Astron. Astrophys. 156, 277–282 (1986)

ASTRONOMY AND ASTROPHYSICS

Two-photon emission in X-ray pulsars

II. Radiation transfer

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Received May 10, accepted July 3, 1985

Summary. A transport equation governing the transfer of radiation in the accretion columns of strongly magnetized X-ray pulsars is derived using a generalization of a method due to Melrose (1981). Processes included are cyclotron absorption and emission, Compton scattering, bremsstrahlung and double Compton scattering. A prescription is derived to deal with terms which scatter particles from the ground state to the first excited state. The equation is used to demonstrate that the resonant part of double Compton scattering may be expected to dominate bremsstrahlung as a source of photons over a wide parameter range of interest. Current models which neglect this effect are inconsistent in their calculation of continuum spectra.

Key words: X-rays: binaries – stars: neutron – accretion plasmas – radiation transfer

1. Introduction

In the preceding companion paper (Melrose and Kirk, 1985, hereinafter, Paper I) we calculate various transition probabilities associated with processes thought to occur in the accretion columns of X-ray pulsars. We now turn to the formulation of a transport equation for radiation, and a discussion of the relative importance of two processes – bremsstrahlung and 2-photon decay of the first Landau level.

The fundamental problem connected with radiation transfer in accretion columns is associated with the very strong magnetic field: it is that the assumption of local thermodynamic equilibrium (LTE) cannot be made. Because of the very rapid spontaneous decay rate of the excited Landau levels, their population is not controlled by collisions in the plasma, but is directly related to the radiation field itself. This problem has been recognized for some years (Bonazzola et al., 1979) and is usually treated by omitting the source term corresponding to cyclotron emission and arguing that this process should be included only as part of a resonant scattering. Such a procedure was put on a firmer theoretical footing by Melrose (1981) who defined a compound probability covering resonant and non-resonant scattering. In such a situation the most obvious source of photons is the bremsstrahlung process, and, indeed, all subsequent treatments of radiation transport in strong magnetic fields have used these two ingredients – Compton (or Thomson) scattering plus bremsstrahlung (Kirk and Mészáros, 1980; Nagel, 1980, 1981a,b; Kaminker et al., 1982a,b., 1983; Langer and Rappaport, 1982; Mészáros et al., 1983; Harding et al., 1984; Soffel et al., 1985; Ventura et al., 1985).

However, bremsstrahlung, being a process quadratic in the particle density, is a rather ineffective source of photons in the relatively thin plasmas envisaged ($n_e \leq 10^{22}\,\mathrm{cm}^{-3}$). As a result, photon transport is dominated by scattering, and the energy flux lies well below that corresponding to the black body spectrum at all frequencies of interest (Felten and Rees, 1972). In view of this, it is reasonable to investigate the possibility that higher order processes linear in the density – such as double Compton scattering – may compete with bremsstrahlung as a source of photons, although they have a much smaller cross-section than does ordinary Compton scattering. For unmagnetized plasmas, Lightman (1981) has shown this possibility to be unlikely. For strongly magnetized plasmas, however, the scattering can be resonant, and we demonstrate here that resonant double Compton scattering can be expected to be the dominant source of photons in the continuum of X-ray pulsars.

First of all, however, we must derive the radiation transport equation including not only scattering and bremsstrahlung but also higher order processes. To do this we generalize Melrose's (1981) approach and show that similar compound probabilities arise. The result is of interest not only because of the new processes which it includes, but also because it serves to remove an ambiguity which arises in the conventional transport equation containing just Compton scattering and bremsstrahlung, namely whether or not ordinary scattering or bremsstrahlung processes in which the electron changes its Landau level should be included. The consensus is that they should be omitted, but no arguments for this have been advanced. (The fact that the number density n_1 of electrons in the first Landau level can be assumed to be much smaller than that in the ground state n_0 is clearly insufficient, since some of these processes are proportional to n_0, not n_1). Here, we show that such processes can be regarded as resonant contributions to double Compton scattering and 2-photon bremsstrahlung respectively. Using our approach, described in Sect. 2, it is possible to write the radiation transport equation solely in terms of the number density of electrons in the lowest Landau level, in which case only those processes arise which do not involve a change of level.

