Astron. Astrophys. 163, 229–238 (1986)

ASTRONOMY AND ASTROPHYSICS

Clumpy Langmuir waves in type III solar radio bursts

D.B. Melrose[1], G.A. Dulk[2], and I.H. Cairns[1]

[1] School of Physics, University of Sydney, Sydney N.S.W. 2006, Australia
[2] Department of Astrophysical, Planetary and Atmospheric Sciences, CB 391, University of Colorado, Boulder, CO 80309, USA

Received October 1, accepted December 31, 1985

Summary. We develop a model for type III radio emission in the interplanetary medium based on recent data on Langmuir waves, associated ion sound waves, density fluctuations in the interplanetary plasma, streaming electrons and radio emission. In our model, Langmuir wave growth is suppressed by refraction in field aligned density irregularities except near density minima where clumps of Langmuir waves form. Quasilinear relaxation limits the growth of the Langmuir waves in the clumps. The radio emission, which is attributed to coalescence of the Langmuir waves with associated ion sound waves, saturates at a brightness temperature equal to the effective temperature of the Langmuir waves, estimated to be between 10^{15} K and 10^{16} K from observational data. The model is consistent with all the relevant data on type III events. In particular, it accounts naturally for observed brightness temperatures of type III bursts.

Key words: Sun: radio radiation – Langmuir waves – type III bursts – interplanetary medium

1. Introduction

In broad outline the qualitative theory for type III Solar radio bursts has not changed since it was first formulated by Ginzburg and Zheleznyakov (1958): electrons generate Langmuir waves through a streaming instability and nonlinear plasma processes lead to escaping radiation near the fundamental and second harmonic of the plasma frequency. However the details of the theory have been modified in a variety of different ways. Ginzburg and Zheleznyakov's theory predated many of the important developments in nonlinear plasma theory, and subsequently the details of the theory needed to be updated, e.g. reviews by Melrose (1980a) and Goldman (1983). Also Sturrock (1963) pointed out that the instability should extract energy so effectively from the stream that it should stop after propagating a very short distance, and to overcome this difficulty a variety of suggestions were made. The most favored suggestions involve some form of nonlinear suppression of the instability (e.g. Kaplan and Tsytovich, 1968; Papadopoulos et al., 1973), but there were also other suggestions including suppression resulting from local plasma inhomogeneities. Direct observations of type III events in interplanetary space have provided qualitative confirmation of the theory in that the electrons, the Langmuir waves and the radio emission have all been observed, and the electrons have been shown to be unstable to the growth of Langmuir waves (Lin et al., 1981). However, these observations have raised further difficulties with the details of the theory. One difficulty is that the Langmuir waves, when they are observed, are found to be very spiky and intermittent (Gurnett and Anderson, 1976, 1977). We refer here to the Langmuir waves occurring in "clumps". Another difficulty arises from the widely accepted assumption that the emission in the interplanetary medium is at the second harmonic of the plasma frequency: this leads to an inconsistency in that observationally the onset of emission (supposedly at $2f_p$) systematically precedes the onset of the Langmuir waves (Melrose, 1982a). This has been resolved by a more detailed study (Dulk et al., 1984) which shows that the bursts are at the fundamental of the plasma frequency at least until their peak. However relatively little attention has been given to the origin of and the implications of the clumpiness of the Langmuir waves.

Here we argue that recent developments in both observation and theory point towards a particular model for type III events in the interplanetary medium. The model involves the following ingredients.

1) Fast electrons in the beams initially have energies of about 30 keV, decreasing with time to about 3 keV. At some point the one-dimensional distribution functions develop a positive slope $\partial f/\partial v > 0$. At that point the Langmuir waves grow at a rate $\gamma \lesssim 1\,\mathrm{s}^{-1}$ (Lin et al. 1981).

