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NONEXISTENCE OF TWO FORMS OF TURBULENT BREMSSTRAHLUNG

J. KUIJPERS
Sterrekundig Instituut, Rijksuniversiteit Utrecht

AND

D. B. MELROSE
School of Physics, University of Sydney
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ABSTRACT

It is shown that the forms of turbulent bremsstrahlung proposed by Tsytovich, Stenflo, and Wilhelmsson and by Nambu do not exist. The proposed mechanisms involve upconversion of ion sound turbulence into Langmuir turbulence, with the ion sound waves being emitted and absorbed resonantly and the Langmuir waves being emitted and absorbed nonresonantly. It is pointed out that a symmetry implicit in a standard QED treatment implies that there is another contribution to turbulent bremsstrahlung in addition to that calculated by Tsytovich, Stenflo, and Wilhelmsson and that the two contributions cancel exactly, leading to the null result. (Our arguments on this point have proved controversial.) Nambu made an approximation inconsistently, and when this approximation is not made, two terms in his analytic treatment cancel exactly. We argue that turbulent bremsstrahlung is related to a radiative correction in which the resonant emission of ion sound turbulence is modified by the nonresonant emission and absorption of Langmuir waves. Physically we interpret the nonexistence of turbulent bremsstrahlung as being due to each emission of a Langmuir quantum being associated with an absorption of an identical Langmuir quantum so that the Langmuir turbulence is unchanged. Proposed astrophysical applications of turbulent bremsstrahlung need to be reconsidered.

Subject headings: radiation mechanisms — turbulence

I. INTRODUCTION

In 1971 Tsytovich proposed that the interaction of fast particles with ion-sound turbulence would lead to the emission of electromagnetic waves (calling the process "plasma-bremsstrahlung"). In 1975 Tsytovich, Stenflo, and Wilhelmsson considered a related process in which the resonant interaction of particles with ion-sound waves produces Langmuir waves; they introduced the name "turbulent bremsstrahlung" (hereafter TB) for such resonant interactions. The process is of second order in the wave energy densities in a weakly turbulent plasma. Processes of this order are obtained from a perturbation solution to the Vlasov-Poisson equations for the induced charge density (Tsytovich, Stenflo, and Wilhelmsson 1975) or the induced current density (Melrose 1982) and involve up to third-order responses in the amplitude of the wave field. Alternatively, the relevant expressions can be obtained from a perturbation solution to the Vlasov cumulant hierarchy (Davidson 1972) up to second order in the energy densities of the wave fields. In this order the following distinct processes exist (e.g., see Davidson 1972, p. 293).

1. Three-wave coupling: the required resonance conditions are

$$\omega_1 = \omega_2 + \omega_3 , \qquad \boldsymbol{k}_1 = \boldsymbol{k}_2 + \boldsymbol{k}_3 . \tag{1}$$

Here ω_j, k_j are, respectively, the angular frequency and wave vector of wave j.

2. Wave scattering on particles (including the double-emission process of Melrose 1982) requiring

$$\omega_1 - \boldsymbol{k}_1 \cdot \boldsymbol{v} = \pm(\omega_2 - \boldsymbol{k}_2 \cdot \boldsymbol{v}) , \tag{2}$$

where $\boldsymbol{v}$ is the velocity of the scattering particle.

3. Turbulent bremsstrahlung arising from the resonance

$$\omega_1 - \boldsymbol{k}_1 \cdot \boldsymbol{v} = 0 \tag{3}$$

for one of the waves partaking in the second-order wave-particle scattering.

The physical justification for the existence of TB seems attractive: particles which are in resonance with the low-frequency waves could be efficiently retarded and radiate a variety of high-frequency wave modes due to their accelerated motion. The analogy with ordinary bremsstrahlung arises from the specific form of the wave-particle interaction: now the electrons would "collide" with resonant waves instead of ions, and the associated emission of the high-frequency waves is nonresonant. Subsequently the expression for the growth rate found by Tsytovich, Stenflo, and Wilhelmsson (1975) was confirmed by Kuijpers (1980*a*) and Melrose (1982).

TB of this form was applied to explain the X-ray background from the resonant interaction of cosmic-ray protons with low-frequency turbulence (Tsytovich 1973), to the generation of Langmuir waves from magnetospheric turbulence (Rönnmark and Stenflo 1976), to the acceleration of electrons in solar flare current sheets (Kaplan, Pikel'ner and Tsytovich 1974; Hoyng 1977; Benz 1977; Kuijpers 1978), to nonlinear absorption during plasma heating, to radiation from pulsars (Tsytovich 1980), and to collisionless perpendicular shocks (Klinkhamer and Kuijpers 1981). The validity of these applications to the problems of acceleration of electrons in flares was questioned by Vlahos and Papadopoulos (1979). They did not question the fundamental existence of TB but rather argued that other second-order processes would dominate the TB effect. However, since the various second-order processes require different resonance conditions (1)–(3), the relative importance of the various processes is determined by the detailed shapes of the wave spectra (Kuijpers 1980*b*; Tsytovich, Stenflo, and Wilhelmsson 1981). Further, Vlahos and Papadopoulos (1979) incorrectly identified the reverse process of TB as the Dawson-Oberman resistivity, which, in fact, is a special form of the three-wave process given in (1) (Dawson and Oberman 1963; Dawson 1968; Kuijpers 1980*b*). An alternative form of TB was calculated by Nambu and Shukla (1979, 1980) and Nambu (1981*a*) who called the processes "induced bremsstrahlung" and later (Nambu 1983) a new "plasma maser effect." These authors found substantially larger growth rate for the process than first found by Tsytovich, Stenflo, and Wilhelmsson (1975) and claimed that the dominant terms had been omitted in previous treatments.

There seem to be two contributions to TB. In the notation of Melrose (1982) these two contributions arise from two contributions $\alpha_{ij}^{\mathrm{NL1}}(k)$ and $\alpha_{ij}^{\mathrm{NL2}}(k)$ to the nonlinear correction to the linear response tensor. In Melrose's notation SI units are used, k denotes $(\boldsymbol{k}, \omega)$, a test field is described in terms of its vector potential $\boldsymbol{A}(k)$ in the temporal gauge $[\phi(k) = 0]$, the inhomogeneous wave equation is written

$$\Lambda_{ij}(k)A_j(k) = -\frac{\mu_0 c^2}{\omega^2} J_i^{\mathrm{ext}}(k) , \tag{4}$$

and the extraneous current $\boldsymbol{J}^{\mathrm{ext}}$ is identified with cubic terms in the weak turbulence expansion

$$J_i(k) = \alpha_{ij}(k)A_j(k) + \int d\lambda^{(2)}\alpha_{ijl}(k, k_1, k_2)A_j(k_1)A_l(k_2) + \int d\lambda^{(3)}\alpha_{ijlm}(k, k_1, k_2, k_3)A_j(k_1)A_l(k_2)A_m(k_3) + \cdots , \tag{5}$$

where

$$d\lambda^{(n)} = \frac{d^4k_1}{(2\pi)^4} \times \cdots \times \frac{d^4k_n}{(2\pi)^4} (2\pi)^4\delta^4(k - k_1 \cdots - k_n) \tag{6}$$

is the n-fold convolution integral. One nonlinear correction $\alpha_{ij}^{\mathrm{NL1}}(k)$ arises from the cubic response $\alpha_{ijlm}(k, k_1, k_2, k_3)$ evaluated at $k_3 = -k_1, k_2 = k$, and integrated over the ion sound waves with wave four-vector k_1. This leads to Tsytovich *et al.*'s form of TB. The other nonlinear correction $\alpha_{ij}^{\mathrm{NL2}}(k)$ arises from an effective cubic response involving two quadratic responses in the form $\alpha_{ija}(k, k_1, k - k_1)\Lambda_{ab}^{-1}(k - k_1)\, \alpha_{blm}(k - k_1, k_2, k_3)$, where Λ^{-1} is the inverse of the operator Λ in equation (4). This leads to Nambu's form of TB. Analogous terms appear in Davidson's kinetic equation for the waves, e.g., equation (2) in Kuijpers (1980*a*, in Gaussian units).

