

# PLASMA AND RADIATION PROCESSES (INVITED REVIEW)\*

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(Received 13 June, 1985)

**Abstract.** The physical processes leading to metre-wave and microwave emissions from the solar corona are reviewed and the possibility of observing analogous phenomena on other stars is discussed. Particular emphasis is placed on three alternative processes for the emission from flare stars: cyclotron maser emission, second harmonic plasma emission, and gyro-synchrotron emission.

## 1. Introduction

There has been a rapid expansion of interest in the radio emissions from stars in recent years, due in large part to the suitability of facilities like the VLA for observing these phenomena (e.g., the reviews by Hjellming and Gibson, 1980; and Gibson, 1983). The emission mechanisms which operate in stars are likely to have solar counterparts, and the interpretation of stellar radio emission has indeed been based almost entirely on the interpretation of solar phenomena. The specific mechanisms which are known to operate in the solar corona are bremsstrahlung, fundamental and second harmonic plasma emission, gyro-synchrotron emission, and electron-cyclotron maser emission. Bremsstrahlung is the basic thermal emission process for a plasma but, although it is important in some stellar emissions (e.g., from stellar winds), it seems unnecessary to discuss it in detail here. In this review we concentrate on those mechanisms which seem most favourable for stellar emissions – i.e., electron-cyclotron maser emission, second harmonic plasma emission, and gyro-synchrotron emission.

Radio observations of the Sun have been made since the 1940's (e.g., the reviews by Wild *et al.*, 1963; Kundu, 1965; and, more recently, by Kundu and Gergely (eds.), 1980 and McLean and Labrum (eds.), 1985). The characteristics of solar radio emission change markedly as the wavelength varies from the microwave range through decimetre wavelengths to metre and longer wavelengths. In the microwave range the dominant emission mechanisms are gyro-synchrotron emission in bursts, and thermal emission in the 'slowly-varying' component from localized hot spots in the corona. A phenomenon thought to be of particular relevance to some stellar radio flares is that of 'spike bursts' which occur at about 1 GHz and which are thought to be due to electron-cyclotron maser emission. Figure 1 shows observations of 'spike bursts' by Slottje (1978). There is a rich variety of other types of emission at decimetric wavelengths (e.g., the review by Kuijpers, 1980). Studies of the decimetre wave phenomena have concentrated largely on fine structures, which could not be detected from a stellar source. The

\* Paper presented at the IAU Third Asian-Pacific Regional Meeting, held in Kyoto, Japan, between 30 September–6 October, 1984.

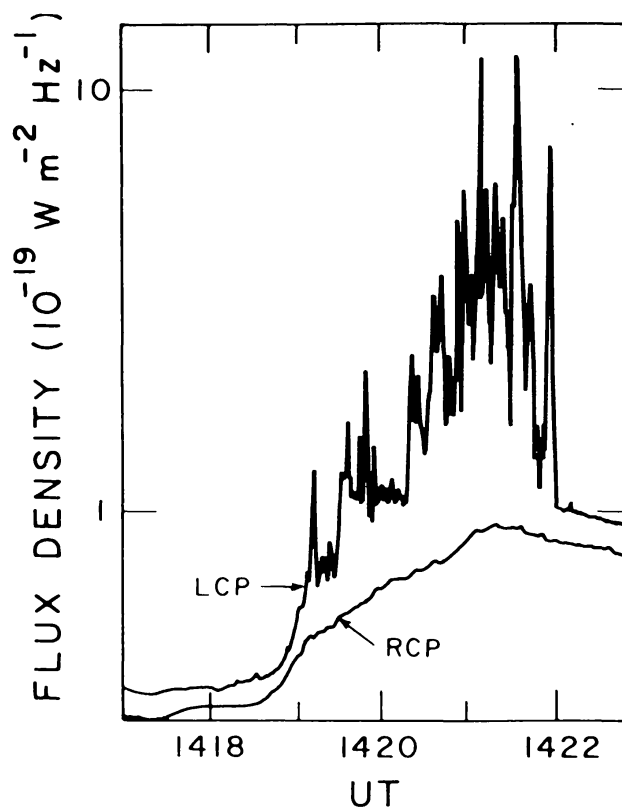


Fig. 1. Solar microwave 'spike bursts' as observed by Slottje (1978).