2) Normally the damping rate for the Langmuir waves due to the presence of ion sound waves (Melrose, 1982; Goldman and DuBois, 1982) and/or to the effect of local plasma inhomogeneities (Escande and de Genouillac, 1978; Muschietti et al., 1985) exceeds the growth rate, and the Langmuir waves do not grow.

3) Clumps of Langmuir waves occur when the local plasma conditions allow growth (Smith and Sime, 1979). Here it is argued that the required condition involves a field aligned density depression with a low level of background ion sound turbulence. Kellogg (1976), based on the rarity of Langmuir wave detections by IMP-6, suggested that "it would seem quite natural for the emissions to be confined in blobs", and "the instability must collect in the bottoms of density troughs." We suggest here that a possible source of density depressions is drift waves.

4) In the clumps the instability approaches saturation, producing Langmuir waves with a characteristic effective temperature T_L determined by the beam parameters.

5) The conversion efficiency from Langmuir waves into transverse waves is greatly enhanced due to the presence of ion sound waves of the kind observed to be associated with the

Send offprint requests to: D.B. Melrose

Langmuir waves (Lin et al., 1986; Cairns, 1984). The process saturates, producing a brightness temperature approximately equal to T_L.

6) A type III source is the envelope of emission from a collection of clumps. Provided that the filling factor for the clumps is adequate, the surface brightness T_B of the envelope would equal that of each clump, i.e. T_L. However, the observed value of T_B is reduced by about a factor of ten due to scattering in the interplanetary plasma (Steinberg et al., 1985).

7) Second harmonic emission should saturate at approximately the same brightness temperature as the fundamental, i.e. at T_L.

Our paper is arranged as follows. In Sect. 2 we discuss the observational data on clumps of Langmuir waves and on other relevant features of type III events in the interplanetary medium. In Sect. 3 we examine four processes which can suppress the growth of Langmuir waves and argue that at least one of these is likely to be effective most of the time. In Sect. 4 the model for clumps is developed with particular emphasis being placed on the determination of relevant parameters from available data. In Sect. 5 the model is extended to treat the emission of radiation, and it is argued that the model provides a natural explanation for the characteristic brightness temperatures observed (Steinberg et al., 1984; Dulk et al., 1984). The results are discussed in Sect. 6.

2. Properties of Langmuir clumps

2.1. Observations of radio and Langmuir waves

Langmuir waves associated with type III events were first reported by Gurnett and Anderson (1976, 1977) using data collected on the Helios spacecraft at about 0.5 AU. A detailed study was presented by Gurnett et al. (1978). Observations of the Langmuir waves at 1 AU with ISEE-3 (Lin et al., 1981) have been reported more recently.

Figure 1 shows the development of a type III burst and related Langmuir waves as recorded by the radio (Knoll et al., 1978) and plasma wave (Scarf et al., 1978) experiments on ISEE-3. The onset times as a function of decreasing frequency lead smoothly to the onset of the spiky Langmuir waves at 30 kHz, leaving little doubt that the initial radiation at frequencies near 30 kHz (and by extrapolation all frequencies) was at the fundamental. The local plasma frequency at the time was about 26 kHz (King, 1983), close enough to the narrowband (3 kHz bandwidth) channel at 30 kHz to allow the Langmuir wave spikes to be seen in the "skirts". Intense Langmuir waves were also observed in the wideband ($\approx$50% bandwidth), high dynamic range channel centered at 17.8 kHz (topmost trace), with both the peak electric field and the average over the sampling period of about 1 min being recorded (the peak corresponds to the maximum field strength in each sampling period). Both the peak and average field strength sometimes vary rapidly between sampling intervals and sometimes remain steady. Peak electric fields in Fig. 1 are approximately 100 times the average, implying a very intermittent nature.