Nambu's version of TB was applied to the generation of whistlers from ion-sound (Nambu *et al.* 1980), of Langmuir waves from ion-sound (Nambu 1981*b*), to magnetically and inertially confined fusion plasma (Nambu 1980*a*, 1983), to the generation of electromagnetic waves from ion cyclotron waves (Nambu 1982), to the production of low-frequency Alfvén waves from high-frequency whistler turbulence (Nambu 1980*b*), to the auroral kilometric radiation (Bujarbarua and Nambu 1983; Bujarbarua, Sarma, and Nambu 1983), to the emission of solar-type III radio bursts (Nambu and Shukla 1983), to the explanation of high-frequency electrostatic bursts (Fujiyama and Nambu 1983), and to radiation below the plasma frequency (Finken and Ackermann 1982), both observed in laboratory experiments. Further, approximate energy and momentum conservation laws were derived for the waves and particles partaking in TB (Nambu and Terasawa 1982), which had been proved already rigorously for any second-order process by Davidson (1972, § 14.4.1). In a second attempt "to reveal the carefully hidden and often completely misunderstood limitations of the theoretical concepts involved," Vlahos and Papadopoulos (1982) used the result of Nambu (1981*a*) without further discussion of its validity.

In this paper we argue that neither of the two forms of TB, namely, those of Tsytovich, Stenflo, and Wilhelmsson (1975) and of Nambu (1981*a*), exist. Our argument on the nonexistence of Tsytovich *et al.*'s form (§ III) has proved controversial. The basis of our argument is a treatment of TB using quantum electrodynamics (QED), as given in § II. The controversial point concerns the imposition of a symmetry property; we discuss the controversy further in a postscript at the end of § V. On imposing this symmetry property one adds a contribution equal and opposite to that retained by Tsytovich *et al.* Our proof of the nonexistence of Nambu's TB mechanism is given in § IV: Nambu made an approximation inconsistently in his calculation, and when this is corrected, two contributions cancel exactly. This is in agreement with Kuijpers (1980*a*) who found the first term in his equation (2), which corresponds to Nambu's most important contribution, to give zero; Tsytovich (1980) and Melrose (1982) also examined the mechanism proposed by Nambu (1981*a*) and found a null result. (Since this paper was first written we have become aware of similar criticisms of Nambu's mechanism by Hirose 1984 and DuBois and Pesme 1984.) Finally, in § V we discuss our result and suggest a possible alternative to TB for the upconversion of ion sound to Langmuir turbulence in the form of the so-called double-emission process.

II. INTERPRETATION OF TURBULENT BREMSSTRAHLUNG

It is helpful to introduce a diagrammatic approach, familiar in QED, in order to understand the distinction between the three types of second-order process described by the resonance conditions (1)–(3), and in order to clarify the distinction between Nambu's and Tsytovich *et al.*'s forms of TB. The diagrams may be used in two different ways. One way corresponds to the amplitude for a process and has the initial state and final state on the right and left, respectively. The other way corresponds to the rate of the process and has the initial and final states at cuts in a closed loop diagram. These two types of diagram are connected, in that a cut separates the loop diagram into two sections corresponding to the two amplitudes whose product gives the rate. The elements in an amplitude are as follows (e.g., Melrose 1980*b*, § 10.4):

1. A solid line represents a particle.

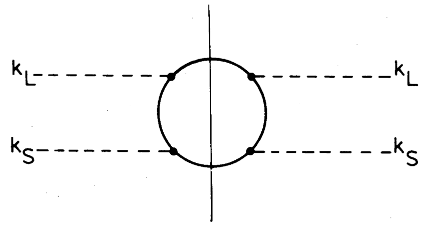

FIG. 1*a*

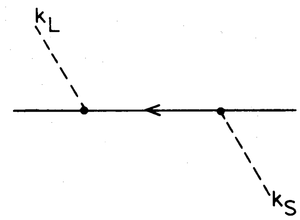

FIG. 1*b*

FIG. 1.—Thomson scattering is described by a single four-vertex loop cut in half (*a*) so that the two halves correspond to the same amplitude diagram (*b*). From a classical viewpoint the position of the photon lines above and below the electron line in (*b*) is of no significance.

2. A dashed line represents a photon in the initial or final state (a "real" photon, i.e., satisfying the relevant linear dispersion relation).
3. A squiggle represents an internal photon line (a "virtual" photon, which need not satisfy a dispersion relation).
4. A hatched circle with n photon lines (real or virtual) emanating from it represents an $n - 1$–fold nonlinear response.
5. A dot denotes a vertex, i.e., a join between a photon line (real or virtual) and a particle line or a circle.

The rules for constructing amplitude diagrams are the following:

1. Particle lines are horizontal.
2. Real photon lines join the particle line at vertices. (In QED the line for a photon in the initial state extends from the bottom of the diagram to the dot, and for a final photon from the dot to the top of the diagram, but this distinction cannot be made so clearly in a classical theory.)
3. Virtual photon lines start and end at dots.
4. The real photon lines are labeled (e.g., with their $\boldsymbol{k}$-vector), and any permutation of photon lines with different labels (changing the order of the dots around a circle and along a line) represents a distinct diagram.

To these we add rules for constructing diagrams corresponding to rates:

5. The particle line is a closed loop which is cut at two points so that the two parts of the diagram are topologically equivalent to the two amplitude diagrams corresponding to the two amplitudes whose product gives the rate.
6. Any uncut loop is to be reinterpreted as a nonlinear response (cf. No. 4 under "elements").

In a field theory a cut in a particle line implies that the particle is on its mass shell (Landau 1959; Cutkosky 1960). Classically the cut determines the resonance condition, which is the Cerenkov condition (eq. [3]) when only the wave 1 is on one side of the cut, and is equation (2) when the waves 1 and 2 are both on the same side of the cut.

We are concerned only with processes of second order in the rate. The processes of this order are Thomson scattering together with nonlinear scattering which interfers with it, three-wave coupling, and TB. Thomson scattering of ion-sound into Langmuir waves can be described by the diagram in Figure 1*a*. The indicated cut, corresponding to the resonance condition $\omega_L - \boldsymbol{k}_L \cdot \boldsymbol{v} = \omega_S - \boldsymbol{k}_S \cdot \boldsymbol{v}$ (cf. eq. [2]), then leads to the diagrams for the amplitude in Figure 1*b*. The diagram for the process of three-wave coupling is given in Figure 2*a* for the case of coalescence of Langmuir waves with ion-sound into Langmuir waves where the cut is associated with the resonance condition (eq. [1]) with $L' = 1$, $L'' = 2$, and $S = 3$. As explained by Melrose (1982), there are three other cuts which can be made in Figure 2*a*, one of which corresponds to nonlinear scattering and the other two to interference between nonlinear and Thomson scattering. These are illustrated in Figures 3*a*–3*e*, respectively. Note that in Figures 3*b* and 3*c* the cuts are asymmetric; in this case the rate involves the product of two different amplitudes, both of which are second order.

The diagrams for TB can be derived from the algebraic expressions obtained from equations (5) and (4): the intrinsic cubic response, which leads to the form of Tsytovich, Stenflo, and Wihelmsson (1975), is represented by the diagram in Figure 4*a* where the

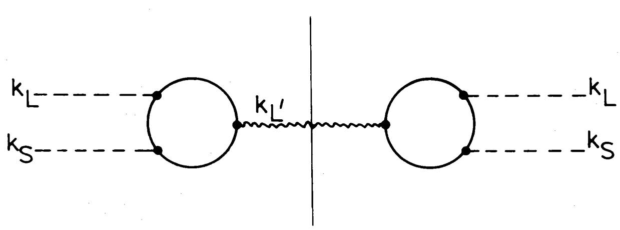

FIG. 2*a*

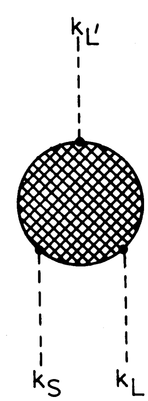

FIG. 2*b*

FIG. 2.—The diagram corresponding to a three-wave coupling involves two three-vertex loops joined by a real photon line which is cut (*a*) to produce two identical amplitude diagrams (*b*).

