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A PHASE-BUNCHING MECHANISM FOR FINE STRUCTURES IN AURORAL KILOMETRIC RADIATION AND JOVIAN DECAMETRIC RADIATION

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Abstract. Fine structures observed in the auroral kilometric radiation (AKR) and in the Jovian decametric radiation (DAM) show remarkable similarities to fine structures in discrete VLF emissions. The feedback model of Helliwell for discrete VLF emissions is modified to apply to ARK and DAM. An important constraint on the model is that the frequency drift must be close to that characteristic of a propagating elect'ron radiating at its cyclotron frequency. For DAM the model fits satisfactorily with the observational data. For AKR it seems necessary to assume that the basic fine structure is unresolved and that the reported fine structures are envelopes of these basic structures drifting at a rate associated with the exciting agency, which is presumed to be a localized parallel electric field. It is argued that these basic fine structures may be drifting rapidly at a rate proportional to the parallel electric field (provided it is > 0.1 mV m^{-1}) and that their bandwidth, determined by the relativistic velocity spread, should be less than ≅ 1 kHz. Triggering of AKR by type III bursts can be explained naturally in terms of the model.

1. Introduction

The electron cyclotron maser theory [Melrose, 1976; Wu and Lee, 1979] for the auroral kilometric radiation (AKR) and for the Jovian decametric radiation (DAM) has been quite successful in accounting for many features of the observed emissions. Notable successes include the prediction that AKR should be in the x mode from a region with plasma frequency ω_p much less than the electron cyclotron frequency Ω_e [Melrose, 1976]; the prediction that AKR can be driven by an upward directed loss cone distribution [Wu and Lee, 1979], as subsequent calculations based on observational data of the electrons confirmed [Melrose et al., 1982; Omidi and Gurnett, 1982]; and the natural explanation [Hewitt et al., 1981] for the observational evidence that DAM has an emission pattern locally confined to the surface of a hollow cone. However in spite of these and other successes, the cyclotron maser theory in its present form remains at best an incomplete theory for AKR and DAM.

One aspect in which the theory is internally unsatisfactory concerns time scales. With measured distribution functions, the calculated growth rates are too small to account for effective amplification in one pass through the source. This is as it should be: the characteristic growth time for the cyclotron maser is quite short, and the maser should saturate (in the case of AKR) in a few tens of milliseconds [e.g., Melrose et al., 1982]. As saturation occurs the distribution function should relax so that the feature associated with the available free energy is smoothed out. The observed distribution functions are measured over a few seconds and must correspond to the relaxed distribution, which may contain little evidence of the features actually driving the instability. Whatever these features are, they need to be generated on a similar time scale to that in which they are smoothed out, i.e., a few tens of milliseconds. Likely mechanisms involve parallel electric fields [e.g., Mozer et al., 1980]; there is observational evidence for a correlation between AKR and the electric field [e.g., Morioka et al., 1981]. However, it is questionable whether the maser mechanism is tenable on the short time scales which are evidently involved. On a sufficiently short time scale. phase coherence effects must be important, and then a maser theory is inapplicable. We now argue on observational grounds that a phase-bunching theory and not the conventional maser theory is appropriate at least for some purposes.

It has long been known that DAM contains fine structures, called S (for "short") bursts, [e.g., Warwick, 1967]. Detailed observations of DAM at at very high resolution were-reviewed by Ellis [1974, 1982]. He found S bursts during about half of the Jupiter noise storms. These bursts have bandwidths as narrow as 2 kHz and are commonly drifting with $|df/dt| \propto f$ where $df/dt \cong 10$ MHz s^{-1} and $f \cong 10$ MHz. Ellis found drifts predominantly from high to low frequency, but reverse drifts and more complicated structures were sometimes observed. Similar fine structures were reported in AKR by Gurnett and Anderson [1981] and Morioka et al. [1981]. Gurnett and Anderson suggested that AKR consists of discrete narrow band structures, rather than of a continuous broadband emission. These structures have bandwidths of 1 kHz or less and drift either predominantly upward or predominantly downward in frequency with a variety of structures. A particular feature of the fine structures, noted by both Ellis [1974] and Gurnett and Anderson [1981], is that they are strongly reminiscent of fine structures in discrete VLF emissions. This suggests that any explanation for these fine structures should be applicable to all three cases, namely AKR, DAM and VLF emissions.

In this paper an attempt is made to develop a theory for fine structures in AKR and DAM by adapting the accepted phenomenological theory [Helliwell, 1967] for discrete VLF emissions. VLF emissions involve electrons resonating with whistlers, and in the model the interaction is assumed to be confined to an "interaction region" (IR) which can drift along the field lines. The important modification which needs to be made to apply this theory to AKR and DAM is to the resonance condition which now applies to x mode waves.

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Also in AKR and DAM, relativistic effects are important, and need to be retained in the resonance condition [Wu and Lee, 1979].

An important motivation for the present investigation is the evidence [Calvert, 1981, 1985a, b; Farrell and Gurnett, 1985] that AKR and Jovian hectometric radiation (HOM) can be triggered by radio bursts originating in the solar wind, specifically type III and type II solar bursts. (Triggering of DAM has yet to be observed; the relevant solar radio bursts would be generated several solar radii from the sun.) Calvert [1985a, b] has argued that such triggering provides strong support for his feedback model [Calvert, 1982] for the generation of AKR. The feedback loop in Calvert's [1982] model involves only the waves, which are assumed to be partially reflected at end points. Fine structures and triggering occur in VLF emissions and are interpreted in terms of a feedback mechanism, but in this case the feedback loop involves both the particles and the waves. Particles entering the IR are phase-bunched by whistlers leaving the IR, and later as these particles approach the other side of the IR they generate a new whistler at a slightly different frequency. This whistler propagates back to phase bunch a new group of particles. This wave-particle feedback loop requires that the particles and waves be propagating in opposite directions through the IR. Potentially this feedback mechanism offers a natural explanation for the triggering of AKR, DAM and HOM by analogy with the well-known triggering of discrete VLF emissions. Triggering is discussed only briefly in the present paper (cf, section 5.4).

Three important ingredients in Helliwell's [1967] model for discrete VLF emission are the "consistent wave condition," the feedback loop and the saturation mechanism. The consistent wave condition selects those particles and waves which remain in resonance for the maximum time in an inhomogeneous system; in practice it relates the frequency drift to the motion of the IR. The application of this condition to x mode radiation is discussed in section 2. (Some of the results of section 2 have been derived by Le Quéau et al. [1985].) The feedback loop requires waves and particles propagating in opposite directions through the IR whereas in Wu and Lee's [1979] maser mechanism, for example, upgoing particles generate upgoing waves so that waves and particles are propagating in the same sense. However, the two theories are compatible, provided the IR has a parallel velocity intermediate between those of the particles and the waves so that the relative motions of particles and waves to the IR are opposite. This requirement on the velocity of the IR imposes a severe restriction on the model in terms of the allowed frequency drift rates (cf. section 6). The saturation mechanism in Helliwell's model is wave trapping, and it is assumed here that this applies also to the x mode. (Le Quéau et al. [1984] have also proposed trapping as the saturation mechanism for AKR.) Trapping should be relevant, rather than saturation through quasi-linear effects [e.g., Wu et al., 1981; Melrose et al., 1982; Pritchett and Strangeway, 1986], provided the emission is sufficiently narrow band.