bursts at metre and longer wavelengths were classified by Wild (Wild and McCready, 1950) as types I, II, and III. Type-III bursts are due to electron streams which can propagate through the corona and the interplanetary medium, and have been studied out to beyond the orbit of Jupiter. Type-II bursts are due to flare-initiated shock waves in the corona. Both of these involve plasma emission at the fundamental and/or the second harmonic. Type-I emission is thought to arise from particles confined in a flux tube near an active region. The emission is at the fundamental of the plasma frequency; its generation is not adequately understood.

The radio emission from stars has been studied most extensively at frequencies  $\gtrsim 1$  GHz; stellar flares, when observed, can be up to  $10^4$  more intense than solar flares at similar wavelengths (Lovell, 1969; Spangler and Moffet†, 1976). Clearly gyro-synchrotron emission and electron-cyclotron maser emission are reasonable candidates for emission mechanisms since they are associated with radiation at  $\gtrsim 1$  GHz from the Sun. Second harmonic plasma emission also needs to be considered because it is sometimes observed at these relatively high frequencies. However, fundamental plasma emission is never observed above a relatively low frequency, say  $\gtrsim 200$  MHz. The likely reason for this is the strong free-free absorption of fundamental plasma emission in relatively dense plasmas. Because of this frequency restriction we will not consider the process further.

In Section 2 we discuss the interpretation of two important parameters for solar and stellar radio emission: the brightness temperature and the degree of polarization. In Sections 3 and 4 we review the ‘coherent’ radiation mechanisms, electron-cyclotron maser emission and second harmonic plasma emission, respectively. Gyro-synchrotron emission is discussed briefly in Section 5.

## 2. Brightness Temperature and Polarization

Solar, and presumably stellar, radio emissions are due to non-relativistic or mildly relativistic electrons. An estimate of the brightness temperature of the source can provide important information on whether the emission mechanism is incoherent or coherent. Brightness temperatures less than about  $10^{10}$  K can be explained in terms of gyro-synchrotron emission, which is an incoherent mechanism, and brightness temperatures much greater than  $10^{10}$  K require a coherent mechanism such as plasma emission or electron-cyclotron maser emission. Similarly the degree of polarization can be useful in distinguishing between mechanisms, with second harmonic plasma emission being weakly polarized and electron-cyclotron maser emission being highly polarized.

The brightness temperature  $T_B(\nu)$  is defined in terms of the specific intensity  $I(\nu)$  by

$$I(\nu) = \frac{2\nu^2}{c^2} k_B T_B(\nu), \quad (1)$$

where  $k_B$  is Boltzmann’s constant. In a non-thermal source  $T_B(\nu)$  is a function of frequency  $\nu$ .

In the solar corona, and stellar coronae and winds, the temperature is typically of order  $10^6$  K. Thermal emission from such a source would have  $T_B(\nu) \lesssim 10^6$  K, or perhaps somewhat higher if it were dominated by hot spots.

For an incoherent emission mechanism such as gyro-synchrotron emission,  $T_B(\nu)$  is limited by the effects of self-absorption, as in the thermal case. If emission at frequency  $\nu$  is due to electrons with a mean energy  $\langle E \rangle$  then self-absorption limits  $T_B(\nu)$  to

$$T_B(\nu) \lesssim \langle E \rangle / k_B. \quad (2)$$

An energy of 1 eV corresponds to a temperature of  $1.16 \times 10^4$  K, and hence mildly relativistic electrons ( $\langle E \rangle \lesssim 1$  MeV say) cannot produce emission with  $T_B(\nu) \gtrsim 10^{10}$  K.

