parameters in Table 1, the magnetic energy starts off

TABLE 1

TYPICAL PARAMETERS IN A FLARE VOLUME

Parameter	Preflare	Soft X-Ray Peak
T (K)	$\approx 3 \times 10^6$	$\approx 2 \times 10^7$
n (cm^{-3})	$\sim 10^9$	$\sim 10^{11}$
nkT (ergs cm^{-3})	~ 0.5	~ 300
B (gauss)	~ 300	~ 300
$B^2/8\pi$ (ergs cm^{-3})	~ 3000	~ 3000
β	$\sim 10^{-4}$	$\sim 10^{-1}$
ω_p/Ω_e	~ 0.3	~ 3

and remains greater than the internal energy, but it need not be so in all parts of the flare or in all flares. The ratio ω_p/Ω_e is important because it governs the operation of the maser, with the maser becoming inoperative when $\omega_p/\Omega_e \gtrsim 1$; given the stated parameters the maser would turn off at or before the soft X-ray peak, i.e., near the end of the impulsive phase. It is interesting to ask what occurs if the condition $\omega_p/\Omega_e \lesssim 0.3$ required for maser emission of fundamental x-mode radiation to occur is not satisfied. This case is currently under investigation, and we comment briefly on it in § VI.

III. OPERATION OF THE MASER

In this section we discuss the operation of the maser in a flux loop; the discussion is based on the idea that the maser saturates in a number of isolated regions, each of which produces what we call an "elementary burst" (EB).

a) *Elementary Bursts*

Suppose maser emission is triggered at a particular point and time. The growth quickly saturates, after a time $t_{\rm sat}$ say; this saturation is local, and as the radiation travels through the unstable volume, it triggers the maser at other places. We assume that emission from this larger region defines an elementary burst with a volume $V_{\rm EB} = A_{\rm EB} L_{\rm EB}$, where $A_{\rm EB}$ and $L_{\rm EB}$ are the cross-sectional area and the length along the field line, respectively. Here we estimate the properties of these EBs.

The length $L_{\rm EB}$ is limited by the relative bandwidth $\Delta\omega/\omega$ of the maser emission and the change of the cyclotron frequency Ω_e with height. Let L_B be the characteristic length over which Ω_e changes. Thus we have

$$L_{\rm EB} = \frac{\Delta\omega}{\omega} L_B . \tag{3.1}$$

The lateral dimensions $l_\perp$ and the duration $t_{\rm EB}$ of an elementary burst are related to each other. The radiation propagates nearly perpendicular to the magnetic field at an angle θ given by $|\cos\theta| \approx v_0/c$ (Hewitt, Melrose, and Rönnmark 1982), where v_0 is a characteristic speed of the radiating electrons, and hence it escapes from a region of height $L_{\rm EB}$ in a time

$$t_{\rm EB} \approx \frac{L_{\rm EB}}{v_0} . \tag{3.2}$$

In this time it propagates a distance $l_\perp \approx ct_{\rm EB}$ in the lateral direction. Thus the area of the EB is

$$A_{\rm EB} \approx \pi L_{\rm EB}^2 \left(\frac{c}{v_0}\right)^2 \tag{3.3}$$

The time (3.2) may be reinterpreted as follows. By the time the maser has saturated throughout $V_{\rm EB}$, all the free energy has been converted into electromagnetic radiation. The resulting relaxed distribution of electrons drifts out of the source region of the EB and is replaced by an unrelaxed distribution with available free energy; the time required is $L_{\rm EB}/v_0$, i.e., the time it takes an electron to propagate a distance $L_{\rm EB}$. From this viewpoint $t_{\rm EB}$ is the minimum time which must elapse before two consecutive EBs can occur in the one localized region. For reasonable parameters, as discussed below, this time is in the range 100 μs to 1 ms, compared with a saturation time $t_{\rm sat}$ for the maser of several microseconds.

b) *Energetics of Elementary Bursts*

We envisage many EBs occuring simultaneously; the total instantaneous power radiated through maser emission is the sum of the powers radiated by each EB. Let this total power radiated $\dot{E}_R$ be a fraction η of the total power $\dot{E}$ released in the flare:

$$\dot{E}_R = \eta \dot{E} , \tag{3.4}$$

Suppose that the flare has an area A which defines a number $N_{\rm EB}$ of EB flux tubes. On average the number of EBs occurring simultaneously is

$$N_{\rm EB} = \frac{A}{A_{\rm EB}} . \tag{3.5}$$

The average power radiated per EB is then

$$\dot{E}_{\rm EB} = \frac{\dot{E}_R}{N_{\rm EB}} . \tag{3.6}$$

We now show that equations (3.4), (3.5), and (3.6) are consistent with a maser operating at an efficiency η.