The paper is set out as follows. In section 2 the consistent wave condition of Helliwell [1967] is discussed and adapted to apply to the x mode case; the effect of a parallel electric field is included in the resulting expression for the frequency drift rate $d\omega/dt$. (The angular frequency is $\omega = 2\pi f$.) Saturation of the growth due to wave trapping is discussed in section 3, and in section 4 possible sources of free energy are considered. Several more minor aspects of the mechanism, including triggering, are discussed in section 5. The applications to DAM and to AKR are explored in section 6, and the conclusions are summarized in section 7.

2. The Frequency Drift

In the theory of discrete VLF emissions there is a condition imposed called the "consistent wave condition" by Helliwell [1967] and the "second-order resonance" by Nunn [1974]. Matsumoto [1979] expressed this condition in terms of derivatives of the relative phase, ψ say, of the wave and the particle. (The relative phase may be identified as the quantity $\Psi - \phi$ appearing in (A8a) and (A9) of the appendix.) The condition $d\psi/dt = 0$ gives the usual resonance condition, and this additional condition is $d^2\psi/dt^2 = 0$. In a homogeneous medium, $d\psi/dt = 0$ implies that the resonant particle experiences systematic acceleration by the wave fields; in principle this acceleration can continue until the particle velocity has changed sufficiently so that it drifts out of resonance. In an inhomogeneous medium the time of the resonant interaction is limited by spatial gradients, which cause a particle and wave initially in resonance to move away from resonance as they propagate. The additional condition $d^2\psi/dt^2 = 0$ selects those particles and waves which do not get out of phase due to the first-order changes. These first-order changes include a frequency drift $d\omega/dt$ as well as the effect of the gradients due to the inhomogeneity. Only particles which satisfy this additional condition are important in the interaction.

An arbitrary resonance, at the sth harmonic ($s = 0, \pm 1, \ldots$), corresponds to the resonance condition

$$\omega - s\Omega_e(1-v_\perp^2/c^2 - v_\parallel^2/c^2)^{1/2} - k_\parallel v_\parallel = 0 \qquad (1)$$

The consistent wave condition is

$$d\{\omega - s\Omega_e(1-v_\perp^2/c^2-v_\parallel^2/c^2)^{1/2} - k_\parallel v_\parallel\} = 0 \qquad (2)$$

In practice. (2) determines the frequency drift $d\omega/dt$ in terms of the spatial gradients and the drift velocity $\mathbf{v}_{IR}$ of the interaction region:

$$\frac{d\omega}{dt}\left(1-v_\parallel \frac{\partial k_\parallel}{\partial\omega}\right) = \mathbf{v}_{IR}\cdot\text{grad}\left\{s\Omega_e(1-v_\perp^2/c^2-v_\parallel^2/c^2)^{1/2} - k_\parallel v_\parallel\right\} \qquad (3)$$

Before applying this to the case of interest here, specifically to x mode waves for $s = 1$, let us note the important details of the application to whistlers

In the case of VLF emissions the whistlers are usually assumed to be propagating parallel to the field lines. Then one has $|k_\parallel| = \omega_p\omega/c\{\omega/(\Omega_e-\omega)\}^{1/2}$, and one may write $|\partial k_\parallel/\partial\omega| = 1/v_g$, where v_g is the group speed. In (1) the relativistic effects are unimportant, and $k_\parallel v_\parallel$ and Ω_e are larger in magnitude than w. The resonance at $s = 1$ then requires $k_\parallel v_\parallel < 0$, i.e., electrons and whistlers propagating in opposite directions along the field lines. In (3) only the component of v_{IR} along the field lines is relevant. As in (1) the relativistic effects are ignored, and the relevant gradients are in Ω_e, ω_p and $v_\parallel$. The parallel velocity is assumed to vary in accord with energy conservation, $v_\perp^2 + v_\parallel^2$ = constant, and conservation of the first adiabatic invariant, $v_\perp^2/\Omega_e$ = constant. A further condition is imposed, but is not particularly restrictive: in order for feedback to occur the propagation velocity of the interaction must be intermediate betveen the propagation velocity of the electrons and that of the waves. Thus if the electrons are moving to the right ($v_\parallel > 0$) and the waves to the left (at $-v_g$), one requires $-v_g < v_{IR} < v_\parallel$. Note that for $v_{IR\parallel} < 0$, the drfft rate $d\omega/dt$ can have the opposite sign to $v_\parallel d\Omega_e/dz$, where z denotes distance along the field line. That is, electrons propagating toward decreasing cyclotron frequencies can emit a rising tone. There is no inconsistency in this because electrons are continuously flowing through the IR, and the electrons radiating at one time are not the same as the electrons radiating at a later time.

For x mode-waves at $s = 1$ the dominant terms in (1) give $\omega \approx \Omega_e$, and the relativistic correction and Doppler term $(k_\parallel v_\parallel)$ are small. The emission must be above the cutoff at $\omega = \omega_x$,

$$\omega_x = \tfrac{1}{2}\Omega_e + \tfrac{1}{2}[\Omega_e^2+4\omega_p^2]^{1/2} \approx \Omega_e + \omega_p^2/\Omega_e \qquad (4)$$

and this requires $k_\parallel v_\parallel > 0$. For present purposes it is sufficient to expand the square root in (1), and also in (2), and retain only the first-order relativistic correction. Then (1) may be replaced by

$$v_\perp^2 + (v_\parallel - v_c)^2 = v_0^2 \qquad (5)$$

with

$$v_c = \frac{k_\parallel c^2}{s\Omega_e}, \quad v_0 = \left[v_c^2 - \frac{2(\omega - s\Omega_e)c^2}{s\Omega_e}\right]^{1/2} \qquad (6)$$

We now set $s = 1$. On assuming $v_\perp^2 + v_\parallel^2$ = constant and $v_\perp^2/\Omega_e$ = constant, one finds that the dominant terms in (3) give $d\omega/dt = \underline{v}_{IR}\cdot\text{grad}\Omega_e$. Thus the variation in $v_\perp$ and $v_\parallel$ is unimportant.

An important constraint on the theory is that feedback must be possible. Consider the frame in which the IR is at rest. By analogy with the case of whistlers, feedback is possible if the parallel velocity of the particles and the parallel group velocity of the waves are oppositely directed in this frame. Only then can the waves emitted by particles at one end of the IR phase bunch incoming particles at the other end (cf. Figure 1). The parallel group velocity $v_{g\parallel}$ of the waves may be approximated by v_c except near the cutoff frequency. More specifically one has $v_{g\parallel} \approx v_c$ whenever one has $\partial\omega N_x/\partial\omega \approx 1$. In the laboratory frame the condition for feedback to operate becomes

$$\min[v_\parallel, v_c] < v_{IR\parallel} < \max[v_\parallel, v_c] \qquad (7)$$

In practice it is likely that $|v_c - v_\parallel|$ is much smaller than either v_c or $v_\parallel$, and then (7) implies a very narrow range for $v_{IR\parallel}$. The sign of $v_c - v_\parallel$ is positive for a loss cone distribution (cf. equation (24)), but more generally one could envisage cases where $v_c - v_\parallel$ is negative. The feedback mechanism can operate for either sign.