Coherent mechanisms are implied by brightness temperatures  $\gtrsim 10^{10}$  K and indirectly by rapid large-scale intensity variations. Such variations indicate small sources and, hence, relatively high brightness temperatures. Thus, for example, the time-scale ( $\lesssim 10$  ms) for the rise and decay of solar ‘spike bursts’ indicates source sizes as small as 30 km to 3000 km and brightness temperatures  $\gtrsim 10^{15}$  K (Slottje, 1978).

In the coronae of stars radio emission splits into two natural modes which propagate independently of each other – i.e., the modes rapidly get out of phase. The modes are referred to as the  $o$  mode and the  $x$  mode and for almost all purposes they may be regarded as oppositely circularly polarized. The  $x$  mode has the same handedness as

a spiralling electron and, hence, is strongly favoured by electron-cyclotron maser emission. Gyro-synchrotron emission also favours the  $x$  mode but nowhere near as strongly. Although fundamental plasma emission can favour the  $o$  mode very strongly, second harmonic plasma emission is intrinsically weakly polarized.

The degree of circular polarization  $r_c$  may be defined in terms of the intensities  $I_o(\nu)$  and  $I_x(\nu)$  in the two modes by

$$r_c = \frac{I_x(\nu) - I_o(\nu)}{I_x(\nu) + I_o(\nu)}. \quad (3)$$

Values of  $|r_c| \cong 1$  from a stellar source are indicative of electron-cyclotron maser emission.

Note that large Faraday rotation should preclude any detection of linear polarization. From time to time linear polarization has been claimed in solar and stellar sources, but experiments which eliminate any possibility of instrumental polarization (e.g., Groggnard and McLean, 1973) have led to negative results for the detection of linear polarization.

### 3. Electron-Cyclotron Maser Emission

Twiss (1958) first showed that electrons with a distribution  $f$  which is an increasing function of the momentum  $p_{\perp}$  perpendicular to the magnetic field lines can lead to negative absorption just below the harmonics  $s = 1, 2, \dots$ , of the nonrelativistic gyro-frequency  $\Omega_e = eB/m$ . Twiss restricted his discussion to the case of zero-parallel momentum,  $p_{\parallel} = 0$ . The full resonance condition for a wave with angular frequency  $\omega$  and wavevector  $\mathbf{k}$ , and an electron with velocity  $\mathbf{v}$  is

$$\omega - k_{\parallel} v_{\parallel} = s\Omega_e \left( 1 - \frac{v_{\parallel}^2}{c^2} - \frac{v_{\perp}^2}{c^2} \right)^{1/2}, \quad (4)$$

and it is obvious that  $\omega < s\Omega_e$  for  $v_{\parallel} = 0$ . There is an extensive literature on electron-cyclotron instabilities but it was not until quite recently that the general case, with  $v_{\parallel} \neq 0$ ,  $k_{\parallel} \neq 0$  and with the relativistic effect retained on the right-hand side, was explored in detail. This general case turns out to be very favourable, allowing electron-cyclotron maser emission to occur under quite mild conditions.

One important requirement is that the plasma frequency  $\omega_p$  be less than or comparable to  $\Omega_e$ , and this condition is not satisfied in most plasmas. It is, however, satisfied above the auroral zones of the Earth, in the magnetospheres of Jupiter and Saturn and in some magnetic flux tubes in the lower solar corona. All these regions are sources of specific radio emissions whose only plausible interpretations involve electron-cyclotron maser emission. These are the auroral kilometric radiation (AKR), Jovian decametric radiation (DAM), Saturnian kilometric radiation, and solar 'spike bursts', respectively. The interpretations have been discussed by Wu and Lee (1979), Lee *et al.* (1980), Holman *et al.* (1980), Hewitt *et al.* (1981), Melrose *et al.* (1982, 1984), Omidi and Gurnett (1982), Melrose and Dulk (1982a, b) amongst others. The emissions are all very

bright, e.g.,  $\gtrsim 10^{17}$  K for DAM and  $\gtrsim 10^{15}$  K for solar ‘spike bursts’ and, with the exception of AKR, are essentially 100% circularly polarized. The most intense AKR bursts are  $\cong 100\%$   $x$ -mode but weaker bursts often have an  $o$ -mode component which tends to be observed only when  $\omega_p/\Omega_e$  is relatively close to unity.