Let n_1 be the number density of energetic electrons with speed $\gtrsim v_0$. Their energy density is $\frac{1}{2}n_1 m_e v_0^2$. In saturated maser emission this energy is converted into radiation with an efficiency η' (equal to the ratio of the free energy density to the total energy density in the energetic electrons). We deduce $\eta' = \eta$ as follows. By hypothesis we have, for the power in the flare,

$$\dot{E} = \tfrac{1}{2} n_1 m_e v_0^3 A , \tag{3.7}$$

and, for the energy in an EB,

$$\dot{E}_{\rm EB} t_{\rm EB} = \eta'(\tfrac{1}{2} n_1 m_e v_0^2)(A_{\rm EB} L_{\rm EB}) . \tag{3.8}$$

On inserting equation (3.2) in equation (3.8) and using equations (3.5), (3.6), and (3.7), one finds $\dot{E}_R = N_{\rm EB} \dot{E}_{\rm EB} = \eta' \dot{E}$, implying $\eta' = \eta$ in view of equation (3.4).

This leads us to a self-consistent picture in which the flare energy is converted with an efficiency η into radiation through saturated maser emission in, at any one time, $N_{\rm EB}$ localized EBs. The source region of each EB has (*a*) an area corresponding to $1/N_{\rm EB}$ of the total cross-sectional area A of the flaring magnetic loops and (*b*) a length $L_{\rm EB}$ at least two orders of magnitude smaller than the total length L of a flux tube.

c) *Brightness Temperatures*

Melrose and Dulk (1982*a*) estimated the maximum possible brightness temperature T_B for the maser; here we calculate T_B for an individual EB. We have

$$W_R = \int \frac{d^3k}{(2\pi)^3} K T_B = \frac{K T_B}{V_c} , \tag{3.9}$$

where W_R is the energy density in the radiation, and V_c is its coherence volume:

$$V_c^{-1} = \left(\frac{\omega}{2\pi c}\right)^3 \frac{\Delta\omega}{\omega} \Delta\Omega \; ; \tag{3.10}$$

$\Delta\Omega \approx 2\pi\Delta \cos\theta$ is the range of solid angles into which the radiation is directed. From the results of Hewitt, Melrose, and Rönnmark (1982) we have $\Delta\omega/\omega \approx (v_0/c)^2$ and $\Delta\Omega \approx 2\pi(v_0/c)^2$ sr, so that equation (3.10) reduces to

$$V_c = \frac{1}{2\pi}\left(\frac{c}{f}\right)^3 \left(\frac{c}{v_0}\right)^4 , \tag{3.11}$$

with $f = \omega/2\pi$ the cyclic frequency.

Assuming that a fraction η of the energy density in the energetic electrons is converted into maser radiation, we have

$$KT_B = \eta \tfrac{1}{2} n_1 m_e v_0^2 V_c \, . \tag{3.12}$$

It is instructive to reexpress equation (3.12) with equation (3.11) in terms of the growth rate for the maser (Melrose, Rönnmark, and Hewitt 1982)

$$\frac{\Gamma}{\Omega_e} \approx \left(\frac{\omega_p}{\Omega_e}\right)^2 \frac{n_1}{n_e} \frac{c^2}{v_0^2} \, . \tag{3.13}$$

We find

$$\frac{KT_B}{m_e c^2} \approx \eta \frac{c}{r_0 \omega} \frac{\Gamma}{\Omega_e} , \tag{3.14}$$

where r_0 is the classical radius of the electron.