The drift of the IR, including possible motion across the field lines, is discussed further in section 5.3.

Suppose the interaction region is drifting along the field lines (i.e., $v_{IR\perp} = 0$). Then (3), when taken together with (7) and $v_\parallel \approx v_c$, implies $d\omega/dt \approx v_{IR\parallel} d\Omega/dz$. This is a very restrictive condition. It is effectively the condition assumed by Ellis [1974] and by Gurnett and Anderson [1981] in interpreting the frequency drifts observed in the fine structures of DAM and AKR, respectively; these authors actually assumed $d\omega/dt = v_\parallel d\Omega_e/dz$. For DAM the observed drifts seem to be consistent with $d\omega/dt \approx v_\parallel d\Omega_e/dz$ for $v_\parallel > 0$ (upward motion toward decreasing Ω_e) with $v_\parallel/c \approx 0.1$ [Ellis, 1974], although rarer drifts inconsistent with this are observed. For AKR the observed drifts would imply $v_\parallel$ between 3 and 300 km s^{-1} [Gurnett and Anderson, 1981], and it seems unlikely that these inferred drift velocities do correspond to $v_\parallel$. Gurnett and Anderson [1981] suggested that the inferred drifts might be characteristic of motion at the ion sound speed, and Morioka et al. [1981] suggested that they may be associated with propagating double layers or electrostatic shocks.

Although a parallel electric field has no effect on the resonance condition, it cannot be ignored in the consistent wave condition. Let the electric potential be $\phi = \phi(z)$. Then conservation of energy and of the first adiabatic invariant imply, respectively,

$$v_\perp^2 + v_\parallel^2 - \frac{2e}{m}\phi = \text{constant} \qquad (8)$$

$$v_\perp^2/\Omega_e = \text{constant} \qquad (9)$$

In view of (5), the condition (3) becomes

$$d\{v_\perp^2 + (v_\parallel - v_c)^2 - v_0^2\} = 0 \qquad (10)$$

and together with (8) and (9) this implies

$$\frac{d\omega}{dt} = \underline{v}_{IR}\cdot\text{grad}\Omega_e + v_{IR\parallel}\frac{d\Omega_e}{dz}\left(\frac{v_c - v_\parallel}{2v_\parallel}\frac{E_\parallel}{E_0}\right) \qquad (11)$$

where some small terms are omitted and where

$$E_0 = -\frac{mc^2}{e}\frac{1}{2\Omega_e}\frac{d\Omega_e}{dz} \qquad (12)$$

is a characteristic electric field. The term involving $E_\parallel = -d\phi/dz$ in (11) arises as follows.

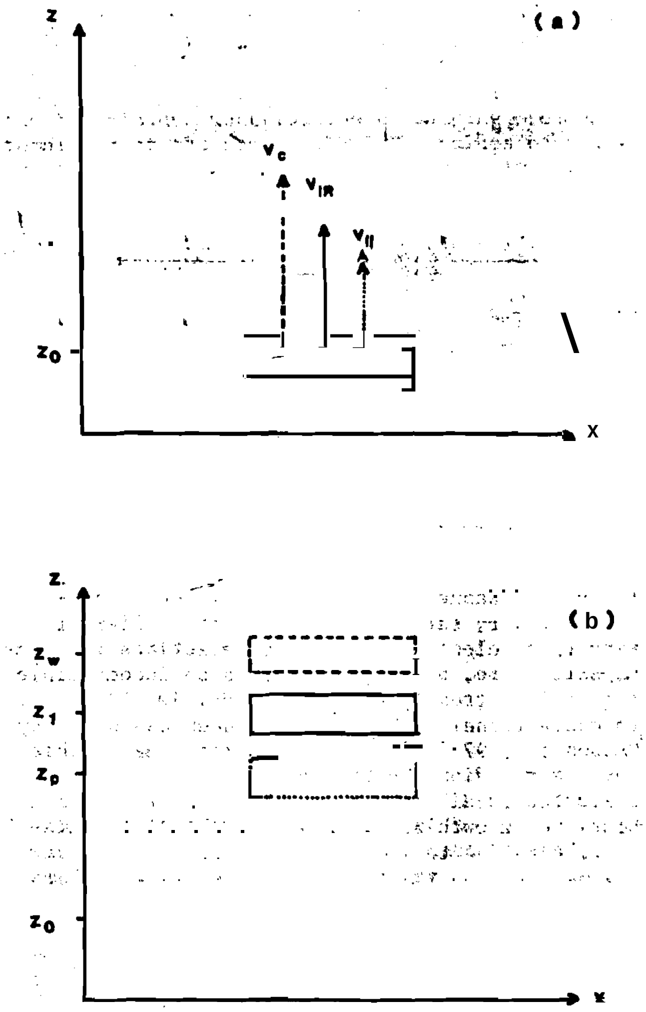

Fig. 1. (a) At an initial time $t = t_0$ a cross section of the IR is centered on a point z_0 along the field lines. The resonant particles, the IR and the emitted radiation have velocity components $v_\parallel$, $v_{IR\parallel}$ and v_c, respectively, along the field lines as indicated. For the case drawn, $v_c > v_\parallel$ is assumed. (b) At a later time $t = t_1$ the IR is centered on a point $z_1 = z_0 + v_{IR\parallel}(t_1-t_0)$. The waves and particles which were inside the IR at $t = t_0$ are now inside the regions marked by dashed lines and dotted lines and centered on $z_w = z_0 + v_c(t_1-t_0)$ and $z_p = z_0 + v_\parallel(t_1-t_0)$, respectively. The particles now in the IR were above the IR at $t = t_0$ and have been phase bunched by waves emitted at $t < t_1$. These particles generate waves which phase bunch particles that enter the IR at $t > t_1$.

The part with $v_c E_\parallel$ arises from $k_\parallel dv_\parallel/dt$, and the part with $-v_\parallel E_\parallel$ arises from the rate of change of the relativistic gyrofrequency $(d/dt)\{\Omega_e(1-v^2/2c^2)\}$, with the change in $v_\parallel$ and v^2 dominated by the effect of the electric acceleration.

In the terrestrial auroral zones one has $E_0 \approx 0.1$ mV m^{-1}, and typical parallel electric fields are stronger than this, e.g., $E_\parallel \approx 1$ mV m^{-1} is common [e.g., Mozer et al., 1980; Weimer et al., 1985]. Hence $d\omega/dt$ should be dominated by the electric effects, and $d\omega/dt$ should be roughly proportional to $E_\parallel$. The sense of drift is from high to low frequencies for an upward motion of the IR, i.e., for $v_{IR\parallel} > 0$ (with $d\Omega_e/dz < 0$). A correlation between $d\omega/dt$ and $E_\parallel$ was suggested by Morioka et al. [1981] based on observational data.

3. Saturation Effects

The model developed above has several further detailed features which require consideration. These are discussed in this section and in the next two sections. However, these further details are not of much practical significance to the applications to DAM and AKR, and the reader interested primarily in the applications would lose little by proceeding directly to section 6.