Two important points were not recognized prior to the work of Wu and Lee (1979). First, the emission can occur above  $\omega = s\Omega_e$  for  $k_{\parallel}v_{\parallel} > 0$ . This is important since it allows fundamental ( $s = 1$ ) emission to occur above the cutoff frequency for the  $x$  mode. Second, the instability can be driven by electrons with  $\partial f/\partial v_{\perp} > 0$  in only a localized region of momentum space and such a distribution is automatically produced during driven precipitation.

These points can be appreciated with the aid of a particularly useful concept, that of a resonance ellipse. Equation (4) implies that resonance is possible only if  $\omega^2 < s^2\Omega_e^2 + c^2k_{\parallel}^2$  and that, if this inequality is satisfied, the resonant electrons are those which lie on an ellipse in  $v_{\parallel} - v_{\perp}$  space. The position and shape of the ellipse are completely determined by  $s$ ,  $\omega$ ,  $\Omega_e$ , and  $k_{\parallel}$  (which in turn is determined by the propagation angle  $\theta$  and the refractive index of the plasma). Depending on the distribution  $f(v_{\parallel}, v_{\perp})$  of electrons in velocity space, the wave either extracts energy from the resonant electrons and grows, or loses energy to them and is damped. In the former case, the intensity of the wave at any point increases exponentially with time, i.e.,  $I \propto \exp(2\Gamma_s t)$ , until the maser saturates. The dominant contribution to the growth rate  $\Gamma_s$  for the  $s$ th harmonic is given by an expression of the form

$$\Gamma_s \cong \int d^3v A_s(\mathbf{v}, \mathbf{k}) \frac{\partial f}{\partial v_{\perp}} \delta(\omega - k_{\parallel}v_{\parallel} - s\Omega_e/\gamma), \quad (5)$$

with  $A_s > 0$  and  $\gamma = (1 - v_{\parallel}^2/c^2 - v_{\perp}^2/c^2)^{-1/2}$ . Because of the delta function this corresponds to an integration around the resonance ellipse. Growth can occur if the resonance ellipse passes through a source of free energy where  $\partial f/\partial v_{\perp} > 0$ . This condition on  $\partial f/\partial v_{\perp}$  is analogous to the requirement of population inversion for the operation of a laser.

The source of free energy for all the maser emissions discussed earlier is believed to be a single-sided loss-cone distribution. This type of distribution is formed when electrons are accelerated downwards along converging magnetic field lines near the surface of a star or planet. Electrons with large pitch angles are reflected by the inhomogeneous magnetic field while those with small pitch angles precipitate. There are no electrons with small pitch angles in the upgoing part of the distribution, i.e., there is an upgoing loss cone. Figure 2 shows a contour map of a Maxwellian distribution with such a loss cone. The figure also shows a resonance ellipse for a typical growing wave passing through the region where  $\partial f/\partial v_{\perp} > 0$ .

The parameters of the resonance ellipse depend strongly on  $\omega$  and  $\theta$ . Small changes in either of these quantities can make  $\omega^2 > s^2\Omega_e^2 + c^2k_{\parallel}^2$  so that resonance is impossible, or can alter the size and position of the ellipse sufficiently for damping rather than growth to occur. The  $s$ th harmonic radiation from the loss cone driven maser has  $\omega$  just