The resulting transport equation is non-linear in the photon occupation number even when induced processes are neglected,

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and so promises to be difficult to solve. Furthermore, we have derived the equation for the simplified situation in which the polarization dependence of the problem is neglected. Consequently, we do not attempt a full solution in this paper and content ourselves in Sect. 3 with an order of magnitude estimate of the relative importance of the bremsstrahlung and double scattering terms. A similar estimate is given by Kirk et al., (1984). The implications of the result are discussed in Sect. 4.

2. The transport equation

For simplicity, two assumptions are made in deriving the transport equation for radiation – that we may neglect the effects of polarization and that we may consider the electrons to be in either the Landau ground state, or in the first excited state. It is difficult to assess the error introduced by the first of these assumptions. Nagel (1980) proposed a line formation mechanism based on the polarization dependence of the scattering cross-section, but later (Nagel, 1981b) suggested that the mechanism is no longer important when the effects of incoherent scattering are included; here we adopt the latter point of view. On the other hand, the neglect of electrons in Landau levels higher than the first should not introduce significant error, provided one has $n_1 \ll n_0$ and is not interested in the spectrum near the second harmonic.

The processes to be included and the Landau levels between which they operate are

1. Cyclotron emission	$1 \to 0$
2. Two-photon emission	$1 \to 0$
3. Collisional de-excitation	$1 \to 0$
4. Compton scattering	$0 \to 0$
5. Compton scattering	$1 \to 0$
6. Double Compton scattering	$0 \to 0$
7. Bremsstrahlung	$0 \to 0$

as well as the inverse of each process. Given the transition probabilities, the standard approach is to write down two kinetic equations – one for the particles and one for the waves (see Melrose, 1980). Our goal is to simplify this system by eliminating the kinetic equation for the particles. In the conventional approach this equation is eliminated by assuming a particular form for the particle distribution, for example that the distribution in parallel momentum is Maxwellian and that all particles are in the Landau ground state. As mentioned in the introduction, the latter assumption turns out to be insufficient; it will be dropped in the following. However, we will still assume that the distribution in parallel momentum can be described by a gas temperature T. Thus, the remaining kinetic equation for particles describes the occupation number of the first Landau level:

$$\frac{dn_1(p_{||})}{dt} = \sum_i B_i(p_{||}) - n_1(p_{||})\Gamma_i(p_{||}) . \qquad (i = 1, 2, 3 \text{ and } 5) \tag{1}$$

where the excitation rates $B_i(p_{||})$ and the decay rates $\Gamma_i(p_{||})$ for the processes 1, 2, 3 and 5 can be written in terms of the probabilities derived in Paper I. For example, the excitation and decay rates for cyclotron emission are:

$$\begin{bmatrix} B_1(p_{||}) \\ \Gamma_1(p_{||}) \end{bmatrix} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} w_1(p_{||}, \mathbf{k}) \begin{bmatrix} n_0(p_{||} - k_{||})N \\ 1 + N \end{bmatrix} \tag{2}$$

where the argument $\mathbf{k}$ of the occupation number N (averaged over polarization) is omitted and the transition probability $w_1(p_{||}, \mathbf{k})$ is given by Eq. (I.27)

$$w_1(p_{||}, \mathbf{k}) = \sum_{t,a} w_{0,1}^{t,a}(\mathbf{k}) . \tag{3}$$

The processes 1, 2, 3 and 5 which enter Eq. (1) can be ordered in terms of three small parameters. The first, and obvious, parameter is the fine-structure constant α, in terms of which 1, 2, 3 and 5 are in the ratio $1 : \alpha : 1 : \alpha$. Closer inspection of the probabilities reveals a dependence on the ratio of the magnetic field B to the critical field B_c ($= 4.414\,10^{13}$ G). However, this dependence does not change the ordering – more important is a small parameter which expresses the dilute nature of the plasma. This parameter can be understood by comparing Γ_1 and Γ_3. From Eq. (I.30) we have, for $p_{||} = 0$.