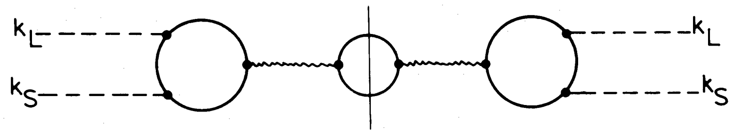

FIG. 3*a*

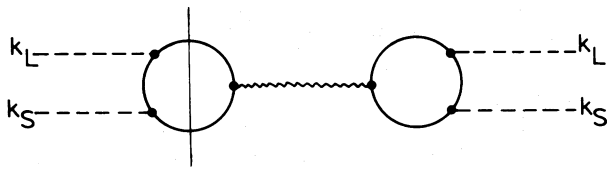

FIG. 3*b*

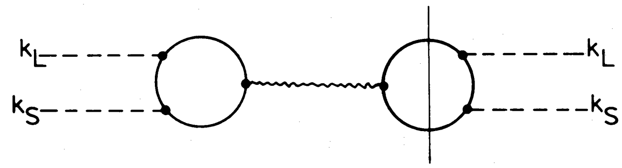

FIG. 3*c*

FIG. 3.—Nonlinear scattering corresponds to a diagram analogous to Fig. 2*a*, but cut at a particle resonance in the photon propagator (*a*), and the interference between Thomson and nonlinear scatterings corresponds to two similar diagrams cut asymmetrically at a three-vertex loop (*b*), (*c*).

cut corresponds to the response condition in equation (3): $\omega_S - \boldsymbol{k}_S \cdot \boldsymbol{v} = 0$. As with the interference between Thomson and nonlinear scattering, the rate is now given, not by the square of one amplitude, but by the product of the different amplitudes represented in Figure 4*b*; i.e., by the product of a first- and a third-order amplitude.

The dominant contribution to the TB of Nambu (1981*a*) arises from the effective cubic response due to two quadratic responses and is represented by the diagrams in Figure 5. Again the cut in the diagram is asymmetric, and the rate is given by a first-order times a third-order amplitude.

The diagrams in Figures 4*a* and 4*b* are closely related to a diagram mentioned by Ross (1969, Fig. 10), who developed a quantum theory for nonlinear plasma processes. Ross's diagram is Figure 6*a*, which corresponds to the two amplitudes in Figure 6*b*. Ross interpreted this diagram as a radiative correction to the emission process, here a radiative correction to the emission of ion sound waves. In our case the "radiative correction" is due to the emission and reabsorption of a real Langmuir quantum rather than a virtual quantum in Ross's case. As with a virtual quantum, it is essential that the wave $\boldsymbol{k}$-vectors of the two Langmuir quanta be equal and opposite. Classically this arises on phase averaging, see equation (12) below, which reduces the number of independent $\boldsymbol{k}$-vectors from four to two. Classically the interpretation of k_L and $-k_L$ in Figure 4*b* is confused by negative and positive frequencies being equivalent, with a real wave described by both positive and negative frequencies. In QED the positive and negative frequencies are associated with absorption and emission, respectively. According to the QED interpretation, Figure 4*b* implies that every time a Langmuir quantum is emitted, an identical one is absorbed. Hence there is no change in the Langmuir turbulence due to the process described by Figure 4*a*.

Tsytovich *et al.*'s form of TB is the same physical process as this radiative correction to the emission of ion sound turbulence, but is viewed from its effect on the Langmuir turbulence. The equivalence of TB and the radiative correction are obvious from the QED viewpoint, in that they are described by the same diagram. We have confirmed this classically by calculating the radiative correction and showing it to involve the same function as in Tsytovich *et al.*'s form of TB; this calculation is not particularly instructive, and we do not include it here. However, a relevant point is that this related process does exist, and our argument that TB does not exist must be compatible with this fact.

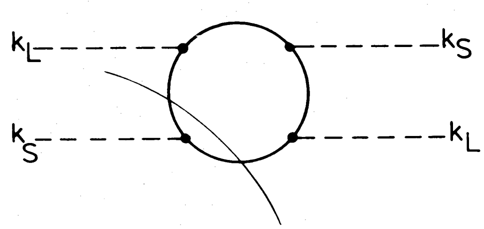

FIG. 4*a*

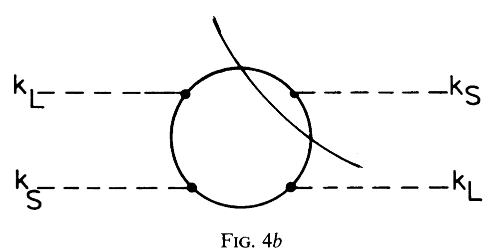

FIG. 4*b*

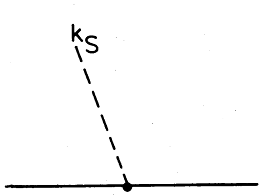

FIG. 4*c*

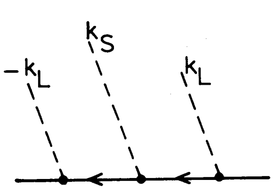

FIG. 4*d*

FIG. 4.—Tsytovich *et al.*'s form of TB corresponds to a four-vertex loop cut around a single vertex corresponding to an ion-sound wave; there are two such diagrams (*a*), (*b*), and the sum of their contributions is to be taken. Once cut, each diagram corresponds to a first-order amplitude (*c*) and a third-order amplitude (*d*). The important symmetry corresponds to interchanging k_L and $-k_L$ in (*d*).

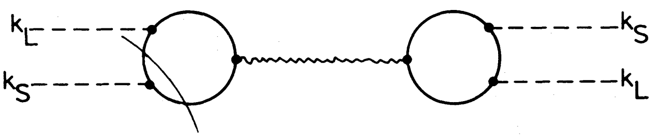

FIG. 5*a*

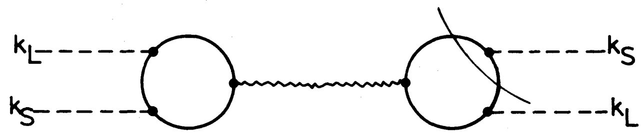

FIG. 5*b*

FIG. 5.—Nambu's form of TB corresponds to two connected three-vertex loops, as in Fig. 2*a*, cut around a vertex corresponding to an ion sound wave; again there are two such diagrams (*a*), (*b*).

The nonexistence of Tsytovich *et al.*'s form of TB follows from the implication that for each emission of a Langmuir quantum an identical Langmuir quantum is absorbed. In the classical theory this balance follows by symmetrizing over the waves described by k_L in Figure 4*a*. This symmetry was not imposed by Tsytovich *et al.*, and when it is imposed, one obtains an additional contribution equal and opposite to the one they retained, as we now show.

III. NONEXISTENCE OF TSYTOVICH, STENFLO, AND WILHELMSSON'S MECHANISM

A symmetry of the $\boldsymbol{S}$-matrix formalism of QED and of an analogous Lagrangian theory for classical electrodynamics (Appendix C) was not imposed by Tsytovich *et al.*, e.g., in their equations (6a) and (7). Here (see also Appendix A) we repeat their calculation in our notation, pointing out where the symmetry applies.

We assume that all waves are longitudinal and are described by the Fourier transform $\phi(k)$ of their electrostatic potential, with k denoting $(\omega, \boldsymbol{k})$. The response of the plasma is described by expanding the charge density $\rho(k)$ in powers of ϕ:

$$\rho(k) = \chi(k)\phi(k) + \int d\lambda^{(2)}\chi(k, k_1, k_2)\phi(k_1)\phi(k_2) + \int d\lambda^{(3)}\chi(k, k_1, k_2, k_3)\phi(k_1)\phi(k_2)\phi(k_3) + \cdots , \tag{7}$$

which is equivalent to the longitudinal part of equation (5). The response tensors χ are calculated in Appendix B using the Vlasov equations. The emission and absorption of radiation may be treated in terms of the time-averaged rate P that work is done on the electric field (e.g., Melrose 1980*a*, pp. 65 and 67), and here this reduces to (Appendix A)

$$P = -\lim_{T\to\infty} \frac{1}{T} \int \frac{d^4k}{(2\pi)^4} \text{Re}\,[i\omega\rho(k)\phi(-k)] , \tag{8}$$

where T is the normalization time and "Re" denotes the real part of the expression. The term of relevance in Tsytovich *et al.*'s mechanism arises from the cubic response in equation (7):

$$P = -\lim_{T\to\infty} \frac{1}{T} \int \frac{d^4k}{(2\pi)^4} \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{d^4k_3}{(2\pi)^4} (2\pi)^4\delta^4(k - k_1 - k_2 - k_3)[i\omega\chi(k, k_1, k_2, k_3)\phi(-k)\phi(k_1)\phi(k_2)\phi(k_3)] . \tag{9}$$

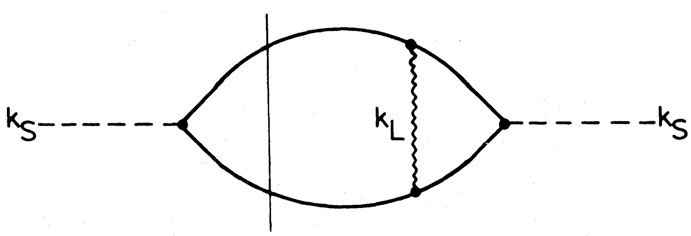

FIG. 6*a*

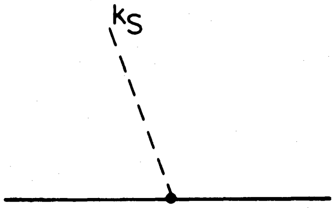

FIG. 6*b*

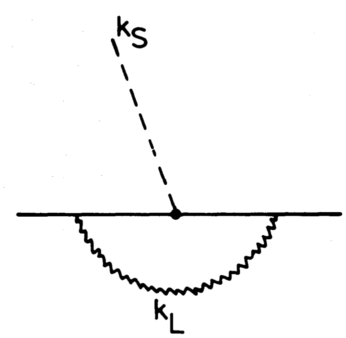

FIG. 6*c*

FIG. 6.—(*a*) That of Ross (1969, Fig. 10). The corresponding amplitude diagrams (*b*), (*c*) are analogous to Fig. 4*c* and 4*d*, except in that the Langmuir wave is virtual rather than real.