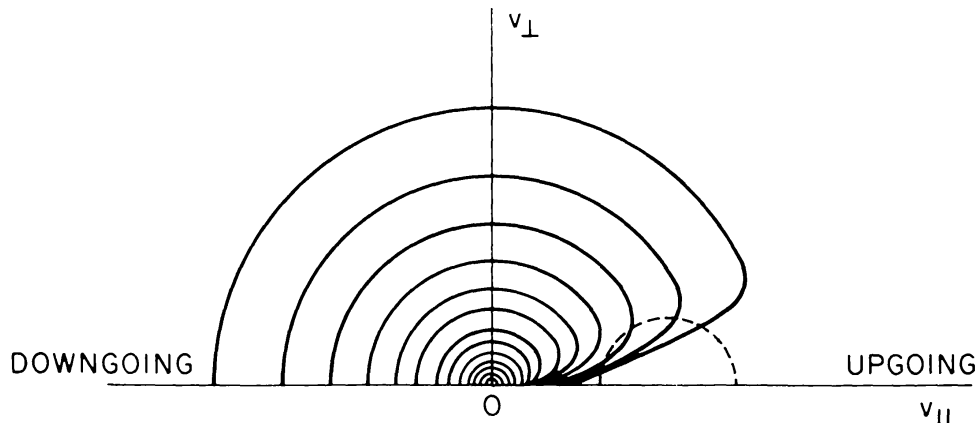


Fig. 2. Contour map in  $v_{\parallel} - v_{\perp}$  space of a Maxwellian distribution of energetic electrons with a loss cone in the upward direction. The superimposed dashed curve is a resonance ellipse for a growing wave.

above  $s\Omega_e$  and a narrow frequency bandwidth, typically  $\cong 0.001\omega$  to  $0.01\omega$ ; it is beamed upwards into a hollow cone with cone half angle  $\cong 70^\circ$  to  $80^\circ$ , cone width  $\lesssim 10^\circ$  and axis aligned with the magnetic field (Hewitt *et al.*, 1982).

In models for the solar corona, the electrons driving the maser typically have energies of 10–100 keV. A colder relatively dense background plasma is also present in the source region and this has a significant effect on the relative growth rates of the various modes because it changes the refractive index and, hence,  $k_{\parallel}$ . For energetic distributions of the type shown in Figure 2, the growth rates for the fundamental ( $s = 1$ )  $x$ -mode are appreciably larger than those for the other modes when  $\omega_p/\Omega_e \lesssim 0.3$ . At higher densities the  $s = 1$ - $x$  mode is suppressed, the  $s = 1$   $o$ -mode becomes dominant when  $0.3 \lesssim \omega_p/\Omega_e \lesssim 1.0$ , the  $s = 2$   $x$ -mode when  $1.0 \lesssim \omega_p/\Omega_e \lesssim 1.4$ . Since a single mode dominates in each range of  $\omega_p/\Omega_e$  the emissions from a localized, reasonably homogeneous source would be highly circularly polarized.

Melrose and Dulk (1984) estimated the brightness temperature of the maser to be

$$\frac{k_B T_B}{mc^2} \cong \eta \frac{c}{r_0 \omega} \frac{\Gamma}{\Omega_e}, \quad (6)$$

where  $r_0$  is the classical radius of the electron and  $\eta$  is the fraction of the energy density of the driving electrons converted to maser radiation. The maximum growth rates for the fundamental  $x$  mode for energetic electron number densities  $\cong 10^{13} \text{ m}^{-3}$  and temperatures  $\cong 10$  keV at frequencies  $\cong 1$  GHz are typically  $\cong 10^{-3} \Omega_e$  (Hewitt *et al.*, 1982); modest values of  $\eta$ ,  $\cong 10^{-3}$  say, lead to brightness temperatures  $\cong 10^{17}$  K.

The above estimates suggest that electron cyclotron maser emission can account satisfactorily for the high brightness temperatures and high degrees of circular polarization seen in solar 'spike bursts' and in some components of stellar emissions. Figure 3, for example, shows observations by Brown and Crane (1978) of flare emissions from the RS CVn-star V 711 Tau at two frequencies, 2.7 and 8.1 GHz. The rapidly varying left hand circularly polarized component at 2.7 GHz is probably due to this mechanism.