For example, for $\Gamma/\Omega_e \approx 10^{-3}$ (Hewitt, Melrose, and Rönnmark 1982), $f = 1$ GHz, and η of order unity, equation (3.14) implies $KT_B/m_e c^2 \approx 10^{10}$ or $T_B \approx 10^{20}$ K.

IV. ABSORPTION AT THE SECOND HARMONIC

Maser radiation emitted at the fundamental *cannot* be reabsorbed at the fundamental. This may be seen by considering how the shape of the resonant ellipse (e.g., as discussed by Melrose, Rönnmark, and Hewitt 1982; Hewitt, Melrose, and Rönnmark 1982; Melrose and Dulk 1982*a*) changes as $\omega - \Omega_e$ decreases: the semimajor axis of the ellipse shrinks to zero (implying that absorption becomes impossible) with its center remaining essentially fixed. Maser radiation propagating toward regions of increasing Ω_e is refracted away without experiencing cyclotron absorption. All the radiation emitted at the fundamental ultimately propagates toward regions of decreasing Ω_e and should be absorbed at the second-harmonic layer, where Ω_e is half its initial value. The geometry of the absorption region is illustrated schematically in Figure 1.

a) The Absorption Rate

To within a factor of order unity, absorption at the second harmonic of the cyclotron frequency occurs at a rate per unit time (e.g., Ginzberg 1964, p. 133; Zheleznyakov 1970, p. 449; Melrose 1980, p. 276)

$$\gamma_2 \approx \frac{\omega_p^2}{\omega} \beta_T \tag{4.1}$$

over a bandwidth

$$\delta\omega_2 \approx \beta_T \omega \, , \tag{4.2}$$

with

$$\beta_T = \left(\frac{KT_e}{m_e c^2}\right)^{1/2} . \tag{4.3}$$

The absorption coefficient quoted in the references cited varies with angle as $\sin^2\theta$, which could lead one to conclude erroneously that it is zero for perpendicular propagation. Actually the usual formulae for the absorption are based on the nonrelativistic approximation to the resonance condition, and for nearly perpendicular propagation it is essential to take relativistic effects into account. One may show that equations (4.1) and (4.2) are replaced by $\gamma_2 \approx \omega_p^2/\omega$ and $\delta\omega_2 \approx \beta_T^2 \omega$ for $|\cos\theta| \lesssim \beta_T$. The net absorption is proportional to the product $\gamma_2 \delta\omega_2$ and is essentially unaffected. Although equations (4.1) and (4.2) do not strictly apply for nearly perpendicular propagation, in practice little error should result from using them. Put another way, on repeating our calculations treating the special case of nearly perpendicular propagation, we find that the net absorption in the second-harmonic layer is the same as that found using equations (4.1) and (4.2) and ignoring the angular dependences.

Let the radiation at the second-harmonic layer have a specific intensity I_2, a bandwidth $\Delta\omega_2$, and be confined to a solid angle $\Delta\Omega_2$. Its energy density is $W_2 = I_2 \Delta\omega_2 \Delta\Omega_2/c$. The absorption causes W_2 to decrease at the rate

$$\frac{dW_2}{dt} = -\gamma_2 W_2 \begin{cases} 1 & \text{for} \quad \delta\omega_2 \gtrsim \Delta\omega_2 \\ \dfrac{\delta\omega_2}{\Delta\omega_2} & \text{for} \quad \delta\omega_2 \lesssim \Delta\omega_2 \end{cases} . \tag{4.4}$$

(Eq. [4.4] follows from $dI_2/dt = -\gamma_2 I_2$ only for radiation within a range $\delta\omega_2$ about $\omega = 2\Omega_e$.) With $\Delta\omega_2 \approx \frac{1}{2}\omega(v_0/c)^2$ and $\delta\omega_2 \approx \omega\beta_T$, we expect $\delta\omega_2 \gtrsim \Delta\omega_2$ for relevant parameters here.