We separate $\phi(k)$ into parts $\phi_L(k)$ and $\phi_S(k)$ describing the Langmuir waves and ion sound waves, respectively, and assume that each is of the form

$$\phi_M(k) = \Phi_M(\boldsymbol{k}) 2\pi\delta[\omega - \omega_M(\boldsymbol{k})] \exp[i\psi_M(\boldsymbol{k})] , \tag{10}$$

with

$$\omega_M(-\boldsymbol{k}) = -\omega_M(\boldsymbol{k}) , \tag{11}$$

and where $\Phi_M(\boldsymbol{k})$ is the amplitude and $\psi_M(\boldsymbol{k})$ is the phase of the wave. In general, we must distinguish between "forward" and "backward" waves before imposing equation (11) (Kuijpers 1980*a*), and we do so explicitly in Appendix A. On substituting equation (10) into equation (7), we get various contributions to $\rho^{(3)}(k)$, where k refers to the Langmuir waves if one is interested in TB of Langmuir waves. Each contribution consists of a different arrangement of the three potentials $\phi(k_1)\phi(k_2)\phi(k_3)$, with one of them describing Langmuir waves and the other two describing ion sound waves. Upon a suitable change of the integration variables k_1, k_2, k_3, we can identify the argument k_2 with the Langmuir wave and k_1 and k_3 with the ion sound wave, provided we now use the expression for χ which is fully symmetrized over the arguments k_1, k_2, and k_3.

We now average (denoted by angle brackets) over the phases of the ion sound waves:

$$\langle \phi_S(k_1)\phi_S(k_3) \rangle = \frac{|\Phi_S(\boldsymbol{k})|^2}{V} 2\pi\delta[\omega_1 - \omega_S(\boldsymbol{k}_1)](2\pi)^4\delta^4(k_1 + k_3) . \tag{12}$$

Then, rewriting $[\delta(\omega - \omega_S(\boldsymbol{k})]^2$ as $T/2\pi$ times $\delta[\omega - \omega_S(\boldsymbol{k})]$, equation (9) gives

$$P = -\int \frac{d^4k}{(2\pi)^4} \frac{d^4k_1}{(2\pi)^4} |\Phi_L(\boldsymbol{k})|^2 |\Phi_S(\boldsymbol{k}_1)|^2 2\pi\delta[\omega - \omega_L(k)] \times 2\pi\delta[\omega_1 - \omega_S(\boldsymbol{k}_1)] \operatorname{Re}[i\omega\chi(k, k_1, k, -k_1)] . \tag{13}$$

The power P as given by equation (13) is related to Tsytovich *et al.*'s analysis as follows: taking the imaginary part of their $\epsilon_k^{\rm NL}$ as given by their equation (6) with equations (6a), (7), or (8), multiplying by their $|E_k^l|^2$, and integrating over their k, one obtains an expression equivalent to P. The existence of TB would imply $P \neq 0$.

The direct calculation of $\chi(k, k_1, k_2, k_3)$, before imposition of any symmetrization, leads to (see eq. [B7] of Appendix B)

$$\chi(k, k_1, k_2, k_3) = -e^4 \int d^3\boldsymbol{p} \frac{1}{\omega - \boldsymbol{k}\cdot\boldsymbol{v}} \boldsymbol{k}_1 \cdot \frac{\partial}{\partial \boldsymbol{p}} \left\{ \frac{1}{\omega_2 + \omega_3 - (\boldsymbol{k}_2 + \boldsymbol{k}_3)\cdot\boldsymbol{v}} \boldsymbol{k}_2 \cdot \frac{\partial}{\partial \boldsymbol{p}} \left[\frac{1}{\omega_3 - \boldsymbol{k}_3\cdot\boldsymbol{v}} \boldsymbol{k}_3 \cdot \frac{\partial f(\boldsymbol{p})}{\partial \boldsymbol{p}} \right] \right\} . \tag{14}$$

As argued above, one should use the expression for χ which is symmetrized over k_1, k_2, and k_3, which is $\frac{1}{6}$ of the sum of the six expressions obtained from equation (14) by permuting the subscripts 1, 2, and 3. Then, provided no resonant parts are taken (denoted "nonres"), the symmetrized (denoted by a bar) form satisfies (Melrose 1972)

$$\text{nonres}\,[\bar{\chi}(k, k_1, k_2, k_3)] = \text{nonres}\,[\bar{\chi}(-k_1, -k, k_2, k_3)] , \tag{15}$$

as well as the imposed symmetries

$$\bar{\chi}(k, k_1, k_2, k_3) = \bar{\chi}(k, k_1, k_3, k_2) = \bar{\chi}(k, k_2, k_1, k_3) .$$

The relevant symmetries here follow from equation (9) with $\phi(-k)\phi(k_1)\phi(k_2)\phi(k_3)$ replaced by $\phi_L(-k)\phi_S(k_1)\phi_L(k_2)\phi_S(k_3)$, and then with $k_3 = -k_1$ and $k_2 = k$. Thus, the symmetries involve the following interchanges of argument: $k_1 \leftrightarrow k_3$ and $-k \leftrightarrow k_2$. The nonresonant part of χ does not contribute in equation (13), and, following Tsytovich, Stenflo, and Wilhelmsson (1975), the relevant resonant (res) part is that at $\omega_3 - \boldsymbol{k}_3 \cdot \boldsymbol{v} = 0$. Equation (14) implies

$$\text{res}\,[\chi(k, k_1, k_2, k_3)] = i\pi e^4 \int d^3\boldsymbol{p}\,\delta(\omega_3 - \boldsymbol{k}_3\cdot\boldsymbol{v})\boldsymbol{k}_3 \cdot \frac{\partial f(\boldsymbol{p})}{\partial \boldsymbol{p}} \boldsymbol{k}_2 \cdot \frac{\partial}{\partial \boldsymbol{p}} \left[\frac{1}{\omega_2 + \omega_3 - (\boldsymbol{k}_2 + \boldsymbol{k}_3)\cdot\boldsymbol{v}} \boldsymbol{k}_1 \cdot \frac{\partial}{\partial \boldsymbol{p}} \frac{1}{\omega - \boldsymbol{k}\cdot\boldsymbol{v}} \right] \tag{16}$$

for this resonant part. Now, on setting $k_3 = -k_1$, one finds that the symmetry $k_1 \leftrightarrow k_3$ implies that the terms which are odd number $k_1 \to -k_1$ do not contribute, as noted by Tsytovich *et al.* Note also that the factor ω in the integrand in equation (13) implies that only the part of $\chi(k, k_1, k, -k_1)$ which is *odd* under $k \to -k$ contributes to P. This point is discussed further in Appendix A.

The symmetry implied by our QED argument requires that we retain on the part of $\chi(k, k_1, k, -k_1)$ which is *even* under $k \to -k$. This part gives zero when inserted in equation (13), and this null result for P implies the nonexistence of Tsytovich *et al.*'s form of TB.

The additional symmetry implied by the *S*-matrix form of QED is not imposed explicitly in any of the standard classical treatments of weak turbulence theory, e.g., Kadomstev (1965), Sagdeev and Galeev (1969), Tsytovich (1970), Davidson (1972), DuBois (1976). In QED the symmetrization over like fields is made explicitly. The classical theories need to be modified by requiring that this symmetry be imposed. We discuss this point in Appendix C where we give a purely classical argument leading to the relevant symmetry.