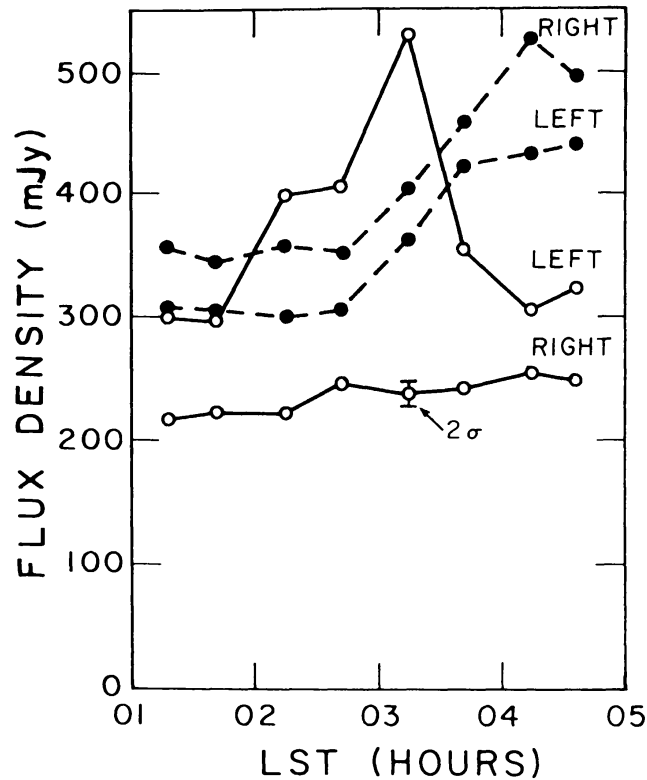


Fig. 3. Circularly polarized flare emissions from the RS CVn star V 711 Tau at two frequencies, 2.7 GHz (hollow circles) and 8.1 GHz (solid circles) (after Brown and Crane, 1978).

The absence of similar activity at 8.1 GHz is consistent with the narrow bandwidth of a maser.

There is, however, a difficulty with solar and stellar applications of this mechanism. There is an average decrease in magnetic field strength with increasing distance from the surface of the Sun or star, and the escaping radiation must, therefore, pass through layers where its frequency is an integer multiple of the local gyrofrequency and where gyromagnetic absorption by thermal electrons occurs. This absorption should prevent the fundamental modes from escaping and would allow the second harmonic modes to escape only under rather special conditions. It is by no means clear how this difficulty can be overcome, because growth at  $s \geq 3$  is certainly too weak to account for observed emissions.

#### 4. Second Harmonic Plasma Emission

Plasma emission was first treated quantitatively by Ginzburg and Zheleznyakov (1958). It involves emission at either the fundamental or the second harmonic of the plasma frequency  $\omega_p$ . Here we concentrate on the latter. The magnetic field plays only a minor role in determining the gross properties of the emissions and can be neglected for most purposes. Plasma emission cannot be treated as simply as the cyclotron maser mechanism since it proceeds in at least two stages, each of which introduces compli-

cations. In the first stage resonant interactions between energetic electrons and Langmuir waves generate high levels of Langmuir turbulence. The waves are longitudinal oscillations of the plasma electrons relative to the ions and propagate at frequencies just above  $\omega_p$ . The most favourable sources of free energy for the interaction are either an energetic electron stream or a double-sided loss cone distribution with a 'gap', i.e., a region with  $\partial f / \partial |v| > 0$  at  $|v| \neq 0$ . The second stage involves a nonlinear interaction between two Langmuir ( $L$ ) waves with wavevectors  $\mathbf{k}_1$  and  $\mathbf{k}_2$ , respectively, and the plasma to create an electromagnetic ( $t$ ) wave with wavevector  $\mathbf{k}$  and frequency just above  $2\omega_p$ . A necessary condition for this coalescence process ( $L + L \rightarrow t$ ) to occur is that  $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}$ . Since  $|\mathbf{k}| \ll |\mathbf{k}_1|, |\mathbf{k}_2|$ ; the Langmuir waves must have  $|\mathbf{k}_1| \cong |\mathbf{k}_2|$  and propagate in essentially opposite directions: i.e.,  $\mathbf{k}_2 \cong -\mathbf{k}_1$ .

An electron stream generates Langmuir waves in a filled cone in the direction of the stream so that for this free-energy source, an intermediate stage is required to produce Langmuir waves travelling in the reverse direction. This greatly complicates the treatment of the mechanism, making it a three-stage one, and effectively precluding a quantitative treatment in which one can have confidence.