b) The Heating Rate

The energy absorbed from the radiation goes into thermal electrons and causes their temperature to increase at the rate

$$\frac{d\beta_T^2}{dt} = \frac{\gamma_2}{n_e m_e c^2} , \tag{4.5}$$

where we assume $\delta\omega_2 \gtrsim \Delta\omega_2$ in equation (4.4). Then equation (4.5) with equation (4.1) implies

$$\frac{d\beta_T}{dt} = \frac{2\pi r_0}{m_e c} \frac{I_2 \Delta\omega_2 \Delta\Omega_2}{\omega} \, . \tag{4.6}$$

Using the relation

$$I = \frac{1}{2\pi} \frac{f^2}{c^2} KT_B$$

between the specific intensity I and the brightness temperature T_B, equation (4.6) becomes

$$\frac{d\beta_T}{dt} = \frac{1}{t_H} = \frac{r_0 f^2}{c} \frac{KT_B}{m_e c^2} \left(\frac{\Delta\omega_2}{\omega}\right) \Delta\Omega_2 \, . \tag{4.7}$$

In estimating the heating time t_H to $KT_e \approx m_e c^2$ (i.e., to $T_e \approx 10^{10}$ K) we assume $f = 1$ GHz, $\Delta\omega_2/\omega = 10^{-2}$, $T_B = 10^{20}$ K ($KT_B/m_e c^2 \approx 10^{10}$), and $\Delta\Omega_2 \approx 10^{-3}$ for a source (an EB) of size 30 km by 300 km at a distance 3000 km from the absorption layer. These numbers give $t_H \approx 1$ s, implying heating to 3×10^7 K in 0.1 s. Put another way, in 1 ms, which is typical of the duration t_{EB} of an elementary burst, the heating causes β_T to increase by $\Delta\beta_T \approx 10^{-3}$, which corresponds to $\Delta T_e \approx 10^6$ K.

The foregoing analysis breaks down for low temperatures, specifically for $\beta_T < \Delta\omega_2/\omega$. In this case equation (4.4) implies an extra factor $\delta\omega_2/\Delta\omega_2$ on the right-hand side of equation

(4.6). It follows that the heating causes T_e to increase exponentially at an e-folding rate $2\omega/\Delta\omega_2 t_H$, which is $\sim 2 \times 10^2 \text{ s}^{-1}$ for the numbers chosen above. This greater rate of heating at lower temperatures is not important in the following discussion.

c) *Role of Collisions*

The energy absorbed by the thermal particles goes initially into a select group of resonant electrons (in a particular region of velocity space), and it is redistributed among all the electrons only through collisions. The collision frequency is

$$\nu \approx \frac{4\pi n_e r_0^2 c}{\beta_T^3} \ln \Lambda , \tag{4.8}$$

with the Coulomb logarithm ln Λ of order 20 here. The electron distribution remains Maxwellian only if ν exceeds the heating rate $\approx \beta_T/t_H$. Thus the electrons remain Maxwellian for

$$\nu > \frac{\beta_T}{t_H} , \tag{4.9}$$

and then the energy is absorbed in a time γ_2^{-1} during which the radiation propagates a distance

$$L_{\text{abs}} \approx \frac{c}{\gamma_2} \tag{4.10}$$

into the second-harmonic absorption layer (of thickness $\approx L_B \omega/\delta\omega_2$ along grad B).

The inequality (4.9) in the form $c\beta_T/\nu < ct_H$ may be interpreted as requiring that the collisional mean free path be less than the distance the radiation propagates in a heating time t_H. There is another restriction implied by the collisional mean free path. The absorbed energy goes initially into energized electrons which propagate along the field lines a distance equal to their collisional mean free path before their excess energy is thermalized through collisions. Radiation with a bandwidth $\Delta\omega_2$ is absorbed over a distance $\approx (\Delta\omega_2/\omega)L_B$ along the field lines. We require

$$\lambda_f < \frac{\Delta\omega_2}{\omega} L_B, \tag{4.11}$$

$$\lambda_f = c\beta_T/\nu = \beta_T^4/4\pi n_e r_0^2 \ln \Lambda , \tag{4.12}$$

for the thermalization of this energy to be localized in the absorption region. The discussion of the heating given above is valid only when inequality (4.11) applies.