$$\int \frac{d^3\mathbf{k}}{(2\pi)^3} \Gamma_1(0, \mathbf{k}) = \tfrac{4}{3}\alpha \left[\frac{B}{B_c}\right] \Omega$$

where Ω is the cyclotron frequency. On the other hand, Eq. (I.50) yields

$$\int \frac{d^3\mathbf{k}}{(2\pi)^3} \Gamma_3(0, \mathbf{k}) \sim (n_i \alpha \lambda_c^3)(B/B_c)^{-5/2} \alpha \Omega$$

where λ_c is the Compton wavelength of the electron ($= 3.9\,10^{-11}$ cm) and n_i is the number density of ions (assumed to be protons). In accretion columns, $n_i < 10^{22}\,\mathrm{cm}^{-3}$ and we may define the small parameter

$$\alpha_c := (n_i \alpha \lambda_c^3) \\ \leq 4\,10^{-12}$$

As a result we arrive at the following ordering for the processes 1, 2, 3 and 5:

$$1 : \alpha(B/B_c)^2 : \alpha_c(B/B_c)^{-7/2} : \alpha(B/B_c)^2$$

where the order of process 5 is obtained by observing that its probability is simply related to that of process 2 (see Paper I). Thus, process 1 is typically a factor of 10^5 times faster than processes 2, 3 and 5, a fact which can be exploited using a two timescale perturbation approach.

Returning to Eq. 1, we seek solutions such that $n_1(p_{||})$ and N are stationary on the fast timescale τ_0 associated with process 1, and vary on the slow timescale τ_1 associated with processes 2, 3 and 5. From the condition

$$\frac{dn_1(p_{||})}{d\tau_0} = 0 \tag{4}$$

one obtains the desired solution:

$$n_1(p_{||}) = \sum_i B_i / \sum_i \Gamma_i \tag{5}$$

where B_i and Γ_i are now functions of time through τ_1 only.

The kinetic equation of the waves can also be split into the contributions from the various processes:

$$\frac{dN}{dt} = \sum_i \left[\frac{dN}{dt}\right]_i$$

where the summation includes all except process 3. As an example, consider the term $i = 1$:

$$\left[\frac{dN}{dt}\right]_1 = \int \frac{dp_{\|}}{2\pi} w_1(p_{\|}, \mathbf{k})[n_1(p_{\|})(1+N) - n_0(p_{\|} - k_{\|})N] \tag{6}$$

Substituting for $n_1(p_{\|})$ according to Eq. (4):

$$\begin{aligned}\left[\frac{dN}{dt}\right]_1 = & -\int \frac{dp_{\|}}{2\pi} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} \bar{w}_4(p_{\|}, \mathbf{k}, \mathbf{k}') \\ & \times [n_0(p_{\|})N(1+N') - n_0(p_{\|} + k_{\|} - k'_{\|})(1+N)N'] \\ & - \int \frac{dp_{\|}}{2\pi} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} \int \frac{d^3\mathbf{k}''}{(2\pi)^3} \bar{w}_6(p_{\|}, \mathbf{k}, \mathbf{k}', \mathbf{k}'') \\ & \times [n_0(p_{\|})N(1+N')(1+N'') - n_0(p_{\|} + k_{\|} - k'_{\|} - k''_{\|}) \\ & \times (1+N)N'N''] + \int \frac{dp_{\|}}{2\pi} \int \frac{dp'_{\|}}{2\pi} n_i \bar{w}_7(p_{\|}, p'_{\|}, \mathbf{k}) \\ & \times [n_0(p'_{\|})(1+N) - n_0(p_{\|})N] - \int \frac{dp_{\|}}{2\pi} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} \\ & \times \int \frac{d^3\mathbf{k}''}{(2\pi)^3} \bar{w}_6(p_{\|}, \mathbf{k}, -\mathbf{k}', \mathbf{k}'')[n_0(p_{\|})NN'(1+N'') \\ & - n_0(p_{\|} + k_{\|} + k'_{\|} - k''_{\|})(1+N)(1+N')N''] \end{aligned} \tag{7}$$

where

$$\bar{w}_4(p_{\|}, \mathbf{k}, \mathbf{k}') := w_1(p_{\|} + k_{\|}, \mathbf{k}) w_1(p_{\|} + k_{\|}, \mathbf{k}') / \sum_i \Gamma_i(p_{\|} + k_{\|})$$

$$\begin{aligned}&\bar{w}_6(p_{\|}, \mathbf{k}, \mathbf{k}', \mathbf{k}'') \\ &\quad := w_1(p_{\|} + k_{\|}, \mathbf{k}) w_2(p_{\|} + k_{\|}, \mathbf{k}', \mathbf{k}'') / \sum_i \Gamma_i(p_{\|} + k_{\|}) \\ &\quad = w_1(p_{\|} + k_{\|}, \mathbf{k}) w_5(p_{\|} + k_{\|}, -\mathbf{k}', \mathbf{k}'') / \sum_i \Gamma_i(p_{\|} + k_{\|})\end{aligned}$$

and

$$\bar{w}_7(p_{\|}, p'_{\|}, \mathbf{k}) := w_1(p'_{\|} + k_{\|}, \mathbf{k}) w_3(p'_{\|} + k_{\|}, p_{\|}) / \sum \Gamma_i(p_{\|} + k_{\|}) . \tag{8}$$