Type-III solar radio bursts are generated by electron streams through plasma emission and it is reasonable to consider the possibility of observing analogous emissions from other stars. There are two classes of type-III events: storm-associated and flare-associated. The storm events start at frequencies  $\lesssim 100$  MHz and drift to lower frequencies; they are, therefore, not relevant to current observations of most flare stars. Although flare type-III bursts can start at frequencies  $\gtrsim 500$  MHz the highest values of  $T_B$  occur at frequencies lower than this. Moreover, the duration of these bursts at the highest frequency is short ( $\cong 0.1$  s). Thus type-III bursts at frequencies  $\gtrsim 500$  MHz could not be observed on other stars with current techniques.

The one plasma emission mechanism we discuss here involves generation of Langmuir waves by a loss cone distribution. Although this is known to be a possible mechanism for solar radio bursts (e.g., Zaitsev and Stepanov, 1983; and references given therein), it is not nearly as familiar as type-III-like plasma emission. It seems more favourable than the latter for scaling up to the higher frequencies and longer time-scales associated with flare star emissions.

A double-sided loss cone distribution with a gap can be formed when a group of accelerated electrons is injected near the top of a magnetic arch connecting two points on the surface of a star and containing a relatively dense thermal plasma. The energetic electrons with small pitch angles precipitate at the feet of the arch producing loss cones in both forward and backward directions. Over a longer time-scale the energetic electrons are scattered by the thermal plasma into the loss cone from which they are rapidly precipitated. A gap results because the scattering is more probable for the slower electrons. This type of distribution can develop only after the injection ceases and can last for only a few collision times (Hewitt and Melrose, 1985).

The loss cone distribution gives positive contributions to the growth rates for Langmuir waves propagating nearly perpendicular to the symmetry axis with phase speeds  $\cong \omega_p/k$  which are less than the speed for which the energetic distribution function

is a maximum. The thermal plasma gives negative contributions which prevent growth when the phase speed is too small. The net growth rates are quite large if the loss cone is large – i.e., loss cone angles  $\gtrsim 45^\circ$  – if the pitch angle distribution falls off rapidly within the loss cone and if the ratio of typical speeds for the energetic and thermal electrons is greater than about 10. Growth is possible for longer periods if the initial injected distribution is relatively hard, i.e., if its distribution function does not fall off rapidly with increasing electron energy. An attractive feature of this source of free energy is that it produces pairs of Langmuir waves satisfying  $\mathbf{k}_2 \cong -\mathbf{k}_1$  directly and no intermediate stage is necessary. The coalescence process produces electromagnetic waves with weak  $x$ -mode polarization.

The brightness temperature of plasma emission is limited by the balance between the coalescence process  $L + L \rightarrow t$  and the inverse decay process  $t \rightarrow L + L$ . In the present case, balance occurs when the effective temperatures of the two wave distributions are equal. These effective temperatures  $T_{\text{eff}}(\mathbf{k})$  are functions of  $\mathbf{k}$  with

$$W = \int \frac{d^3\mathbf{k}}{(2\pi)^3} k_B T_{\text{eff}}(\mathbf{k}), \tag{7}$$

representing the energy density in the respective waves. For transverse waves  $T_{\text{eff}}$  is just the brightness temperature  $T_B$ , which is a constant along a ray.

We may estimate the maximum  $T_B$  as follows. The observed  $T_B$  must be no greater than  $T_{\text{eff}}$  for the transverse waves at the source, and this can be no greater than  $T_{\text{eff}}$  for the Langmuir waves producing the transverse waves. If  $W_{\text{free}}$  is the free-energy density available to drive the instability generating the Langmuir waves, and  $\Delta k$ ,  $\Delta\Omega$  are, respectively, the ranges of wavenumbers and solid angles to which the Langmuir waves are confined, then

$$T_{\text{eff}} < \frac{(2\pi)^3 W_{\text{free}}}{k_B k^2 \Delta k \Delta\Omega}. \tag{8}$$