A more formal discussion of the symmetry properties of the nonlinear response tensors is given elsewhere (Melrose and Kuijpers 1984). Hitherto the following has been assumed: the resonant part of $\chi(k, k_1, k_2, k_3)$ satisfies a symmetry opposite to that of the nonresonant part; i.e., for the resonant part there is a minus sign on the right-hand side of the counterpart of equation (15). However, this is not correct in general and is not correct in the case of relevance here. The response function $\chi(k, k_1, k_2, k_3)$ has resonances at $\omega - \boldsymbol{k}\cdot\boldsymbol{v} = 0$, $\omega_1 - \boldsymbol{k}_1\cdot\boldsymbol{v} = 0$, $\omega_2 - \boldsymbol{k}_2\cdot\boldsymbol{v} = 0$, and $\omega_3 - \boldsymbol{k}_3\cdot\boldsymbol{v} = 0$, and also other resonances at $(\omega - \boldsymbol{k}\cdot\boldsymbol{v}) - (\omega_1 - \boldsymbol{k}_1\cdot\boldsymbol{v}) = 0$ and so on, which are of no interest here. Let us denote these as "res_0", "res_1" "res_2", and "res_3", respectively. The result found by Melrose and Kuijpers (1984) is equivalent to the following. Consider $k \leftrightarrow -k_1$, as in equation (15): then for "res_2" and "res_3" the symmetry property has the same sign as in equation (15), and for "res_0" and "res_1" the opposite sign applies. One important point is that there is no unique symmetry property for the nonresonant parts; different nonresonant parts have different symmetries. In brief, if neither of the interchanged k's (k and k_1 in eqn. [15]) is resonant, then the sign is as in equation (15), and if either is resonant, then the opposite sign applies. The cases of interest here are for $\mathrm{res}_1\ \chi(k, k_1, k_2, k_3)$ and $\mathrm{res}_3\ \chi(k, k_1, k_2, k_3)$ under interchange of the nonresonant k's. The appropriate symmetries have the positive sign:

$$\mathrm{res}_1\ \chi(k, k_1, k_2, k_3) = \mathrm{res}_1\ \chi(-k_2, k_1, -k, k_3)\ ,$$

$$\mathrm{res}_3\ \chi(k, k_1, k_2, k_3) = \mathrm{res}_3\ \chi(-k_2, k_1, -k, k_3)\ .$$

Together these imply

$$\mathrm{res}_1\ \chi(k, k_1, k, -k_1) = \mathrm{res}_1\ \chi(-k, k_1, -k, -k_1)\ , \tag{17a}$$

which is the symmetry property relevant for TB. Thus, $\mathrm{res}_1\ \chi(k, k_1, k, -k_1)$ is even under $k \to -k$, but because only the odd part contributes to P in equation (13), the net result is null; i.e., $P = 0$. On the other hand, the other two resonant parts have symmetries with the opposite sign and combine to give

$$\mathrm{res}_0\ \chi(k, k_1, k, -k_1) = -\mathrm{res}_0\ \chi(-k, k_1, -k, -k_1)\ . \tag{17b}$$

A resonance in $\boldsymbol{k}$, i.e., in the "response" $\rho(k)$, is not usually considered in classical theories. If we relabel k as k_1 and k_1 as k in equation (17b) and then insert the resulting expression in equation (13), we obtain a nonzero result for P, which now refers to the power in ion sound waves. The corresponding physical process may be identified as a radiative correction to the Cerenkov emission of ion sound waves, which is the process that Ross (1969) mentioned.

IV. NONEXISTENCE OF NAMBU'S MECHANISM

The dominant term in Nambu (1981*a*) arises from an incorrect evaluation of the last term in his equation (12). Apart from the one-dimensional form and different normalization the imaginary part of this term, arising from the resonance $\omega_S - \boldsymbol{k}_S \cdot \boldsymbol{v} = 0$, is essentially equal to the last term in equation (2) of Kuijpers (1980*a*) which Kuijpers found to give zero, i.e., not to contribute to TB. Also, Melrose (1982) indicated that no relevant contribution arose from his nonlinear response tensor $\alpha_{ij}^{\mathrm{NL2}}$ (his eq. [A5]), and Tsytovich (1980; see comments preceding his eq. [14]) came to the same conclusion, thus anticipating a null result for the term isolated by Nambu. In evaluating the last term in his equation (12), Nambu made the approximations $(\omega_S - \boldsymbol{k}_S \cdot \boldsymbol{v})^{-1} + [\omega_L - \omega_S - (\boldsymbol{k}_L - \boldsymbol{k}_S) \cdot \boldsymbol{v}]^{-1} \approx (\omega_S - \boldsymbol{k}_S \cdot \boldsymbol{v})^{-1}$ and $(\boldsymbol{k}_S \cdot \boldsymbol{v} - \omega_S)^{-1} + (\omega_L - \boldsymbol{k}_L \cdot \boldsymbol{v})^{-1} \approx (\boldsymbol{k}_S \cdot \boldsymbol{v} - \omega_S)^{-1}$ (see his eqs. [21] and [22]) which are correct for the imaginary part of each factor but not for the real part. If these approximations are not made and one proceeds along Nambu's line, the result obtained is zero, as shown in Appendix D (see also Hirose 1984 and Du Bois and Pesme 1984).

The nonexistence of Nambu's mechanism may also be deduced in terms of symmetry properties of nonlinear response tensors. Suppose we take the quadratic response $\rho^{(2)}(k - k_1)$ to fields at k_2 and k_3, find the associated $\phi^{(2)}(k - k_1)$ by inverting the relation $\rho(k) = \chi(k)\phi(k)$ in equation (7) expressing the linear response, and then take the quadratic response to $\phi^{(2)}(k - k_1)$ and $\phi(k_1)$. The result is of the same form as the cubic response in equation (7), but with an effective response tensor

$$\chi^{\mathrm{eff}}(k, k_1, k_2, k_3) = \frac{\chi(k, k_1, k - k_1)\chi(k - k_1, k_2, k_3)}{\chi(k - k_1)} \tag{18}$$

involving two quadratic responses, as discussed in § I. As before, we set $k_3 = -k_1$, $k_2 = k$, take the resonance $\omega_3 - \boldsymbol{k}_3 \cdot \boldsymbol{v} = 0$, and then impose the symmetrizations $k_1 \leftrightarrow k_3$, $-k \leftrightarrow k_2$. In this case we have the symmetry properties

$$\chi(k - k_1, k, -k_1) = \chi[-k, -k_1, -(k - k_1)] = \chi^*(k, k_1, k - k_1)\ , \tag{19}$$

where the asterisk (*) denotes complex conjugation; the nonresonant parts of $\chi(k, k_1, k_2)$ and $\chi(k)$ are real, and the resonant parts are imaginary. It then follows that the resonances at k_1 and k_3 in equation (18) gives equal and opposite contributions. The implied cancellation is shown explicitly in Appendix D using Nambu's notation.

V. DISCUSSION AND CONCLUSIONS

We conclude that TB does not exist in the forms currently proposed in the literature. The specific process which does not exist is TB defined as follows: (i) the upconversion of low-frequency waves to high-frequency waves through (ii) a second-order process (same order as Thomson scattering) in weak-turbulence theory in which (iii) the particles are resonant with the low-frequency waves. The conclusion that Nambu's version of TB does not exist has been stated independently by DuBois and Pesme (1984) and Hirose (1984); it was also implicit in earlier work (Kuijpers 1980*a*; Tsytovich 1980; Melrose 1982). Our conclusion that Tsytovich *et al.*'s mechanism does not exist is likely to be controversial, especially in view of the rapidly expanding interest in various possible applications, as summarized in § I, and in view of increasing theoretical interest in the mechanism (e.g., DuBois and Pesme 1984).

Our formal arguments for the nonexistence of TB centers around a symmetry property which is imposed explicitly in QED and which is not imposed in standard versions of weak-turbulence theory (e.g., Tsytovich 1970; Davidson 1972; DuBois 1976). QED is the modern-day theory for electrodynamics, and this inconsistently implies that weak turbulence theory, as currently formulated, is deficient. It is elementary to rectify this deficiency by imposing the relevant symmetry. We give a classical argument in Appendix C, implying that this symmetry should be imposed. The symmetry is over like fields when one corresponds to a "disturbance" and the other to a "response" in weak turbulence theory; e.g., when k_2 and k, respectively, in equation (5) or (7) describe the same field (as they do in TB). The net and only effect of imposing this symmetry is to add to the calculated growth rate for TB an equal and opposite contribution, leading to the null result.