The quantities w_2 and w_3 are those probabilities associated with the process 2 and 3, i.e.

$$\begin{bmatrix} B_2(p_{\|}) \\ \Gamma_2(p_{\|}) \end{bmatrix} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} w_2(p_{\|}, \mathbf{k}, \mathbf{k}') \begin{bmatrix} n_0(p_{\|} - k_{\|} - k'_{\|})NN' \\ (1+N)(1+N') \end{bmatrix}$$

and

$$\begin{bmatrix} B_3(p_{\|}) \\ \Gamma_3(p_{\|}) \end{bmatrix} = n_i \int \frac{dp'_{\|}}{(2\pi)} w_3(p_{\|}, p'_{\|}) \begin{bmatrix} n_0(p'_{\|}) \\ 1 \end{bmatrix}$$

In terms of the probabilities derived in Paper I, w_2 is given by Eq. (I.32) when a sum over polarizations is performed; as indicated in Eq. (I.34)

$$w_2(p_{\|}, \mathbf{k}, \mathbf{k}') = \sum w_{0,1}^{\alpha\alpha'\beta\beta'}(\mathbf{k}, \mathbf{k}') .$$

Similarly,

$$\begin{aligned} w_3(p_{\|}, p'_{\|}) = (2\pi)^2 w_{1,0}(p_{\|}) \delta[(p'_{\|} + \sqrt{p_{\|}^2 + 2eB} \\ + \delta(p'_{\|} - \sqrt{p_{\|}^2 + 2eB})] \end{aligned}$$

from Eq. (I.49), so that $\bar{w}_4$, $\bar{w}_5$ and $\bar{w}_6$ are just the compound probabilities discussed in Paper I.

Thus, Eq. (6), representing the contribution to the transport equation of cyclotron absorption and emission has been converted using Eq. (5) into four terms which are identical to resonant contributions from three processes in which the electron begins and ends in the ground state ($0 \to 0$). These four terms are, in order of their appearance in Eq. (7). Compton scattering $0 \to 0$, double Compton scattering $0 \to 0$, bremsstrahlung $0 \to 0$, and another resonant contribution to double Compton scattering $0 \to 0$.

Such a transformation of a $1 \to 0$ process into resonant $0 \to 0$ processes is easily pictured in terms of modified Feynman diagrams. In Fig. 1 we illustrate schematically the $1 \to 0$ processes which, with their inverses, contribute to the kinetic equation for $n_1(p_{\|})$ and the $0 \to 1$ processes contributing with their inverses to the kinetic equation for N. The effect of applying equation (5) is to produce terms in the equation for N which are simply combinations of one process from the left column with one from the right. More precisely, one obtains that resonant contribution to the combined process which is associated with an intermediate state in the position of the label '1' in Fig. 1. The terms written out in Eq. (7), for example, are obtained by combining process 1 in the N-column with each process in the n_1-column. Contributions which appear as double Compton scattering arise from the processes 2 and 5a,b. They produce the second and fourth terms in Eq. (7). However, these are not all the resonances encountered in double Compton scattering. In fact there are two more which, in their turn arise from the combination of process 1 in the n_1-column with processes 2 and 5a,b in the N-column. It is easy to check that these contributions are obtained when one substitutes for n_1 in the expression for $(dN/dt)_2$ and $(dN/dt)_5$.

In summary, the application of Eq. (5) eliminates the kinetic equation for particles and gives the following rules for the construction of the transport equation

1. All processes of the type $0 \to 1$ or $1 \to 0$ are to be omitted.
2. Resonances in the $0 \to 0$ processes are to be treated by incorporating the compound probabilities discussed in Paper I.

It is interesting to note that the compound probabilities are themselves functions of the radiation field through $\Gamma_i(p_{\|})$. Only in the case of small occupation numbers (the so-called "Wien region") can one make the approximation

$$\sum_i \Gamma_i(p_{\|}) \approx \int \frac{d^3\mathbf{k}}{(2\pi)^3} w_1(p_{\|}, \mathbf{k})$$

which removes this nonlinearity.