This can be rewritten in the form

$$T_{\text{eff}} < \eta \frac{mv^2}{2k_B} \frac{8nv^3}{v^3} \frac{n_1}{n} \left( \frac{\Delta k}{k} \Delta\Omega \right)^{-1}, \tag{9}$$

where  $W_{\text{free}} = \eta^{1/2} n_1 mv^2$ ,  $n_1$  and  $n$  are the number densities of the energetic and thermal electrons, respectively,  $v = \omega_p/k$  is the resonant speed and  $\nu = 2\omega_p/2\pi$  is the frequency of observation. For 10–100 keV energetic electrons,  $n \cong 10^{15} \text{ m}^{-3}$  and  $\nu \cong 1 \text{ GHz}$  we have  $mv^2/2k_B \cong 10^8\text{--}10^9 \text{ K}$  and  $8nv^3/v^3 \cong 10^{11}$ . Except for quite large values of  $n_1/n$  (e.g.,  $\gtrsim 10^{-4}$ ) these numbers suggest that it is unlikely that this mechanism could produce radiation with a very high brightness temperature. The observed maximum  $\cong 10^{13} \text{ K}$  for plasma emission from the solar corona is unlikely to be exceeded to any significant extent in other sources.

## 5. Gyro-synchrotron Emission

Electrons spiralling in a magnetic field radiate at frequencies  $\omega$ , given by (4), with positive integral values of  $s$ . Simple analytic treatments for this emission have long been known in the nonrelativistic or ‘cyclotron’ limit and in the ultrarelativistic or ‘synchrotron’ limit. In the intermediate energy range from about 10 keV to 1 MeV, neither of these approximate treatments is valid, and this is just the range of relevance to gyro-synchrotron emission. Considerable progress has been made over the past few years in treating this intermediate range. One useful approach is to calculate various quantities exactly and use curve-fitting techniques to derive relatively simple functional forms for the results. This approach has been pursued by Dulk and Marsh (1982) whose formulas have already been used extensively in semi-quantitative applications. The other approach is analytic. Petrosian (1981) showed that one could derive quite accurate results for gyro-synchrotron emission from a distribution of electrons by carrying out the integrals over pitch angle and energy by the method of steepest descents. This method has been extended and generalized by Petrosian and McTiernan (1983) and by Robinson and Melrose (1984). Although this approach leads to particularly useful simplified formulas for Maxwellian distributions, it is more convenient to use Dulk and Marsh’s (1982) numerically-derived formulas for power-law distributions.

In the gyro-synchrotron regime, radiation emitted at harmonics  $\omega \cong s\Omega_e$  with  $10 \lesssim s \lesssim 100$  merges to form a continuum. Dulk and Marsh (1982) found that, for a power-law distribution function proportional to  $E^{-\delta}$  with  $E > 10$  keV and  $2 \lesssim \delta \lesssim 7$ , the brightness temperature was bounded above by

$$T_B \lesssim 2.2 \times 10^9 \times 10^{-0.31\delta} (\sin \theta)^{-0.36 - 0.06\delta} (\omega/\Omega_e)^{0.50 + 0.085\delta}, \quad (10)$$

where  $\theta \gtrsim 20^\circ$ . This implies that brightness temperatures are typically  $\lesssim 10^{10}$  K, with the largest values occurring for small values of  $\delta$  and  $\Omega_e$  – i.e., for relatively flat distributions and weak magnetic fields. Dulk and Marsh also gave formulas similar to (10) for the emission and absorption coefficients, the degree of circular polarization, and for the frequency where the brightness temperature is largest. The radiation pattern has a broad maximum perpendicular to the magnetic field (i.e.,  $\theta = 90^\circ$ ) where the degree of circular polarization is typically  $< 40\%$ . There are no values of  $\delta$ ,  $\Omega_e$ , and  $\theta$  for which  $T_B(\nu)$  and  $|r_c|$  are both large. Gyro-synchrotron emission is the best candidate for the non-flaring microwave background emissions from the Sun and other stars.

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