Saturation of the wave growth is attributed to wave trapping. It is postulated that the bunching time (or the bounce or trapping time) is equal to the resonance time, which is the time spent by a resonant electron in the IR of length L. The bandwidth of the emission is limited by the trapping frequency ω_T. In this section we first estimate the length of the IR, and then we consider the bunching time and the bandwidth.

Let Δ be the frequency mismatch away from resonance:

$$\Delta = \omega - s\Omega_e(1-v_\perp^2/c^2-v_\parallel^2/c^2)^{1/2} - k_\parallel v_\parallel \quad (13)$$

To a first approximation we require $\Delta = 0$ and $d\Delta = 0$ inside the IR, and these conditions in effect locate the center of the IR. Consider a frame comoving with the IR, and let quantities in this frame be denoted by primes. In the IR frame, Δ' is not a function of t', and if z' measures distance from the center of the IR, we may approximate Δ' by $z'd\Delta/dz$, where $d\Delta/dz$ is to be determined at the center of the IR. An electron with velocity $v_\parallel$ in the laboratory frame has a velocity $v_\parallel - v_{IR\parallel}$ in the IR frame, and hence we have $dt' = dz'/|v_\parallel-v_{IR\parallel}|$. The length of the IR is determined by setting L/2 equal to the distance over which the phase changes by π [Helliwell, 1967]. Thus we set

$$\int dt'\Delta' = \pi \quad (14)$$

where the integral is from $t' = 0$ to $t' = L/2|v_\parallel-v_{IR\parallel}|$. On integrating across the IR, (14) leads to the condition

$$L = 2\left[2\pi|v_\parallel-v_{IR\parallel}|/\left|\frac{d\Delta}{dz}\right|\right]^{1/2} \quad (15)$$

Using (11) we may replace (15) by

$$L = \left[\frac{2\pi|v_\parallel-v_{IR\parallel}|\, v_{IR\parallel}}{|d\omega/dt|}\right]^{1/2} \quad (15')$$

where we ignore drift of the IR across the field lines.

The resonance time is the time spent by an electron in the IR:

$$t_R = \frac{L}{|v_\parallel-v_{IR\parallel}|} \quad (16)$$

The time required for bunching is estimated in the appendix. One has

$$t_B = \frac{|\Delta|}{\omega_T^2} \tag{17}$$

where

$$\omega_T = \left[\frac{eE}{mc^2} v_\perp \Omega_e\right]^{1/2} \tag{18}$$

is the trapping (or bounce) frequency. with E the electric amplitude of the waves. The saturation amplitude is assumed to be determined by the condition $t_R = t_B$ in Helliwell's [1967] model, with $|\Delta|$ in (17) implicitly assumed to be equal to ω_T. A justification for the latter assumption is that the minimum spread in frequency (e.g., the minimum bandwidth) is **determined** by ω_T due to the forced motion of electrons induced by the wave. Thus the saturation amplitude is determined by $t_R\omega_T = 1$, giving

$$E = \frac{mc^2}{8\pi e}\frac{|v_\| - v_{IR\|}|}{|v_{IR\|}|} \tag{19}$$

If we ignore drift across the field lines and use (11) with (12), then (19) becomes

$$\frac{E}{E_0} = \frac{1}{4\pi}\frac{|v_\| - v_{IR\|}|}{v_\perp} \tag{20}$$

The bandwidth can be no narrower than ω_T, implying

$$(\Delta\omega)_{min} = \omega_T = \frac{1}{t_R} = \left[\frac{1}{8\pi}\frac{|v_\| - v_{IR\|}|}{v_{IR\|}}\left|\frac{d\omega}{dt}\right|\right]^{1/2} \tag{21}$$

The factors involving the velocities in (20) and (21) can be determined only when a specific model for the free energy is assumed, as discussed in the next section.

4. The Source of Wave Growth

A peculiar feature of the phase–bunching model is that no reference needs to be made to the nature of the wave growth, nor to the source of free energy which is driving it. Questions relating to the nature of the growth mechanism apply to discrete VLF emissions, as well as to the case of interest here, and these questions continue to be the source of controversy.

It has long been known that whistlers can be driven unstable by suprathermal electrons with a loss cone anisotropy. A theory by Kennell and Petschek [1966], for example. provides a satisfactory explanation for the average properties of such electrons in the magnetosphere. In this theory the free energy is due to the loss cone anisotropy, and the vave growth involves a maser instability. From an observational viewpoint an obvious deficiency with this theory is that the observed whistlers are the discrete VLF emissions whose generation involves phase–coherent effects, whereas the maser theory is explicitly phase-independent. Helliwell's [1967] phase–bunching theory and Kennel and Petschek's [1966] quasilinear theory satisfactorily explain different aspects of electron–whistler interactions in the magnetosphere, but they appear to be incompatible theories. Attempts at relating Helliwell's theory to conventional mechanisms have been discussed by Matsumoto [1979]. The most favored idea is that the distribution function of the electrons is distorted locally, so as to increase $\partial f/\partial v_\|$ and hence the growth rate. However, this local distortion is due to whistlers, and it is not clear how net free energy can be supplied to whistlers in this way.

What appears to be required to make the two theories compatible is a mechanism whereby phase bunching can enhance wave growth. A possible resolution of chis impasse has been suggested by Winglee [1985] who proposed a transient form of wave growth due to a postulated initial phase bunching of the particles. Although this growth mechanism is different from that in the random phaae approximation, it can be driven by the same sources of free energy as can a maser mechanism. In the following discussion it is assumed that the growth mechanism is analogous to that proposed by Winglee.

For AKR there is a variety of possible sources of free energy. Melrose [1976] propoaed a source due to a beam (on theoretical grounds), and Wu and Lee [1979] identified a more effective source due to a loss cone anisotropy. These were discussed critically by Gurnett and Anderson [1981] who identified a further possible source of free energy in a "hole" in the distribution function. More recent evidence for "electron conics" [Menietti and Burch, 1985] offers yet a further possibility.

In principle the phase–bunching mechanism is not dependent on which of these or other sources of free energy actually drives the growth. However, the parameters $v_\|$, $v_\perp$, v_c and v_0 do depend on the form of the free energy. The source of free energy is localized in velocity space, and $v_\|$, $v_\perp$ are to be identified with this location. The parameters v_c, v_0 are those of a resonant semicircle (cf. equation (5)) and must be such that the semicircle passes through the source of free energy. In addition, the free energy is

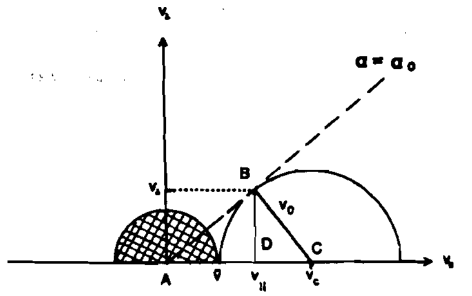

Fig. 2. For an idealized loss cone distribution there are no particles between the line $a = \alpha_0$ (α_0 = loss cone angle) and the $v_\|$ axis, and there is a sharp gradient in f near the edge of the loss cone. The resonance circle drawn (center $(v_c, 0)$ and radius v_0) passes through a point $(v_\|, v_\perp)$ on the edge of the loss cone, such that the circle lies outside the shaded circles or radius v where the thermal electrons are located.

associated with a gradient in velocity space, and there is an optimum resonant semicircle which weights the maximum contribution to wave growth from the arc through the source of free energy and the minimum contribution to damping from the remainder of the semicircle. The values of v_c, v_0 are to correspond to this optimum semicircle.