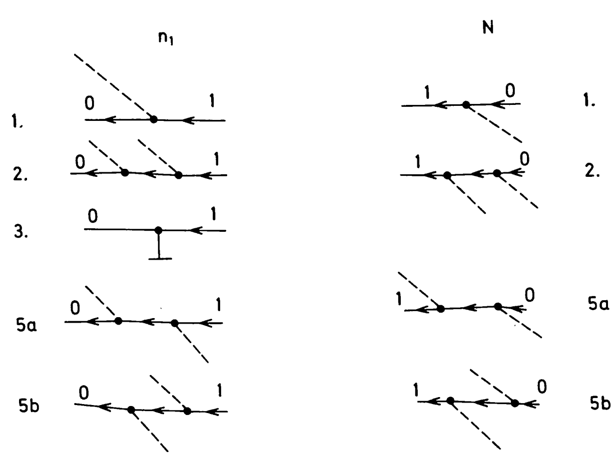

Fig. 1. The processes included in the kinetic equations for the particles (n_1-column) and photons (N-column) which involve a change in the Landau level of the electron

Including all processes 1–7, the transport equation constructed according to the above rules reads:

$$\begin{aligned}\frac{dN}{dt} = & -\int \frac{dp_{||}}{2\pi} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} w_4(p_{||}, \mathbf{k}, \mathbf{k}') \\ & \times [n_0(p_{||})N(1+N') - n_0(p_{||}+k_{||}-k'_{||})(1+N)N'] \\ & - \int \frac{dp_{||}}{2\pi} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} \int \frac{d^3\mathbf{k}''}{(2\pi)^3} \\ & \times \{w_6(p_{||}, \mathbf{k}, \mathbf{k}', \mathbf{k}'')[n_0(p_{||})N(1+N')(1+N'') \\ & - n_0(p_{||}+k_{||}-k'_{||}-k''_{||})(1+N)N'N''] \\ & + 2w_6(p_{||}, \mathbf{k}; -\mathbf{k}', \mathbf{k}'') \\ & \times [n_0(p_{||})NN'(1+N'') - n_0(p_{||}+k_{||}+k'_{||}-k''_{||})(1+N) \\ & \times (1+N')N'']\} + \int \frac{dp_{||}}{2\pi} \int \frac{dp'_{||}}{2\pi} w_7(p_{||}, p'_{||}, \mathbf{k})[n_0(p_{||}) \\ & \times (1+N) - n_0(p'_{||})N]\end{aligned} \tag{9}$$

w_4, w_6 and w_7 are the transition probabilities for Compton scattering, double Compton scattering and bremsstrahlung (all $0 \to 0$). Expressions for w_4 applicable away from the resonance have been given by Herold (1979) and for w_7 by Kirk and Mészáros (1980) (see also Eqs. (I.21) and (I.43)). Electron-electron bremsstrahlung can be included if higher accuracy is desired (see Paper I). To our knowledge, no complete computation of w_6 has been performed. However, close to any of the four resonances, the compound probability $\bar{w}_6$ can be written down from Paper I. Two of these appear already in Eq. (8), the two remaining are simply constructed using the crossing symmetries referred to in Sect. 3.2.4 of Paper I. Bussard et al., (1985) have given expressions for the resonance which corresponds to Compton scattering $0 \to 1$ followed by radiative decay of the excited state.

3. Continuum spectrum

Equation (9) contains one scattering term, which preserves the number of photons, and two terms which create and destroy photons. Under usual accretion column conditions the scattering term dominates, in the sense that the mean free path of a photon between scatterings is much smaller than that between emission and absorption. Nevertheless, it is essential to include in the equation of transfer a term which can act as a photon source. In all treatments to date, this photon source has been taken to be bremsstrahlung. Here we show that the second term in Eq. (9) – namely that corresponding to double Compton scattering is in fact a much more effective source than bremsstrahlung (which is represented in the third term).

Such a result can already be guessed from the discussion of the previous section, where the processes 2 and 3 are shown to be in the ratio $\alpha : \alpha_c(B/B_c)^{-1/2}$ or approximately $1 : 10^{-4}$. Of course, these processes enter into the transport equation in different ways, but, provided the photon occupation number is not everywhere very small this result can be expected to carry through.