Before discussing the implications of inequality (4.11) not being satisfied, there is one interesting possibility worth mentioning: in principle intense narrow-band radiation can "burn a hole" in the absorption layer and escape through it. A "hole" is formed when all the resonant electrons gain energy and move from the resonant to a nonresonant region of velocity space. For such a hole (in velocity space) to lead to escape of radiation it must (*a*) exist everywhere (in coordinate space) along the ray path through the absorption layer and (*b*) be formed in less than a collision time. A semiquantitative analysis leads one to conclude that such a hole could form under conditions considered here, but only for radiation with a bandwidth $\Delta\omega/\omega \ll \beta_T$. This final proviso is not adequately satisfied for maser radiation.

d) *The Final Temperature*

As the heating proceeds, β_T increases, and the left-hand side of inequality (4.11) increases proportional to β_T^4. At a sufficiently high temperature inequality (4.11) must cease to apply. Further heating then leads to deposition of energy in a region whose length along the field lines is λ_f, rather than $(\Delta\omega_2/\omega)L_B$, and this length increases as β_T^4. The rate of temperature increase then slows dramatically. Let V_{hot} be the volume of heated plasma: we have $V_{\text{hot}} \propto \lambda_f$ for $\lambda_f > (\Delta\omega_2/\omega)L_B$. Hence an energy E absorbed leads to a temperature

$$KT_e = \frac{E}{n_e V_{\text{hot}}} \propto \begin{cases} E , & \lambda_f < \dfrac{\Delta\omega_2}{\omega} L_B \\ E^{1/3} , & \lambda_f > \dfrac{\Delta\omega_2}{\omega} L_B \end{cases} . \tag{4.13}$$

On the basis of equations (4.13) we conclude that the temperature corresponding to $\lambda_f = (\Delta\omega_2/\omega)L_B$, i.e.,

$$\frac{KT_e}{m_e c^2} \approx \left(4\pi \ln \Lambda \frac{\Delta\omega_2}{\omega} n_e r_0^2 L_B\right)^{1/2} , \tag{4.14}$$

is a characteristic temperature beyond which heating becomes less effective in increasing T_e. For $L_B = 3 \times 10^3$ km and $n_e = 3 \times 10^{10} \text{ cm}^{-3}$, this temperature is 10^7 K, increasing to 3×10^7 K for $n_e = 3 \times 10^{11} \text{ cm}^{-3}$. Such temperatures are characteristic of the soft X-ray emitting plasma.

V. INTERPRETATION OF OBSERVATIONAL FEATURES

Observed features of soft X-ray sources include their time developments, sizes, line widths, and shifts and their intensities (implying emission measures). Here we comment on the interpretation of these features in terms of the RF heating model and in terms of an evaporation model.

a) *Rapid Heating*

In the impulsive phase the intensity of soft X-rays rises rapidly, in ~10 s (Doschek 1983) and varies roughly as the time-integral of the intensity of the hard X-rays (Tanaka *et al.* 1982). The latter feature suggests that the soft X-ray plasma is heated either directly or indirectly by the electrons which produce the hard X-ray emission. With the RF heating model the causal connection is through the maser-generated RF radiation. The time delay between the hard X-rays and the soft X-rays is the time required for the heating of the soft X-ray emitting plasma. This can be quite short in small volumes (0.1 s is estimated in § IV), but in the overall soft X-ray source it is limited by energetic considerations to about 10 s. The time delay expected with an evaporation model is short if the heated chromosphere radiates *in situ*. However, soft X-rays appearing promptly high in the corona are difficult to explain in terms of evaporation from the chromosphere to great heights.

b) *Source Sizes*

There is evidence that soft X-ray sources are larger than hard X-ray sources (Duijveman, Hoyng, and Machado 1982); certainly the lower frequency microwave sources, representing the locations of the lower energy electrons, are larger than those at higher frequencies (Marsh and Hurford 1982). If the heating of the soft X-ray plasma is a secondary process (and not due to local dissipation of magnetic energy, for example), then these data imply that the energy is transported across field lines. In an evaporation model the region of the chromosphere which is heated is in the same flux tube as the precipitating electrons which produce the hard X-rays. It is difficult to see how the soft X-ray source can be larger than or displaced from the hard X-ray source. On the other hand, the RF heating model accounts naturally for transport of energy across field lines.