For illustrative purposes let us assume that the source of free energy is an idealized loss cone distribution, as illustrated in Figure 2. The source of free energy is along the line $\alpha = \alpha_0$, where α_0 is the loss cone angle. In practice the distribution function decreases with increasing v, and there is a region near the origin where the loss cone is filled by cold electrons. The optimum semicircle in this case is illustrated in Figure 2. For this semicircle one has, from triangle ABC in Figure 2

$$v_0 = v_c \sin\alpha_0 \tag{22a}$$

$$(v_\parallel^2+v_\perp^2)^{1/2} = v_c \cos\alpha_0 \tag{22b}$$

and, from triangle DBC in Figure 2,

$$v_c - v_\parallel = v_0 \sin\alpha_0 \tag{23a}$$

$$v_\perp = v_0 \cos\alpha_0 \tag{23b}$$

In addition for the semicircle not to intersect the region $v < \bar{v}$, to which the cold electrons are assumed to be confined, one requires

$$v_c - v_0 \geq \bar{v} \tag{24}$$

with the equality sign for the optimum semicircle in this idealized case.

Using the relations (22) and (23) we may eliminate v_c and $v_\parallel$ in (11) in favor of α_0 through

$$\frac{v_c - v_\parallel}{2v_\parallel} = \tfrac{1}{2}\tan^2\alpha_0 \tag{25}$$

The functions of velocity which appear in (20) and (21) simplify, provided we further assume that $v_{IR\parallel}$ is approximately equal to v_c. This assumption provides minimum values of E and $\Delta\omega$. The relevant results are

$$\frac{E}{E_0} = \frac{1}{4\pi}\tan\alpha_0 \tag{26}$$

and

$$(\Delta\omega)_{min} = \sin\alpha_0 \left[\frac{1}{8\pi}\left|\frac{d\omega}{dt}\right|\right]^{\frac{1}{2}} \tag{27}$$

respectively.

The detailed results (25) to (27) are illustrative and are not used below. The important point is that the parameters in the theory are to be determined by the form of the distribution function, and specifically by the location of the source of free energy in velocity space.

5. Discussion of the Mechanism

In this section we discuss several aspects of the model, concentrating on the differences between the applications to discrete VLF emissions and to AKR.

5.1. Axial Versus Azimuthal Bunching

A notable qualitative distinction between cases with $N^2\cos^2\theta > 1$ and $N^2\cos^2\theta < 1$ is that in the former case phase bunching is predominantly axial and in the latter case it is predominantly azimuthal [e.g., Chu and Hirshfield, 1978; Winglee, 1983]. This is demonstrated explicitly in the appendix. Whistlers have $N^2\cos^2\theta \gg 1$, and the x mode waves of interest here have $N^2\cos^2\theta \ll 1$. While the distinction between axial and azimuthal bunching is important in understanding the mechanisms in detail, it is of no significance for the model outlined above.

5.2. The Frequency Mismatch

The treatment given in the appendix shows that phase bunching is effective slightly off resonance. Specifically, the frequency mismatch

$$\Delta = \omega - s\Omega_e(1-v^2/c^2)^{\frac{1}{2}} - k_\parallel v_\parallel \tag{28}$$

needs to be greater than the growth (or damping) rate in order for phase bunching to occur with little energy transfer. In addition, phase bunching occurs only for $(1-N^2\cos^2\theta)\Delta > 0$, so that the required sign of Δ is different for whistlers and for x mode waves. Thus on entering the IR the electrons should have a nonzero Δ of the appropriate sign for phase bunching, and as they drift through the IR, Δ should decrease in magnitude until the electrons are close enough to resonance to transfer energy to the waves. This requirement has been included in the VLF case by Winglee [1985]. There is no difficulty in accommodating the required change in Δ, provided that the consistent wave condition $d\Delta/dt = 0$ remains a valid first approximation. With Δ changing by less than about ω_T in a resonance time, which is assumed to be of the order of ω_T^{-1}, this requires $|d\Delta/dt| \lesssim \omega_T^2$. Nonlinear corrections have been ignored in the resonance condition, and it is consistent to neglect them here, provided the actual frequency drift rate $|d\omega/dt|$ exceeds the required $|d\Delta/dt|$.

5.3. Lateral Extent and Lateral Drift of the IR

In the case of whistlers it is reasonable to assume that the waves are propagating along the field lines ($|\cos\theta| = 1$), and then the lateral extent of the IR is of only secondary importance compared with its length along the field lines. The angular distribution of the radiation in the case of AKR is likely to be similar to that which is thought to occur for maser emission from a loss-cone distribution. Thus the radiation should be confined to the surface of a hollow cone with half-angle $\theta = \arccos(v_c/c)$ [e.g., Hewitt et al., 1982] with its apex at the point of emission. In order for radiation emitted by particles at one end of the IR to phase bunch particles at the other

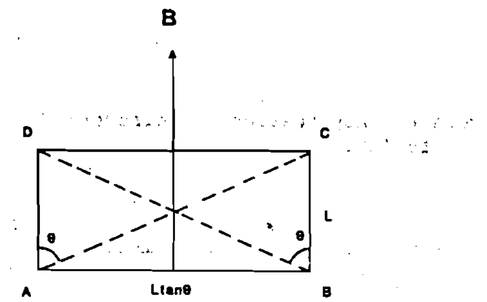

Fig. 3. Electrons leaving the bottom of the IR at points A and B radiate on the surface of cones with half-angle θ. The lateral extent of the IR should be such that these cones pass through the top rather than the sides of the IR. The minimum width is $L\tan\theta$, such that rays emitted at A and B pass through D and C, respectively.

end of the IR, a distance L along the field lines away, the lateral extent of the IR needs to be greater than $L\tan\theta$, as illustrated in Figure 3.

There is a strong tendency for the IR to spread across the field lines in all directions due to the large angle θ of emission. The lateral extent of the IR should adjust quickly to fill the entire cross-sectional area occupied by electrons with available free energy. However, regions with cross-sectional area greater than about $(L\tan\theta)^2$ can operate independently of each other, and it is probable that the source is composed of individual subsources of roughly cylindrical shape with height to diameter in the ratio $|\cos\theta| \approx v_c/c$. A region of larger lateral extent than this is likely to break up due to phase coherence gradually being lost over lateral distances greater than about $L\tan\theta$.

5.4. Triggering

The feedback mechanism can be initiated by a triggering wave. This wave must have the appropriate properties to resonate with the electrons with the available free energy, and the amplitude of this wave must be adequate to cause phase bunching in the time available. Once the phase bunching has been initiated, a self-sustaining emission can continue and drift away in frequency from the triggering wave.

An x mode emission due to an upward directed loss-cone distribution [e.g., Wu and Lee, 1979; Melrose et al., 1982; Omidi and Gurnett, 1982] could be triggered by another x mode wave with properties similar to those of the emitted wave. For example, AKR could be triggered by a type III burst as follows. The incoming type III radiation splits into local x mode and o mode components. Provided the x mode component is incident on the cutoff layer $\omega \approx \omega_X$ at an angle $\theta \approx \arccos(v_c/c)$ to the magnetic field, then just after reflection the wave has appropriate conditions to trigger an emission. That is, this wave has the appropriate value of ω and θ to resonate with the relevant electrons and to phase bunch them.