The basic difficulty in comparing the resonant double scattering process with bremsstrahlung is that the former makes the transport equation nonlinear. As a result it is expedient to pose the following question instead: 'Is it consistent to assume that bremsstrahlung is the dominant absorption process?' An approximation to the radiation field can easily be found if bremsstrahlung is indeed dominant, and then the magnitude of the double scattering term can be found. In particular, one can compare the relative probability that a photon entering such a radiation field be absorbed in a double scattering event or in a bremsstrahlung event. Consistency demands this quantity be small; we find it to be large in the parameter range of interest.

The first step is the reduction of the transport Eq. (9) to an easily solved form. Although this results in a very simple radiation field (Eq. 12), the form of which could be taken from the work of Felten and Rees (1972), it is helpful to perform the reduction explicitly, so that a similar process can later be applied to the more complicated double scattering term. First of all, we neglect electron recoil on scattering and on absorbing/emitting a photon by setting $k_{||}$, $k'_{||}$ and $k''_{||}$ to zero wherever they appear in the argument of $n_0(p_{||})$ in the scattering and bremsstrahlung terms of Eq. (9). Neglect of recoil also implies that $w_4(p_{||}, \mathbf{k}, \mathbf{k}')$ is proportional to $\delta(\mathbf{k} - \mathbf{k}')$, whereas $w_7(p_{||}, \rho'_{||}, \mathbf{k})$ is still proportional to $\delta(|\mathbf{k}| - p_{||}^2/2m + p_{||}'^2/2m)$, because the momentum absorbed by the ion in bremsstrahlung cannot be neglected. Then, an equation in the diffusion approximation can be obtained either by expanding $N(\mathbf{k})$ in Legendre polynomials and retaining only the first term, or by assuming the symmetric part of $N(\mathbf{k})$ under the transformation $\mathbf{k} \to -\mathbf{k}$ to be much larger than the antisymmetric part (Kaminker et al., 1982). The first method, although more restrictive in its assumption about $N(\mathbf{k})$ can be used also when frequency shifts upon scattering are permitted, whereas the latter demands coherent scattering. In each case an equation is obtained of the form

$$D\frac{d^2J(\omega)}{dz^2} = \kappa_b(J(\omega, z) - B(\omega)) \tag{10}$$

where

$$J(\omega, z) = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} N(\mathbf{k}) \sin\theta \, d\theta d\phi$$

with θ and ϕ defining the direction of $\mathbf{k}$ in spherical polars, and $\omega = |\mathbf{k}|$. The distribution in parallel momentum $p_{||}$ of electrons in the ground state has been assumed to be Maxwellian with temperature T, so that the source function is

$$B(\omega) := (e^{\omega/T} - 1)^{-1}$$

Equation (10) has been written for slab geometry with z the coordinate normal to the slab. κ_b and D are the bremsstrahlung absorption coefficient and the diffusion coefficient. κ_b includes a factor allowing for stimulated emission:

$$\begin{aligned}\kappa_b = n_i & \int_0^{\pi} d\theta \int_0^{2\pi} d\phi \sin\theta \int \frac{dp_{||}}{2\pi} \int \frac{dp'_{||}}{2\pi} w_7(p_{||}, p'_{||}, \mathbf{k}) n_0(p_{||}) \\ & \times [\exp(\omega/T) - 1]\end{aligned}$$

and the definition of D depends on which approach to the diffusion equation is used. Since these details are unimportant for the present argument, we shall content ourselves by replacing both κ_b and D by their values in a plasma with zero magnetic field. At a frequency equal to one half of the electron cyclotron frequency, such a replacement is correct to order of magnitude, and leads to the simple expressions

$$\kappa_b = \frac{4(2\pi)^{5/2}\alpha^2(n_e n_i \lambda_c^5 \alpha)}{3\sqrt{3}T^{1/2}\omega^3}[1 - \exp(-\omega/T)]$$

and

$$D = (3n_e\sigma_T)^{-1} = (3\kappa_c)^{-1} \tag{11}$$

where T is in units of mc^2, ω is in units of $mc^2/\hbar$, n_e is the electron number density and σ_T the Thomson cross-section. Equation (11) is valid provided $\kappa_c \gg \kappa_b$.

Consider the case in which the boundary between a uniform semi-infinite plasma and vacuum lies at $z = 0$, with vacuum at $z < 0$ and plasma at $z > 0$. The solution of Eq. (10) subject to the boundary conditions $J(\omega, 0) = 0$ and $J(\omega, \infty)$ finite is

$$J(\omega, z) = B(\omega)[1 - \exp(-z/z_T)] \tag{12}$$

where z_T is the thermalization length:

$$z_T = [3\kappa_c\kappa_b]^{-1/2}$$

Equation (12) is applicable for frequencies far enough from the cyclotron resonance that the non-magnetic opacities constitute a reasonable order of magnitude estimate.