Another way of expressing our argument is based on Figures 4*a* and 4*b* which are Feynman-like diagrams for a radiative correction to the Cerenkov emission of the low-frequency waves (Ross 1969). The cuts in these diagrams define the initial and final states, and the difference in the energy of the particle between these two states must be the same irrespective of which way around the diagram one proceeds. Suppose, proceeding from the initial (*i*) to the final (*f*) state, the shorter way corresponds to a low-frequency quantum being absorbed: then conservation of energy implies $\epsilon_f - \epsilon_i = \hbar\omega_S$. Proceeding the other way, we must get the same result: $\epsilon_f - \epsilon_i = \hbar\omega'_L + \hbar\omega_S - \hbar\omega_L$, where the prime applies to $\omega_L(k_2)$ with $k_2 = k$, and hence we must have $\omega'_L = \omega_L$. Thus the high-frequency quanta have equal and opposite frequencies, implying that one is absorbed and that the other is emitted. In other words, the high-frequency quanta are emitted and absorbed in identical pairs. Therefore there can be no net change in their distribution. We claim that TB, as proposed by Tsytovich, Stenflo, and Wilhelmsson (1975), corresponds to this process but without the correct symmetry being imposed, so that the required emission and absorption in identical pairs is erroneously violated.

A question which has been posed by Nambu (1984, private communication) is whether our results apply to TB in the presence of a magnetic field. Although we have not discussed the magnetized case explicitly, we believe that our arguments are unaffected by a magnetic field. The argument is (*a*) only the part of $\chi(k, k_1, k, -k_1)$ which is an odd function of k contributes in equation (13), and (*b*) the symmetry discussed above requires that we symmetrize so that $\chi(k, k_1, k, -k_1)$ is an even function of k. Hence, the inclusion of a magnetic field should not alter the conclusion that TB does not exist.

In § I a physical interpretation of TB is mentioned in which an electron radiates the high-frequency waves due to its accelerated motion in the fields of the low-frequency waves. From a quantum mechanical viewpoint, this process can exist only as a higher order process in which $N = \omega_L/\omega_S$ low-frequency quanta coalesce into one high-frequency quantum. This process is a familiar one in quantum optics (e.g., Bloembergen 1965). However, it requires an Nth-order response of the medium; it is not possible as a second-order process in weak-turbulence theory. We suggest that this physical interpretation is not relevant to TB.

The implications of the nonexistence of TB are obvious for many of the suggested applications: the suggestions must be rejected. However, upconversion of low-frequency waves to high-frequency waves is possible by other processes, notably by the scattering/double-emission process (Melrose 1982; Goldman and DuBois 1982). The process requires a high level of anisotropic ion-sound turbulence, as arises, for example, in current-driven ion sound in the laboratory (Stenzel 1978). A possible application is radiation from quasi-perpendicular collisionless shock waves, replacing the TB mechanism in Klinkhamer and Kuijpers (1981). A constraint on this alternative mechanism is that the growth rate of the nonlinear process is to be added to the linear growth rate of the ion sound waves (Goldman and DuBois 1982), and the result must be positive. This constraint appears to be quite severe, requiring continuously driven ion sound turbulence with the electrons having a second source of free energy to drive the nonlinear (double-emission) instability.

We conclude that upconversion of ion sound turbulence to Langmuir turbulence cannot be by the (nonexistent) processes called turbulent bremsstrahlung, and that the efficacy of upconversion by the alternative scattering/double-emission process has yet to be established.

Note added in manuscript.—The reader should be made aware that in the refereeing of this paper our central argument has not been found convincing. A major difference of opinion has not been resolved. It is recognized that there are two alternative types of formalism, which might be called "response" and "Lagrangian" formalisms. All standard classical theories, and also DuBois's (1967) version of QED, fall into the "response" class in which the response is treated differently from the disturbances. In the Lagrangian theories, which include the form of QED adopted here and the classical counterpart of it discussed in Appendix C, one treats the response and the disturbances as equivalent and symmetrizes over them. The only important differences between these theories concern TB: the former allows it, the latter forbids it. We believe that the Lagrangian formalism is the appropriate one in general, but it is on this point that opinions differ. It is not clear how this controversy can be resolved other than by experiment.

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APPENDIX A

NONLINEAR DAMPING PROCESSES

The power radiated P is found as follows. Identify P as the volume integral of the rate per unit volume $-\boldsymbol{J}(\boldsymbol{x}, t) \cdot \boldsymbol{E}(\boldsymbol{x}, t)$ at which work is done on the electromagnetic by the current density $\boldsymbol{J}(\boldsymbol{x}, t)$ and average it over an arbitrarily long time T; use the power theorem to write the integral over Fourier space; insert

$$\boldsymbol{E}(k) = -i\boldsymbol{k}\phi(k) \tag{A1}$$

and use the equation of continuity

$$\boldsymbol{k} \cdot \boldsymbol{J}(k) = \omega\rho(k) \tag{A2}$$

to derive equation (8).

The wave properties are determined by the longitudinal dielectric constant

$$\epsilon^L(k) = \frac{\chi(k)}{\epsilon_0 |\boldsymbol{k}|^2} . \tag{A3}$$

Any solution $\omega = \omega_M(\boldsymbol{k})$ of the dispersion equation $\epsilon^L(k) = 0$ may be chosen to satisfy equation (11). However, real waves are described by both positive and negative frequencies, and we require two solutions $\omega_{M\pm}(\boldsymbol{k})$ to describe "forward" and "backward" waves (Kuijpers 1980*a*). We write (Melrose 1980*a*, p. 47)

$$R_{M\pm}(\boldsymbol{k}) = \left\{ \left[\omega \frac{\partial \epsilon^L(k)}{\partial \omega} \right]^{-1} \right\}_{\omega = \omega_{M\pm}(\boldsymbol{k})} \tag{A4}$$

for the ratio of electric to total energy in the waves. With $\gamma_{M\pm}(\boldsymbol{k})$ the absorption coefficient, the power given to waves in the mode M may be written as

$$P = -\sum_{\pm} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{\gamma_{M\pm}(\boldsymbol{k}) |\Phi_{M\pm}(\boldsymbol{k})|^2}{R_{M\pm}(\boldsymbol{k})} . \tag{A5}$$

Then using equation (13), now summed over the forward and backward modes on the right-hand side, we identify

$$\gamma_{L\pm}(\boldsymbol{k}) = \sum_{S\pm} \int \frac{d^3\boldsymbol{k}_1}{(2\pi)^3} |\Phi_{S\pm}(\boldsymbol{k}_1)|^2 \operatorname{Re}\left[i\omega\chi(k, k_1, k, -k_1)\right] , \tag{A6}$$

with $k = [\omega_{L\pm}(\boldsymbol{k}), \boldsymbol{k}]$ and $k_1 = [\omega_{S\pm}(\boldsymbol{k}_1), \boldsymbol{k}_1]$ implicit on the right-hand side. The relation (11) between positive and negative frequencies for each mode implies that only the part of $\chi(k, k_1, k, -k_1)$ which is odd under $k \leftrightarrow -k$ contributes. The symmetry property discussed in § III implies that only the part which is even under $k \rightarrow -k$ is to be retained, leading to the null result.