One can envisage more complicated processes which could trigger x mode emissions. An example is that for the o mode component of the incoming type III emission. This component can resonate with upgoing electrons ($k_\parallel < 0$, $v_\parallel < 0$) at $\omega < \Omega_e$ and can phase bunch them. These upgoing electrons can then radiate in the x mode due to this phase bunching, and thereby initiate a self-sustaining x mode emission.

The details of how triggering might occur warrant a careful investigation. The discussion given here indicates only that triggering is possible in principle and that it is plausible that it occurs in practice.

6. Application to DAM and AKR

The main predictions of the feedback model for fine structures in AKR and DAM are that the drift rate should lie in the range $\min[v_\parallel, v_c] < v_{IR\parallel} < \max[v_\parallel, v_c]$, and that the minimum bandwidth is approximately the trapping frequency.

The frequency drift rates in Jovian S bursts were analyzed by Ellis [1974] under the assumption that the emission is at the local cyclotron frequency for an electron propagating along a magnetic dipole field line. The observed drift rates fit quite well with this model for electrons with speed $\approx$ 0.1 c. In the feedback model the drift rate $v_{IR\parallel}$ of the IR is restricted to close to the parallel velocity of the electron. It follows that the predicted drift rate (cf. equation (11)) is nearly the same as in the model assumed by Ellis, provided that the parallel electric field is unimportant. Thus for DAM the feedback model provides an acceptable explanation for the characteristic observed frequency drifts. The usual drift is from high to low frequency, implying an upward drifting IR and hence a source of free energy in upward propagating electrons. A loss cone distribution of reflected electrons is plausibly the source of such free-energy. There are, however, rarer examples of drifts from low to high frequency, and drifts which reverse their sense. One may account for drifts from low to high frequency by postulating a source of free energy in downward propagating electrons. However, it is difficult to account for drifts which reverse their sense because this suggests that $v_{IR\parallel}$ passes through zero, which is not possible for x mode emission.

Ellis [1974] found some fine structures in DAM with bandwidths at the minimum instrumental limit of a 2 kHz at 10 MHz. Setting $(\Delta\omega)_{min} \approx \omega_T$, and assuming $\Omega_e/2\pi \approx 10$ MHz, $v_\perp \approx 0.1c$, the electric field strength implied by (18) is $E \approx 40$ mV m^{-1}. There is no simple way of relating this parameter to observed quantities. By way of illustration, let us suppose that the source region for DAM consists of many elementary source regions each associated with an IR, and that these IRs fill some fraction of the volume of the average source. The average brightness temperature for DAM is $\gtrsim 10^{17}$ K [Dulk, 1970]. The brightness temperature from each IR can be estimated from E, the assumed solid angle $\Delta\Omega$ of emission and the assumed relative bandwidth $\Delta f/f$. For $E \approx 40$ mV m^{-1} one finds $T_B \approx 3 \times 10^{13}/\Delta\Omega(\Delta f/f) \approx 10^{17}/\Delta\Omega$, where we set $\Delta f/f \approx (2$ kHz$)/10$ MHz) to obtain the final estimate. The emission is highly directional, implying $\Delta\Omega \ll 1$. Provided the filling factor is not too small, this estimate of T_B is compatible with the observed brightness temperatures. Another test, which is not independent of the foregoing one, arises from the estimate that the source size is

$\leq$ 400 km [Dulk, 1970]. If this is assumed to be the lateral extent of the IR, then with $v_c/v \approx v_\parallel/c \approx 0.1$, the implied length L of the IR from $L\tan\theta < 400$ km is $L < 40$ km. On estimating L from (15') with $|v_\parallel - v_{IR\parallel}|$ and $v_{IR\parallel}$ of the order of 0.1 c and $d\omega/dt = 2\pi \times 10$ MHz s^{-1}, one finds $L \approx 10$ km. These estimates suggest that the reported observations of DAM and its fine structures are consistent with the feedback model.

The comparison of the predicted and observed drift rates is far less satisfactory for AKR than for DAM. Gurnett and Anderson [1981] analyzed the observed drift rates in terms of motion of electrons emitting at the cyclotron frequency, i.e., using the same model as Ellis [1974]. The drift velocities so calculated were between 3 and 300 km s^{-1}, which they tentatively related to the ion sound speed. Morioka et al. [1981] obtained qualitatively similar results and argued that the velocity increases with height in the auroral zone. They inferred that the deduced source motion is closely related to the parallel electric field. These inferred velocities are at least a factor of 10 smaller than the typical speeds of the electrons thought to have available free energy. This poses a serious difficulty. For the x mode it is essential that $k_\parallel v_\parallel$ in (1) be positive, such that the requirement $\omega > \omega_x$ (cf. equation (4)) is satisfied. For $\omega_p/\Omega_e \ll 1$ one requires $|\cos\theta| \geq \omega_p/\Omega_e$ [e.g., Hewitt et al., 1982] and hence $v_{IR\parallel} \approx v_c \approx v_\parallel \geq \omega_p c/\Omega_e$. Arbitrarily slow drift rates $v_{IR\parallel}$ are not consistent with the kinematics for x mode emission.

To see how this difficulty might be overcome, consider how the expression (11) for the frequency drift rate might lead to $|d\omega/dt| \ll |v_\parallel d\Omega_e/dz|$, where $v_\parallel d\Omega_e/dz$ is the rate assumed in the model used by Ellis [1974], Gurnett and Anderson [1981] and Morioka et al. [1981]. We have already argued that $v_{IR\parallel}$ must lie between v_c and $v_\parallel$ and that $v_c v_\parallel$ is positive and cannot be arbitrarily small for the x mode. Thus it does not seem possible to account for the low drift rates in terms of $|v_{IR\parallel}| \ll |v_\parallel|$, i.e., in terms of a slowly drifting IR, while retaining the assumption that the emission is in the x mode. In (11) there is also a contribution from the drift of the IR across the field lines. Motion of the IR toward the pole increases $|d\omega/dt|$ and motion toward the equator decreases $|d\omega/dt|$ compared with motion of the IR along the field lines. Although in principle the IR could drift fast enough across the field lines to give $|d\omega/dt| \ll |v_\parallel d\Omega_e/dz|$ due to this effect, this is an unlikely explanation because of the very narrow range of latitudes to which the relevant ("inverted V") electrons are confined at any one time.