c) *Plasma Motions*

The observed soft X-ray emitting plasma has upward motions which can reach several hundred kilometers per second, and there are also turbulent motions at speeds up to ≈ 300 km s^{-1} (Doschek *et al.* 1980; Antonucci *et al.* 1982; Tanaka *et al.* 1982). Here we argue that these motions can be caused by localized heating leading to rapid expansion at up to the ion sound speed $v_s \approx 300 T_7^{1/2}$ km s^{-1} with $T_7 = T_e/10^7$ K. First, however, we show that an alternative suggestion can be ruled out: this is that the motions are due to momentum transferred from the maser radiation itself.

The maser emission is directed at an angle θ to B, and a momentum component $(E/c)\cos\theta$ along B is transferred to the absorbing electrons when an energy E is absorbed. Let ρ be the plasma density. Assuming that the ions move with the electrons, a heated region of volume V attains a speed

$$v = \frac{E \cos\theta}{V\rho c} . \tag{5.1}$$

The electrons are heated to a temperature such that $n_e K T_e V$ equals E, and then $\rho = m_i n_e$ and $v_s = (K T_e/m_i)^{1/2}$ imply

$$v = \frac{v_s^2}{c} \cos\theta . \tag{5.2}$$

This speed is always $\ll v_s$ and so cannot account for motion at $\approx v_s$.

d) *Expanding Hot Regions*

The absorption of the maser radiation due to very many EBs is likely to lead to inhomogeneous heating in the second-harmonic layer. Suppose a localized region is heated much more than its surroundings. Then the heated electrons tend to propagate out of this region. Charge balance requires either that the escaping electrons drive a return current or that they drag the ions with them. A return current involves cold electrons flowing back to replace the escaping electrons in the heated region. One expects a return current to balance the direct current only when the direct current flows into a region where the density of cold electrons is much greater than that of hot electrons. Otherwise, to balance the direct current, the cold electrons would need to flow so fast that they would excite microinstabilities. When the supply of cold electrons is inadequate in this sense, the hot electrons must drag the ions with them to maintain charge and current neutrality. This leads to expansion of the locally overheated region at up to $v_s \approx 300$ km s^{-1}, which is the relevant sonic speed in the present context (i.e., for hot electrons and colder ions).

The RF heating model leads one to expect turbulent motions in the heated region at up to the ion sound speed, and this is consistent with the motions inferred from observation.

e) *The Volume Heated*

There is no problem in principle with the RF heating model in accounting for the required energy and power transfer to the soft X-ray emitting plasma. However, to account for the observed emission measure, the volume heated must be quite large, and this requirement is not automatically satisfied. Emission measures between 10^{48} and 10^{50} cm^{-3} have been estimated (Antonucci *et al.* 1982; Tanaka *et al.* 1982), and, as summarized in Table 1, densities vary from 10^9–10^{10} cm^{-3} before the flare to 10^{11}–10^{12} cm^{-3} after the flare. Volumes up to 10^{26} cm^{-3} are implied.

The volume of plasma heated at the second-harmonic layer may be estimated in two superficially different (but actually equivalent) ways for a given EB. First, the region irradiated by a single EB is an annulus of height $\Delta\theta L_B$ at a distance L_B (from the maser source) and hence of area $2\pi\Delta\theta L_B^2$. Radiation with a bandwidth $\Delta\omega$ is absorbed over a distance $(\Delta\omega/\omega)L_B$ in which Ω_e changes by $\Delta\omega$. This gives a volume

$$\delta V \approx 2\pi\delta\theta \frac{\Delta\omega}{\omega} L_B^3 . \tag{5.3}$$

The alternative method for estimating δV is to equate the energy absorbed $\delta V n_e m_e c^2 \Delta\beta_T^2$ with the energy $E_{\rm EB}$ in an EB. Using detailed results derived in §§ III and IV, this method reproduces equation (5.3).