APPENDIX B

NONLINEAR RESPONSE TENSORS

The Vlasov equation for longitudinal fields, after Fourier transforming, gives

$$-i(\omega - \boldsymbol{k} \cdot \boldsymbol{v}) f(\boldsymbol{p}, k) + \int d\lambda^{(2)} ie\phi(k_1)\boldsymbol{k}_1 \cdot \frac{\partial}{\partial \boldsymbol{p}} f(\boldsymbol{p}, k_2) = 0 , \tag{B1}$$

where $f(\boldsymbol{p}, k)$ is the Fourier transform of the electron distribution function. On expanding in powers of ϕ, we write $f^{(n)}(\boldsymbol{p}, k)$ for the nth-order term with

$$f^{(0)}(\boldsymbol{p}, k) = f(\boldsymbol{p})(2\pi)^4\delta^4(k) . \tag{B2}$$

Then equation (B1) gives

$$f^{(n)}(\boldsymbol{p}, k) = \int d\lambda^{(2)} \frac{e\phi(k_1)}{\omega - \boldsymbol{k} \cdot \boldsymbol{v}} \boldsymbol{k}_1 \cdot \frac{\partial}{\partial \boldsymbol{p}} f^{(n-1)}(\boldsymbol{p}, k_2) . \tag{B3}$$

The nth-order response is given by

$$\rho^{(n)}(k) = -e \int d^3\boldsymbol{p} f^{(n)}(\boldsymbol{p}, k) , \tag{B4}$$

with $\rho^{(1)}, \rho^{(2)}, \rho^{(3)}$ identified with the three terms on the right-hand side of equation (7). One identifies

$$\chi(k) = -e^2 \int d^3\boldsymbol{p} \frac{1}{\omega - \boldsymbol{k} \cdot \boldsymbol{v}} \boldsymbol{k} \cdot \frac{\partial f(\boldsymbol{p})}{\partial \boldsymbol{p}} , \tag{B5}$$

$$\chi(k, k_1, k_2) = -e^3 \int d^3\boldsymbol{p} \frac{1}{\omega - \boldsymbol{k} \cdot \boldsymbol{v}} \boldsymbol{k}_1 \cdot \frac{\partial}{\partial \boldsymbol{p}} \left[\frac{1}{\omega_2 - \boldsymbol{k}_2 \cdot \boldsymbol{v}} \boldsymbol{k}_2 \cdot \frac{\partial f(\boldsymbol{p})}{\partial \boldsymbol{p}} \right] , \tag{B6}$$

$$\chi(k, k_1, k_2, k_3) = -e^4 \int d^3\boldsymbol{p} \frac{1}{\omega - \boldsymbol{k} \cdot \boldsymbol{v}} \boldsymbol{k}_1 \cdot \frac{\partial}{\partial \boldsymbol{p}} \left\{ \frac{1}{\omega_2 + \omega_3 - (\boldsymbol{k}_2 + \boldsymbol{k}_3) \cdot \boldsymbol{v}} \boldsymbol{k}_2 \cdot \frac{\partial}{\partial \boldsymbol{p}} \left[\frac{1}{\omega_3 - \boldsymbol{k}_3 \cdot \boldsymbol{v}} \boldsymbol{k}_3 \cdot \frac{\partial f(\boldsymbol{p})}{\partial \boldsymbol{p}} \right] \right\} . \tag{B7}$$

APPENDIX C

SYMMETRIZATION OVER LIKE FIELDS: A CLASSICAL VERSION OF THE QED ARGUMENT

In standard classical treatments of weak turbulence theory, a nonlinear response is found to a test field, as described by equation (5) or (7), for example. If ϕ in equation (7) is composed of two or more fields, $\phi = \phi_A + \phi_B + \cdots$, say, then the only physically significant part of $\rho(k)$ is the part which is symmetrized over the like fields. For an initially unsymmetrized cubic response tensor χ and for fields ϕ_L and ϕ_S the relevant source term for TB is

$$\rho(k) = \int d\lambda^{(3)}\bar{\chi}(k, k_1, k_2, k_3)\phi_S(k_1)\phi_L(k_2)\phi_S(k_3) , \tag{C1}$$

with

$$\bar{\chi}(k, k_1, k_2, k_3) = \tfrac{1}{2}[\chi(k, k_1, k_2, k_3) + \chi(k, k_3, k_2, k_1)] . \tag{C2}$$

Our QED argument implies that we should symmetrize over k and k_2, and the reason for this is not apparent when the theory is formulated directly in terms of a source of the form (C1).

In QED this additional symmetry arises from the form of the interaction Lagrangian or Hamiltonian. Although it is not common practice, classical weak turbulence theory can be developed using a Lagrangian approach (e.g., Low 1958; Galloway and Kim 1971; Dougherty 1970, 1974; Dewar 1972, 1977). The potential energy density is $\rho(x)\phi(x)$, and hence the action S resulting from the cubic response is

$$\begin{aligned} S &= \int d^4x\rho(x)\phi(x) = \int \frac{d^4k}{(2\pi)^4}\,\rho(k)\phi(-k) \\ &= \int \frac{d^4k}{(2\pi)^4}\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\,(2\pi)^4\delta^4(k-k_1-k_2-k_3)\chi(k, k_1, k_2, k_3)\phi(-k)\phi(k_1)\phi(k_2)\phi(k_3) , \end{aligned} \tag{C3}$$

where we insert the cubic response from equation (7) and assume that no symmetry is imposed on χ. Now writing $\phi = \phi_L + \phi_S$ and retaining only the term $\phi_L(-k)\phi_S(k_1)\phi_L(k_2)\phi_S(k_3)$ relevant to TB, it is apparent that the only part which contributes to the action is the part which is symmetric under $-k \leftrightarrow k_2$. That is, in this case we should replace χ by

$$\bar{\bar{\chi}}(k, k_1, k_2, k_3) = \tfrac{1}{2}[\bar{\chi}(k, k_1, k_2, k_3) + \bar{\chi}(-k_2, k_1, -k, k_3)] , \tag{C4}$$

where we assume that the symmetrization [eq. [C2]) has already been carried out.

A more formal procedure for deriving the symmetry property (eq. [C4]) is the standard procedure in field theory for calculating the interaction term in the equation of motion from the Lagrangian. This involves the following steps: (i) starting from equation (C3), write all frequencies as positive with $\phi(k) = \phi(\omega, \boldsymbol{k})$ for $\omega > 0$ and $\phi(k) = \phi^*(-\omega, -\boldsymbol{k})$ for $\omega < 0$; we now omit the arguments and regard ϕ_L, ϕ_L, ϕ_L^*, ϕ_S, and ϕ_S^* as independent fields. (ii) Construct the effective Lagrangian $L(k)$ by writing

$$S = \int \frac{d^4k}{(2\pi)^4}\,L(k) \tag{C5}$$

and regard $L(k)$ as a functional of the fields. (iii) Calculate the relevant response by taking the appropriate functional derivative; here $\rho(k)$ is found by differentiating $L(k)$ with respect to ϕ_L^*. (iv) Because ϕ_L^* appears in both $\phi_L(-k)$ (for $\omega > 0$) and in $\phi_L(k_2)$ (for $\omega_2 < 0$), there are two terms, which may be combined as in equation (C4) to obtain

$$\rho(k) = \int d\lambda^{(3)}\bar{\bar{\chi}}(k, k_1, k_2, k_3)\phi_S(k_1)\phi_L(k_2)\phi_S(k_3) . \tag{C6}$$

The essence of our argument is that one should use equation (C6) rather than equation (C1). This change has no effect in the treatment of all second-order processes other than TB, and for TB it leads to an additional term which exactly cancels the one retained by Tsytovich, Stenflo, and Wilhelmsson (1975).

APPENDIX D

NAMBU'S APPROXIMATION

Consider the following term from Nambu's (1981*a*) equation (12):

$$\begin{aligned} \epsilon_k(K, \Omega) = \frac{\omega_{pe}^2}{K^2}\frac{e^2}{m^2}\int dv \sum_k \frac{1}{\Omega - Kv}\,\frac{k}{K-k}\,\frac{|\Phi_l(k, \omega)|^2}{\epsilon_0(K-k, \Omega-\omega)}\,\frac{\partial}{\partial v}\left(\frac{1}{\omega - kv} + \frac{1}{\Omega - \omega - (k-k)v}\right)\frac{\partial f_{0e}}{\partial v} \\ \times\, \omega_{pe}^2 \int dv\,\frac{1}{\Omega - \omega - (K-k)v}\,kK\,\frac{\partial}{\partial v}\left(\frac{1}{Kv - \omega} + \frac{1}{\Omega - Kv}\right)\frac{\partial f_{0e}}{\partial v} , \end{aligned} \tag{D1}$$

where the notation is Nambu's. Apart from notation, $\epsilon_k(K, \Omega)$ is a one-dimensional form of our $\chi^{\text{eff}}(k, k_1, k, -k_1)$, with k replaced by (Ω, K) and k_1 by (ω, k). Nambu writes $\epsilon_k(K, \Omega)$ as a product AB. Let us write

$$\chi(K, \Omega; k, \omega; K - k, \Omega - \omega) = -\frac{e^3}{m_e} n_e \int dv \, \frac{k(K-k)}{\Omega - Kv} \frac{\partial}{\partial v} \left\{ \frac{1}{\omega - kv} + \frac{1}{\Omega - \omega - (K-k)v} \right\} \frac{\partial f_{0e}}{\partial v} , \tag{D2}$$

which is the one-dimensional nonrelativistic form of equation (B6) with Nambu's normalization for f_{0e}. Then we have