There are two other possibilities. One involves the parallel electric field. It follows from (11) that we can have $|d\omega/dt| \ll |v_\parallel d\Omega_e/dz|$, provided the condition

$$\frac{v_c - v_\parallel}{2v_\parallel}\frac{E_\parallel}{E_0} \approx -1 \qquad (29)$$

is satisfied. However, although one could envisage this condition being satisfied incidentally in specific events, there is no apparent reason why it should normally be satisfied. Indeed, for the expected values of $v_c - v_\parallel$, $v_\parallel$, $E_\parallel$ and E_0 in this AKR source region, it is likely that the left-hand side of (29) is positive and ≥ 1. The other possibility is that the emission occurs very close to the cutoff frequency. Then we can have $v_{g\parallel} = \partial\omega/\partial k_\parallel \ll v_\parallel$, and the drift of the IR may be slow, i.e., one may have $v_{IR\parallel} \approx v_{g\parallel} \ll v_\parallel$. Then, according to (3) one has

$$\frac{d\omega}{dt} \approx -\frac{v_{IR\parallel} v_{g\parallel}}{v_\parallel}\frac{d\Omega_e}{dz} \qquad (30)$$

where we ignore a term from $dk_\parallel/dz$ and assume $v_{IR\perp} = 0$. This possibility presents difficulties due to $d\omega/dt$ and $v_\parallel d\Omega_e/dz$ having opposite signs. Individual emissions are necessarily restricted to the narrow range of frequencies $\ll \omega_p^2/\Omega_e$ where N_x is much less than unity.

It may be concluded that none of these possibilities is favorable. In order to overcome the dilemma associated with the slow drift rates observed in AKR, there seem to be three options: (1) the drifts are as the theory implies ($d\omega/dt \approx v\, d\Omega_e/dz$), and the observations require interpretation, (2) the feedback mechanism proposed here is incorrect, or (3) the emission is not at s = 1 in the x mode. We now argue for option 1 and comment on option 2. Option 3 is not considered further.

Option 1 needs to be explored from an observational viewpoint. Consider the situation with DAM. The high drift rates observed [Ellis, 1974] are seen only at the highest resolution. At lower resolution the rapidly drifting bursts have envelopes which have much slower drift rates. It is conceivable that the measured drift rates for fine structures in AKR correspond to such envelopes, and that on a fine scale these envelopes will be seen to consist of many rapidly drifting, presently unresolved bursts. A superficial inspection of the records published by Gurnett and Anderson [1981] shows a fuzziness in the slowly drifting structures which is suggestive of their being envelopes of such more rapidly drifting structures.

Option 2 would require major rethinking of the feedback mechanism. Here the feedback is due to the relative motion of waves and particles through the IR. Calvert [1982] has developed a feedback model for AKR in which reflection of the waves (on both sides of a source region) feeds them back to interact with the particles. Thus Calvert's feedback mechanism involves only the waves. A further possibility would be to have a feedback of the particles. For example, a parallel electric field can trap electrons between a magnetic mirror ($\sin\alpha = 0$) below and the reflection ($v = 0$) due to the electric field above. Phase-bunched electrons could reenter the IR after a pair of such reflections. Further possibilities arise if one allows more than one wave-particle interaction, e.g., one could have two complementary IRs, one for upgoing and the other for downgoing electrons in the foregoing model, with the phase bunching in one causing growth in the other after the electrons have reflected. However, we are forced to consider these alternatives only if option 1 turns out to be excluded by observation.

Let us therefore return to the suggestion that the observed slow drifts correspond to the envelope of unresolved rapidly drifting bursts. It is natural to attribute the slow drift to that of an electric field structure. A local electric field can distort the distribution function locally on a short time scale, making free energy available on this time scale, as should be the case according to an argument presented in section 1. This complements the suggestions made on observational grounds [Gurnett and Anderson, 1981; Morioka et al., 1981] in favor of a source related to an electric field structure. There is also evidence for this in modulation of AKR at a frequency characteristic of ion waves [Grabbe, 1982], as these waves are known to be associated with electric field structures [Mozer et al., 1980].

The postulated fine structures should be drifting either at about $d\omega/dt \simeq v_\parallel d\Omega_e/dz$ or at $d\omega/dt \approx (d\Omega_e/dz)(v_c - v_\parallel)\, E_\parallel/E_0$ if the latter is the larger, as seems likely. The bandwidth may be narrower than can currently be resolved. A bandwidth of $\simeq$ 1 kHz has been quoted [Gurnett and Anderson, 1981] for the observed fine structures. If these were the basic fine structures then theory would imply $\omega_T/2\pi \approx 1$ kHz. The implied electric amplitude from (18) with $\omega_T/2\pi = 1$ kHz, $\Omega_e/2\pi = 300$ kHz, $v_\perp = 10$ m s^{-1} is $E = 1$ V m^{-1}, which is very high. If the true fine structures have an intrinsic bandwidth $\approx$ 100 Hz then the electric field is $\approx$ 10 mV m^{-1}, and if the bandwidth is $\simeq$ 10 Hz then the electric field is $E \approx 0.1$ mV m^{-1}. The saturation value of E may be estimated from the theory, e.g., using (20), or the corresponding result (26) for an idealized loss cone distribution, with $E_0 \approx 0.1$ mV m^{-1} in the auroral zones. For a field $E \approx 1$ mV m^{-1} the intrinsic bandwidth is only 30 Hz, for example, and this is probably smaller than the Doppler spread. The actual relative bandwidth $\Delta\omega/\Omega_e$ expected would then be of the order of $\Delta v_\parallel v_c/c^2$, where $\Delta v_\parallel$ is the range of $v_\parallel$ for the electrons driving the wave growth. For electrons with an energy around 1 keV this Doppler spread implies a bandwidth of several hundred hertz.

7. Discussion and Conclusions

A general conclusion of this investigation is that the ideas originally developed for discrete VLF emissions [Helliwell, 1967] may be adapted to apply to discrete x mode emissions. Qualitatively this makes it plausible that the similarities in the fine structures observed in AKR, DAM and VLF emissions are due to the underlying emission mechanisms being similar.

The most important difference between the applications to whistlers and to x mode waves concerns the drift velocity and the associated drift rate in frequency. For whistlers the drift rate of the IR is only weakly constrained. It must lie between the group velocity of the whistlers and the parallel velocity of the resonant electrons, but these have opposite signs. The analogous requirement for x mode waves is quite restrictive. Again the velocity of the IR must lie between the parallel velocity of the waves and of the particles (i.e., between v_c and $v_\parallel$) in order for the feedback process to be possible, but in the x mode case, v_c and $v_\parallel$ must have the same sign, and in practice they are roughly equal. In the absence of a parallel electric field the implied frequency drift rate is $d\omega/dt \approx v_\parallel d\Omega_e/dz$. However, a parallel electric field can have an important effect on the drift rate, as is apparent from (11), for a parallel electric field $E_\parallel$ in excess of a characteristic electric field E_0 (cf. equation (12)), which is of the order of 0.1 mV m^{-1} in the auroral zone. If the effect of the electric field dominates then the drift rate is proportional to $E_\parallel$.

The observed drift rates in DAM fit quite well with the theory, but the drift rates in AKR are too small. It is suggested that the finest structures in AKR have yet to be resolved, and that the observed drift rates are those of envelopes of much more rapidly drifting narrow-band emissions.

One of the motivations for this investigation is the evidence that AKR (and Jovian HOM) can be triggered by solar radio bursts [Calvert, 1981, 1985a, b]. Although no detailed model for such triggering has been developed here, it is highly plausible that discrete AKR emissions can be triggered if they are generated in a way analogous to discrete VLF emissions, whose ability to be triggered is well established. We suggest that incoming solar radio emission splits into x mode and o mode components, and that the triggering is probably due to the x mode component just after it has been reflected.