For $L_B = 3 \times 10^8$ cm, $\Delta\theta = 0.1$, and $\Delta\omega/\omega = 10^{-2}$, the volume δV is of order 10^{23} cm^3, which is too small by a factor $\sim 10^{-3}$. This is because the volume heated must be the envelope of a large number of smaller volumes δV heated in individual EBs. Suppose the maser operates over a range of heights in the source region, and hence over a range $\Delta\Omega_e$ of cyclotron frequencies, and that in the source region the local direction of B varies over a wide range of angle $\Delta\chi$. Then the total volume heated is

$$\delta V_{\rm hot} \approx \pi \frac{\Delta\Omega_e}{\Omega_e} \Delta\chi L_B^3 . \tag{5.4}$$

With $\Delta\Omega_e/\Omega_e \approx 1$ and $\Delta\chi \approx 1$, equation (5.4) gives $\delta V \approx 10^{26}$ cm^3 for $L_B \approx 3 \times 10^8$ cm.

With these assumptions we can account for a sufficiently large volume of heated gas.

VI. DISCUSSION

Our purpose here has been to investigate the details of an RF heating model (Melrose and Dulk 1982*b*) for soft X-ray emission in flares. The basic model involves cyclotron maser generation of RF radiation which is subsequently absorbed at the second harmonic, i.e., in a region where the cyclotron frequency is half its value in the source region. The following points have been discussed.

1. The maser operates in localized regions for short durations, leading to radiation in the form of many elementary bursts (EBs). Each EB comes from a source with a height $L_{\rm EB} \approx (\Delta\omega/\omega)L_B$, where $\Delta\omega/\omega$ is the relative bandwidth of the maser emission, and $L_B = B/|\text{grad } B|$ is the scale length over which the magnetic field changes in magnitude, a duration $t_{\rm EB} \approx L_{\rm EB}/v_0$ and an area $A_{\rm EB} \approx \pi(c t_{\rm EB})^2$. For example, for a small flare from a loop of length ~ 1 arcsec (Marsh and Hurford 1982), corresponding to a flare with an area $A \approx 3 \times 10^{16}$ cm^2, we would expect $L_B \approx 300$ km, and then $\Delta\omega/\omega \approx (v_0/c)^2 \approx 10^{-2}$ (Hewitt, Melrose, and Rönnmark 1982) implies $t_{\rm EB} \approx 100$ μs, $A_{\rm EB} \approx 3 \times 10^{13}$ cm^2, and $V_{\rm EB} = L_{\rm EB} A_{\rm EB} \approx 10^{19}$ cm^3. For the maser to operate efficiently there would be $N_{\rm EB} = A/A_{\rm EB} \approx 3 \times 10^4$ EBs occurring simultaneously, and to produce a total power $\dot{E} = 10^{27}$ ergs s^{-1}, each EB would radiate a power $\dot{E}_{\rm EB} \approx 3 \times 10^{22}$ ergs s^{-1} and a total energy $E_{\rm EB} \approx 3 \times 10^{18}$ ergs. For a larger flare, say $A \approx 10^{18}$ cm and $L_B \approx 3000$ km, these numbers would be replaced by $t_{\rm EB} \approx 1$ ms, $L_{\rm EB} \approx 30$ km, $A_{\rm EB} \approx 3 \times 10^{15}$ cm^2, $N_{\rm EB} \approx 300$, $\dot{E}_{\rm EB} \approx 3 \times 10^{27}$ ergs s^{-1}, and $E_{\rm EB} \approx 3 \times 10^{24}$ ergs.

2. The brightness temperature of the maser radiation may be estimated using equation (3.14) for each EB. This implies $T_B \approx 10^{20}$ K.

3. The available free energy in the source region is radiated away as a result of the continual reestablishment and reduction of an upward-directed loss-cone anisotropy. The establishment has been discussed by White, Melrose, and Dulk (1983) in a model with continuous injection, and the reduction is due to the operation of the maser.

4. The energy in each EB is absorbed in a second-harmonic layer, i.e., where the cyclotron frequency is half its value in the source region. In the absorption volume for a given EB the temperature is increased only slightly (by $\Delta T_e \approx 10^6$ K) for each EB. The overall heated region is the envelope of the absorption volumes for very many EBs.

5. Temperatures in excess of a characteristic value of a few $\times\ 10^7$ K are difficult to achieve since the collisional mean free path grows and eventually exceeds the nominal dimension of the absorption volume, and then distributes the energy through a larger volume.