$$\epsilon_k(K, \Omega) = \frac{1}{\epsilon_0 K^2} \sum_k \frac{|\Phi_l(k, \omega)|^2}{\chi(K-k, \Omega-\omega)} \chi(K, \Omega; k, \omega; K-k, \Omega-\omega)\chi(K-k, \Omega-\omega; -k, -\omega; K, \Omega) , \tag{D3}$$

which is obviously analogous to our equation (19). In his equations (21) and (22) Nambu introduces factors A and B, and apart from constants which are irrelevant in the following discussion, these are

$$A = \chi(K, \Omega; k, \omega; K - k, \Omega - \omega) , \tag{D4}$$

$$B = \chi(K - k, \Omega - \omega; -k, -\omega; K, \Omega) . \tag{D5}$$

Nambu then neglected, without comment, the term $[\Omega - \omega - (K - k)v]^{-1}$ in A (see eqns. [D4] and [D2], and an analogous term $(\Omega - Kv)^{-1}$ in B (see eqs. [D5] and [D2]). These omissions are unimportant in evaluating the imaginary parts, which are

$$\text{Im } A = -\frac{\pi e^3 n_e}{m_e} \int dv \delta(\omega - kv) \frac{\partial f_{0e}}{\partial v} k(K-k) \frac{\partial}{\partial v} \frac{1}{\Omega - Kv} , \tag{D6}$$

$$\text{Im } B = \frac{e^3 n_e}{m_e} \int dv \delta(\omega - kv) \frac{\partial f_{0e}}{\partial v} kK \frac{\partial}{\partial v} \frac{1}{\Omega - \omega - (K-k)v} . \tag{D7}$$

Evaluation of the derivatives in equations (D6) and (D7) leads to the identity

$$\text{Im } A = -\text{Im } B . \tag{D8}$$

If we retain the terms neglected by Nambu, then explicit evaluation of the real parts gives

$$\text{Re } A = \text{Re } B = \frac{e^3 n_e}{m_e} kK(K-k) \int dv \frac{\partial f_{0e}/\partial v}{(\omega - kv)(\Omega - Kv)[\Omega - \omega - (K-k)v]} . \tag{D9}$$

One then finds

$$\text{Im } \epsilon_k(K, \Omega) \propto \text{Im } AB = \text{Im } A \text{ Re } B + \text{Im } B \text{ Re } A = 0 . \tag{D10}$$

Nambu's approximations led him to conclude incorrectly that Re A and Re B are different and hence that Im AB is nonzero. The vanishing of Im $\epsilon_k(K, \Omega)$ implies that Nambu's mechanism does not exist.

REFERENCES

Benz, A. O. 1977, *Ap. J.*, **211**, 270.
Bloembergen, N. 1965, *Nonlinear Optics* (New York: Benjamin).
Bujarbarua, S., and Nambu, M. 1983, *J. Phys. Soc. Japan*, **52**, 2285.
Bujarbarua, S., Sarma, S. N., and Nambu, M. 1984, *Phys. Rev. A*, **29**, 2171.
Cutkosky, R. E. 1960, *J. Math. Phys.*, **1**, 429.
Davidson, R. C. 1972, *Methods in Nonlinear Plasma Theory* (New York: Academic).
Dawson, J. 1968, in *Advances in Plasma Physics*, Vol. **1**, ed. A. Simon and W. B. Thompson (New York: Interscience), pp. 1–66.
Dawson, J., and Oberman, C. 1963, *Phys. Fluids*, **6**, 394.
Dewar, R. L. 1972, *J. Plasma Phys.*, **7**, 267.
———. 1977, *Australian J. Phys.*, **30**, 553.
Dougherty, J. P. 1970, *J. Plasma Phys.*, **4**, 761.
———. 1974, *J. Plasma Phys.*, **11**, 331.
Dubois, D. F. 1967, *Lectures in Theoretical Physics*, Vol. **9**, ed. W. E. Brittin (New York: Gordon & Breach), p. 469.
———. 1976, *Phys. Fluids*, **19**, 1746.
DuBois, D. F., and Pesme, D. 1984, *Phys. Fluids*, **27**, 218.
Finken, K. H., and Ackerman, U. 1982, *Physica*, **113B**, 135.
Fujiyama, H., and Nambu, M. 1983, *Experimental Study on Induced Bremsstrahlung Radiation from a Plasma* (*16th Internat. Conf. Ionized Gases*, Düsseldorf).
Galloway, J. J., and Kim, H. 1971, *J. Plasma Phys.*, **6**, 53.
Goldman, M. V., and DuBois, D. F. 1982, *Phys. Fluids*, **25**, 1062.
Hirose, A. 1984, *Comments Plasma Phys. Controlled Fusion*, **8**, 117.
Hoyng, P. 1977, *Astr. Ap.*, **55**, 31.
Kadomstev, B. B. 1965, *Plasma Turbulence* (New York: Academic).
Kaplan, S. A., Pikel'ner, S. B., and Tsytovich, V. N. 1974, *Phys. Rev.*, **15**, 1.
Klinkhamer, F. R., and Kuijpers, J. 1981, *Astr. Ap.*, **100**, 291.
Kuijpers, J. 1978, *Astr. Ap.*, **69**, L9.
———. 1980*a*, *Astr. Ap.*, **83**, 201.
———. 1980*b*, *Ap. J.* (*Letters*), **238**, L165.
Landau, L. D. 1959, *Nucl. Phys.*, **13**, 181.
Low, F. E. 1958, *Proc. Roy. Soc. London, A*, **248**, 282.
Melrose, D. B. 1972, *Plasma Phys.*, **14**, 1035.
———. 1980*a*, *Plasma Astrophysics*, Vol. **1** (New York: Gordon & Breach), pp. 65, 67.
———. 1980*b*, in *Plasma Astrophysics*, Vol. **2** (New York: Gordon & Breach), p. 163.
———. 1982, *Australian J. Phys.*, **35**, 67.
Melrose, D. B., and Kuijpers, J. 1984, *J. Plasma Phys.*, **32**, 239.
Nambu, M. 1980*a*, *J. Phys. Soc. Japan*, **49**, 2349.
———. 1980*b*, *Phys. Scripta*, **22**, 81.
———. 1981*a*, *Phys. Rev. A*, **23**, 3272.
———. 1981*b*, *J. Phys. Soc. Japan*, **50**, 11.
———. 1982, *Phys. Fluids*, **25**, 1196.
———. 1983, *Laser and Particle Beams*, **1**, 427.
Nambu, M., Bujarbarua, S., Shukla, P. K., and Spatschek, K. H. 1980, *J. Plasma Phys.*, **23**, 483.
Nambu, M., and Shukla, P. K. 1979, *Phys. Rev. A*, **20**, 2498.
———. 1980, *Phys. Rev. Am* **22**, 787.
———. 1983, *Ap. J.* (*Letters*), **271**, L35.
Nambu, M., and Terasawa, T. 1982, *J. Phys. Soc. Japan*, **51**, 2385.
Rönnmark, K., and Stenflo, L. 1976, *Planet. Space Sci.*, **24**, 904.
Ross, D. W. 1969, *Phys. Fluids*, **12**, 613.
Sagdeev, R. Z., and Galeev, A. A. 1969, *Nonlinear Plasma Theory* (New York: Benjamin).
Stenzel, R. L. 1978, *Phys. Fluids*, **21**, 99.

Tsytovich, V. N. 1970, *Nonlinear Effects in Plasma* (New York: Plenum).
———. 1971, in *Proc. 10th Internat. Conf. Phenomena in Ionized Gases* (Oxford: Oxford University Press), p. 333.
———. 1973, *Ann. Rev. Astr. Ap.*, **11**, 363.
———. 1980, *Soviet J. Plasma Phys.*, **6**, 608.
Tsytovich, V. N., Stenflo, L., and Wihelmsson, H. 1975, *Phys. Scripta*, **11**, 251.
———. 1981, *Ap. J.* (*Letters*), **246**, L89.
Vlahos, L., and Papadopoulos, K. 1979, *Ap. J.* (*Letters*), **234**, L217.
———. 1982, *Ap. J.* (*Letters*), **252**, L75.

J. KUIJPERS: Sterrekundig Instituut, Rijksuniversiteit Utrecht, Sterrewacht "Sonnenborgh," Zonnenburg 2, 3512 NL Utrecht, The Netherlands

D. B. MELROSE: School of Physics, University of Sydney, Sydney NSW 2006, Australia