The model outlined here is essentially kinematic. The nature of the growth mechanism involved has yet to be agreed upon, and remains controversial for the VLF case. Here we have argued qualitatively in favor of a mechanism proposed by Winglee [1985] for enhanced growth due to phase bunching; the essentially new feature (in this context) in Winglee's theory is the relaxation of the time asymptotic condition in kinetic theory. This allows a new type of growth which is transient in that it is associated with the initial conditions (e.g., due to phase bunching) and which dies away due to phase mixing. This type of wave growth requires a source of free energy analogous to that required for growth due to conventional cyclotron maser action. As remarked in section 1, the model based on the cyclotron maser theory itself requires features in the distribution function which drive the instability and persist for only a few tens of milliseconds in the case of AKR. The viewpoint at which we now arrive is that these fine structures drive discrete emissions through a phase-bunching mechanism. These fine structures in the distribution seem (on both observational and theoretical grounds) to be related to parallel electric fields.

Appendix: Phase Bunching

A simple model for the interaction between a spiraling electron, with momentum

$$\mathbf{p} = (p_\perp \cos\phi,\ p_\perp \sin\phi,\ p_\parallel) \tag{A1}$$

and position

$$\mathbf{x} = \mathbf{x}_0 + (R\sin\phi,\ -R\cos\phi,\ v_\parallel t) \tag{A2}$$

and a wave in the x-z plane with amplitude

$$\mathbf{E} = \mathbf{e}\varepsilon(t)\, e^{-i\psi(t,\mathbf{x})} + \text{c.c.} \tag{A3}$$

(c.c. = complex conjugate) is as follows. After substituting (A1), (A2) and (A3) in the equation of motion, and writing

$$\psi(t,X) = \Psi - k_\perp R\sin\phi \quad (A4)$$

with

$$\Psi = -k.x_0 + (\omega - k_\parallel v_\parallel)t \quad (A5)$$

one can expand in Bessel functions to find

$$\frac{dp_\perp}{dt} = \sum_{s=-\infty}^{\infty} -e\varepsilon(t)\left[e^{-i(\Psi-s\phi)}\left\{(1-N\beta_\parallel\cos\theta)\left(e_x \frac{s}{k_\perp R} J_s - ie_y J_s'\right) + N\beta_\parallel \sin\theta e_z \frac{s}{k_\perp R} J_s\right\} + c.c.\right] \quad (A6a)$$

$$\frac{dp_\parallel}{dt} = \sum_{s=-\infty}^{\infty} -e\varepsilon(t)\left[e^{-i(\Psi-s\phi)}\left\{\left(1-\frac{s\Omega}{\omega}\right)e_z J_s + N\beta_\parallel\cos\theta\left(e_x \frac{s}{k_\perp R} J_s - ie_y J_s'\right)\right\} + c.c.\right] \quad (A6b)$$

$$\frac{d\phi}{dt} = \Omega + \sum_{s=-\infty}^{\infty} -\frac{e\varepsilon(t)}{p_\perp}\left[ie^{-i(\Psi-s\phi)}\left\{(1-N\beta_\parallel\cos\theta)\left(-e_x J_s' + ie_y \frac{s}{k_\perp R} J_s\right) + iN\beta_\perp \sin\theta e_y J_s - N\beta_\parallel \sin\theta e_z J_s'\right\} + c.c.\right] \quad (A6c)$$

where the argument of the Bessel functions is $k_\perp R$, with $R = v_\perp/\Omega$, $\Omega = eB/\gamma m$, and with $N = kc/\omega$, $k_\perp = k\sin\theta$, $k_\parallel = k\cos\theta$, $p_\perp = \gamma m v_\perp = \gamma m\beta_\perp c$, $p_\parallel = \gamma m v_\parallel = \gamma m\beta_\parallel c$.

In the small gyroradius limit $k_\perp R \ll 1$, only $s = 0, \pm 1$ contribute. Here we concentrate on the case $s = 1$ and assume that the frequency mismatch

$$\Delta = \omega - \Omega - k_\parallel v_\parallel \quad (A7)$$

is smaller than other relevant frequencies. Then (A6) may be reduced to

$$\frac{d(\Psi-\phi)}{dt} = \Delta - \frac{e\varepsilon(t)}{2mc}\left[\frac{i}{\gamma\beta_\perp} e^{-i(\Psi-\phi)}\left\{(1-N\beta_\parallel\cos\theta)(e_x - ie_y) + N\beta_\parallel\sin\theta e_z\right\} + c.c.\right] \quad (A8a)$$

$$\frac{d\Delta}{dt} = -\frac{e\varepsilon(t)}{2mc}\left[\frac{\beta_\perp}{\gamma} e^{-i(\Psi-\phi)}\left\{\omega(1-N^2\cos^2\theta) - \Delta\right\}(e_x - ie_y) + c.c.\right] \quad (A8b)$$

$$\frac{d\gamma}{dt} = -\frac{e\varepsilon(t)}{2mc}\left[\beta_\perp e^{-i(\Psi-\phi)}(e_x - ie_y) + c.c.\right] \quad (A8c)$$

The bounce frequency is identified by approximating $d^2(\Psi-\phi)/dt^2$ by $d\Delta/dt$ from (A8a), and omitting the term Δ in the braces in (A8b) to find

$$\frac{d^2(\Psi-\phi)}{dt^2} = -\omega_T^2\cos(\Psi-\phi) \quad (A9)$$

with

$$\omega_T^2 = \frac{e\varepsilon\beta_\perp\omega(1-N^2\cos^2\theta)(e_x - ie_y)}{\gamma mc} \quad (A10)$$

where ε is assumed constant for this purpose.

The phase bunching of relevance here may be described as follows. We assume that $\varepsilon(t)$ may be regarded as constant. We use (A6c) to write $\phi = \Omega t$ to a first approximation, finding $\Psi - \phi = -k.x_0 + \Delta t$. Then on integrating (A8b), the integral is of the form

$$\Delta(t) - \Delta(0) = A\int_0^t dt'\left[e^{i\psi_0} e^{-i\Delta t'} + c.c.\right] = A\left[e^{i\psi_0}\frac{i}{\Delta}(e^{-i\Delta t} - 1) + c.c.\right]$$

where A is a constant and where ψ_0 is the initial phase $k.x_0$. On substituting this expression for $\Delta(t)$ into (A8a) and integrating, there are various terms which describe oscillating phase changes, and one term of the form

$$\Psi - \phi = -\frac{e\varepsilon}{mc}\frac{\omega\beta_\perp(e_x - ie_y)}{\gamma\Delta}\sin\psi_0(1-N^2\cos^2\theta)\,t \quad (A11)$$

For $\Delta(1-N^2\cos^2\theta) > 0$ this causes electrons with relative initial phase $\psi_0 > 0$ to drift toward $\psi = 0$, and causes electrons with initial phase $\psi_0 < 0$ also to drift toward $\psi = 0$, leading to phase bunching about $\psi = 0$. The rate at which this phase bunching occurs is ω_T^2/Δ. The analysis breaks down if $\varepsilon(t)$ varies secularly with time over a time scale $< 1/|\Delta|$, i.e., when $|\Delta|$ is less than about the growth or damping rate.

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