6. Turbulent motions are attributed to expansion at up to the ion sound speed v_s of localized regions which are heated more rapidly than their surroundings.

In this model the gross energetics are taken into account by adjusting the rate at which EBs occur. With a postulated (and plausible) high efficiency of energy transfer from precipitating electrons to maser radiation and effectively 100% efficiency of conversion of this radiation into heat at the second-harmonic layer, there is no difficulty in principle in accounting for the energetics of soft X-ray emission. Particularly favorable features of the RF mechanism include the very short time scale (~ 10 s) in which heating to 2–3 $\times\ 10^7$ K can occur, and the efficient transport of energy across field lines.

One feature which at first glance favors an evaporation model is the apparently large increase in density of the soft X-ray emitting plasma as a flare develops. Actually our model contains an element of evaporation because the parallel energy of the electrons in the flaring flux tubes is enough to "boil off" part of the chromosphere. At other locations, where RF heating occurs in a region with initial density 10^9 cm $^{-3}$ say, downward conduction from the heated region would cause plasma to evaporate off the chromosphere. In the absence of spatially resolved observations we do not know whether the blueshifted components of soft X-ray lines originate mainly in the flaring flux tubes or in all parts of the soft X-ray source.

A necessary condition for RF heating is that electron-cyclotron maser emission be efficient. This should occur for $\omega_p/\Omega_e \lesssim 0.3$ when the maser operates in the fundamental x-mode. For $\omega_p/\Omega_e \gtrsim 0.3$ it is likely that the z-mode is the dominant mode of the maser, and then escaping radiation can be produced at the second harmonic by coalescence of two z-mode waves (Melrose, Hewitt, and Dulk 1984). This may be a more favorable mechanism than direct maser emission at $\sim 2\Omega_e$ for production of microwave spike bursts (cf. Melrose and Dulk 1982*a*). Our discussion here presupposes $\omega_p/\Omega_e < 0.3$ in the source region. However, many features of the model may continue to be relevant for $\omega_p/\Omega_e \gtrsim 0.3$. Then a significant fraction of the energy may go into radiation at $2\Omega_e$, most of which would be absorbed at the third-harmonic layer. A different but related point is that maser emission can occur for downgoing electrons (White, Melrose, and Dulk 1983) and can lead to emission at $\omega < \Omega_e$ (in the z-mode or the o-mode) as well as at $\omega > \Omega_e$. Modifications to the maser mechanism as discussed here may be needed and are currently under investigation.

One further point concerns the possible increase in ω_p/Ω_e in the flaring region due to an increase in plasma density. Such an increase is inferred in the soft X-ray source and may also occur in the flaring flux tubes themselves. An increase in ω_p/Ω_e from less than 0.3 to greater than 0.3 will suppress the dominant fundamental emission, and a further increase to $\gtrsim 1.0$ will suppress the maser emission entirely. The electrons may then remain trapped for a longer time, but this is not obviously the case because, associated with the density increase, there is an increase in the collision frequency, and collisions can cause them to precipitate or to become a "coronal hard X-ray source." We have not explored this point in detail.

Another point which we need to discuss is the assumption that the acceleration mechanism for the 10–100 keV in a flare does not lead to highly collimated electrons. For stochastic acceleration mechanisms there is no reason to expect the electrons to become anisotropic. Indeed the only mechanism which might be expected to lead to collimated electrons is acceleration by a parallel potential drop, e.g., as in a double layer. However, in the case of auroral electrons, which are thought to be accelerated in this way, a substantial perpendicular component remains, and indeed it is just these electrons which are thought to produce the auroral kilometric radiation by the cyclotron maser mechanism we invoke here (Wu and Lee 1979; Moser *et al.* 1980). Thus one expects the accelerated electrons to have a substantial perpendicular energy and part of this energy to become available as free energy to drive the maser.

It is of interest to examine the most recent observations in order to test the predictions of the present RF heating theory. Signatures of RF heating which would distinguish it from evaporation are (i) transfer of energy across field lines, and (ii) heating which correlates with the hard X-ray emission with a short (~ 10 s) time delay.

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