Having arrived at an approximate solution to the transport problem we now proceed to compute the relative probability for absorption by double scattering and by bremsstrahlung. According to Eq. (9), the probability of absorption of a photon by bremsstrahlung in a path length ds is given by

$$p_b\, ds = \int \frac{dp_{||}}{2\pi} \int \frac{dp'_{||}}{2\pi} w_7(p_{||}, p'_{||}, \mathbf{k}) n_0(p'_{||})\, ds$$

Using the same simplifying arguments as above one obtains

$$p_b = \kappa_b$$

On the other hand, the probability of absorption by double Compton scattering is given by

$$\begin{aligned} p_{2c}\, ds = \int \frac{dp_{||}}{2\pi} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} \int \frac{d^3\mathbf{k}''}{(2\pi)^3} &\{w_6(p_{||}, \mathbf{k}; \mathbf{k}', \mathbf{k}'') n_0(p_{||}) \\ &\times (1 + N')(1 + N'') \\ &+ 2w_6(p_{||}, \mathbf{k}; -\mathbf{k}', \mathbf{k}'') n_0(p_{||}) N'(1 + N'')\}\, ds \end{aligned}$$

Since ω is, by assumption, far from the cyclotron resonance, only two of the four possible resonances in w_6 can occur. One of these corresponds to Compton scattering of the photon $\mathbf{k} \to \mathbf{k}'$ with electron transition $0 \to 1$ followed by decay $1 \to 0$ with emission of the photon $\mathbf{k}''$. This resonance is of interest only when ω lies above the cyclotron frequency. Here we consider ω to be below the cyclotron frequency; in which case the other resonance – corresponding to two-photon absorption $0 \to 1$ followed by single photon decay – is of importance. Selecting only this dominant part we rewrite w_6 according to Eq. (8):

$$\begin{aligned} w_6(p_{||}, \mathbf{k}; -\mathbf{k}', \mathbf{k}'') &= w_6(p_{||} + k_{||} + k'_{||} - k''_{||}, \mathbf{k}''; \mathbf{k}, \mathbf{k}') \\ &= w_1(p_{||} + k_{||} + k'_{||}, k''_{||}) w_2(p_{||} + k_{||} + k'_{||}, \mathbf{k}, \mathbf{k}') / \\ &\quad \sum_{i=1}^{6} \Gamma(p_{||} + k_{||} + k'_{||}) \end{aligned}$$

where the principle of detailed balance has been used. Therefore,

$$p_{2c} = \int \frac{dp_{||}}{2\pi} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} \int \frac{d^3\mathbf{k}''}{(2\pi)^3} \times \frac{w_1(p_{||}, \mathbf{k}'') w_2(p_{||}, \mathbf{k}, \mathbf{k}') n_0(p_{||})[N' + N'N'']}{\sum_{i=1}^{6} \Gamma_i(p_{||})} \tag{13}$$

where electron recoil has been neglected in line with the simplifications leading to (12). In (13), the leading term in the denominator Γ_1 is retained, and the $p_{||}$ dependence of w_2 neglected, so that, in the diffusion approximation

$$p_{2c} = n_e \int \langle w_{01}(\mathbf{k}, \mathbf{k}')\rangle J(\omega', z) \frac{d^3\mathbf{k}'}{(2\pi)^3}$$

where $\langle w_{01}(\mathbf{k}, \mathbf{k}')\rangle$ is defined in Eq. (I.37). Thus we may define an absorption coefficient for resonant double Compton scattering

$$p_{2c} =: \kappa_{2c} J(\Omega - \omega)$$

with

$$\kappa_{2c} = n_e \frac{c^2}{\Omega^2} \frac{4\pi}{45} \alpha^2 (B/B_c)^3 F(y)/y^2$$

and $F(y)$ as defined in Paper I.

Let the probability that a photon entering the plasma be absorbed by bremsstrahlung before diffusing to a depth z be P_b, and the corresponding quantity for double scattering be P_{2c}. Then, since the path length s which is traversed in diffusing to z is given by

$$s = \kappa_c z^2,$$

the quantities P_b and P_{2c} fulfil the equations

$$\frac{dP_b}{dz} = (1 - P_b - P_{2b}) 2p_c\kappa_c z$$

$$\frac{dP_{2c}}{dz} = (1 - P_b - P_{2c}) 2p_{2c}\kappa_c z$$

which have the solution

$$P_b = \tfrac{2}{3} \int_0^{z/z_T} dx\, x \exp[a - (a + \tfrac{2}{3})x^2/2 - a(1 + x)e^{-x}]$$

$$P_{2c} = 1 - P_b - \exp[a - \tfrac{1}{2}(a + \tfrac{2}{3})(z/z_T)^2 - a(1 + z/z_T)e^{-z/z_T}]$$

where

$$a = \frac{2}{3} \frac{\kappa_{2c}}{\kappa_b} B(\Omega - \omega)$$

and the boundary condition $P_b = P_{2c} = 0$ at $z = 0$ has been imposed.

For conditions typical of accretion columns on X-ray pulsars i.e. $B/B_c = 0.1$, $T = 0.02$, $n_e = 10^{22}\,\mathrm{cm}^{-3}$ one finds for a frequency equal to one half of the cyclotron frequency

$$a = 2.4\, 10^3$$

and

$$\frac{P_{2c}}{P_b} \to 3.7\, 10^3 \quad \text{as} \quad z \to \infty \tag{14}$$

This means that a photon entering a plasma with radiation field given by Eq. (12) is nearly four thousand times more likely to be absorbed by double scattering than by bremsstrahlung, and the answer to the question posed at the beginning of this section is clear: it is inconsistent to assume that bremsstrahlung is the dominant absorption process in accretion columns.

4. Discussion

The transport Eq. (9) represents an improvement on previous formulations in several respects. Although it has been recognized for some time that radiative transitions and not collisions determine the number of electrons in the excited Landau levels, the only formal derivation of a transport equation to implement this condition was that of Melrose (1981), which contained only emission and absorption (resonant scattering) terms. For bremsstrahlung, Langer and Rappaport (1982) used an approximate method of treating the resonant part as collisional excitation followed by radiative decay. Also, Nagel and Ventura (1983) wrote an equation similar to Eq. (5) but without any momentum dependence and including only resonant bremsstrahlung as an absorption process. As well as including higher order processes, the derivation of Eq. (9) shows explicitly how processes such as Compton scattering $0 \rightarrow 1$ are to be treated, a question not addressed by previous investigations. One further advantage of Eq. (9) is that it applies both in the cyclotron line and in the continuum – it requires no artificial separation of the spectrum into cyclotron photons and continuum photons. At the same time, our approach has several limitations. Thus, the two-timescale perturbation method works only when the radiation field is slowly varying in space and time compared to the mean free path for cyclotron absorption and the decay rate of the first excited state. This may not be the case near the plasma boundary. Further, we have chosen to simplify the problem by assuming that most electrons reside in the Landau ground state, an assumption which no longer holds when the photon occupation number in the vicinity of the cyclotron frequency is comparable to unity.

The inclusion of double scattering terms has, of course, introduced further nonlinearities into the transport equation. These are such that they cannot be eliminated simply by omitting induced effects, and they promise to make the solution of the transport problem a difficult undertaking. However, the estimates presented in Sec. 3 demonstrate that it is inconsistent to omit the double scattering terms. Indeed, the margin by which these terms dominate bremsstrahlung according to Eq. (14) is so large that it seems unlikely that bremsstrahlung plays any role in producing continuum photons in X-ray pulsars. The implications for current models which aim to explain the spectrum of these sources below the cyclotron frequency are serious; radically different results could well emerge when double scattering terms are included. In this case, our picture would be as follows: cyclotron photons are created by resonant bremsstrahlung (collisional excitation followed by radiative decay) and subsequently diffuse through the plasma undergoing many resonant scatterings (absorptions and re-emissions). Sometimes, the radiative decay proceeds by the two-photon process and produces the continuum below the cyclotron frequency, sometimes the excited electron produced by the absorption encounters a soft photon and scatters it into the continuum above the cyclotron frequency. In this way the shape of the line is intimately connected with the shape of the continuum. However, more precise statements must await further investigations of the transport equation.

Acknowledgements. JGK wishes to thank the Department of Theoretical Physics, University of Sydney for their generous hospitality during his stay there, which was supported by the Australian Research Grants Scheme. We also thank Michelle Allen for useful discussions.

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