Observed brightness temperatures (measured for a set of 120 relatively intense and isolated bursts) are typically 10^{12} to 10^{14} K (Steinberg et al., 1984; Dulk et al., 1984). Figure 2 shows brightness spectra for two bursts, the events described by Lin et al. (1985). The upper one is unusually bright and has approximately the same brightness temperature at all frequencies. Occasionally bursts atain brightnesses above 10^{15} K at the higher frequencies observed by ISEE-3 ($\approx$300 to 2000 kHz) but never or hardly ever at the lower frequencies. These observed brightness temperatures are

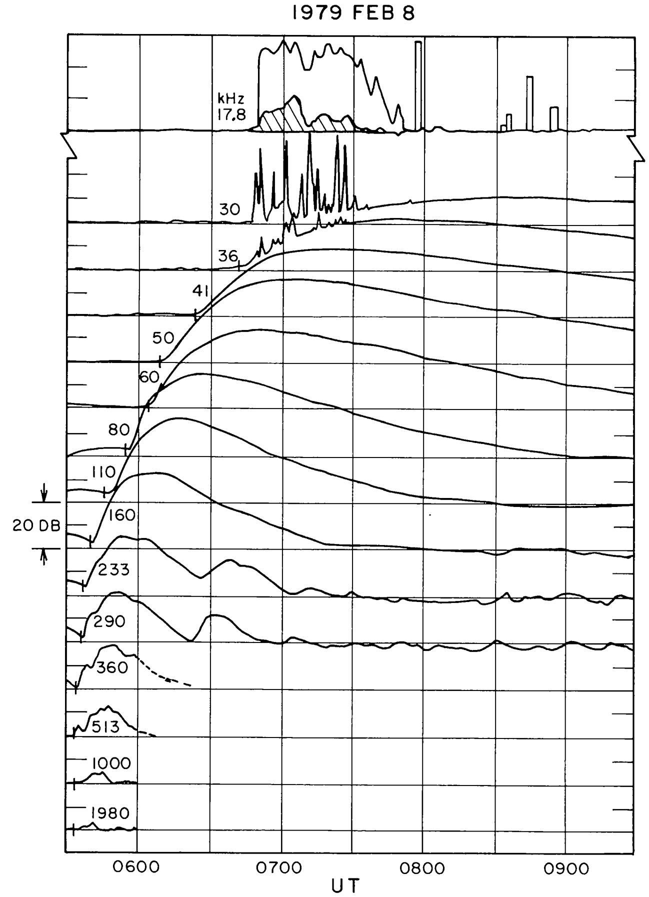

Fig. 1. Example of a type III solar radio burst with Langmuir waves observed at 1 AU by ISEE-3 at a time when the local plasma frequency was $f_p = 26$ kHz (King 1983). The data at 30 kHz and higher were recorded by the radio astronomy experiment (Knoll et al., 1978) while that at 17.8 kHz were recorded by the plasma wave experiment (Scarf et al., 1978). The large ratio of the maximum (solid line) to the average (hatched area) of the 17.8 kHz trace indicates that the Langmuir waves are characterized by many brief bursts

probably lower by a factor $\lesssim$10 than the actual brightness temperatures; this is because scattering of the radio waves increases the apparent source size by a factor of two to three over the true source size (Steinberg et al., 1985).

There is some uncertainty whether these observed brightness temperatures apply to the fundamental or to the harmonic. Quoted values pertain to the time of peak flux density when there is probably a mixture of fundamental and harmonic radiation (Dulk et al., 1984). Preceding the peak, when the radiation is primarily fundamental, the flux density is lower but the brightness temperature is not much different because the source size is smaller. After the peak when the radiation becomes primarily harmonic, the flux density decreases relatively slowly. It is therefore likely the brightness temperatures of the two components are roughly equal, i.e., to within a factor of about two.

Figures 3 and 4 display Langmuir wave events on a finer time scale, 0.1 to 0.5 s. While some spikes seem to be present on the shortest scales, the most prominent structures have time scales of about 1 s.

Few data on the statictical properties of the Langmuir waves are available and we must infer the likely properties only from samples of the data such as those illustrated in Figs. 1, 3 and 4.

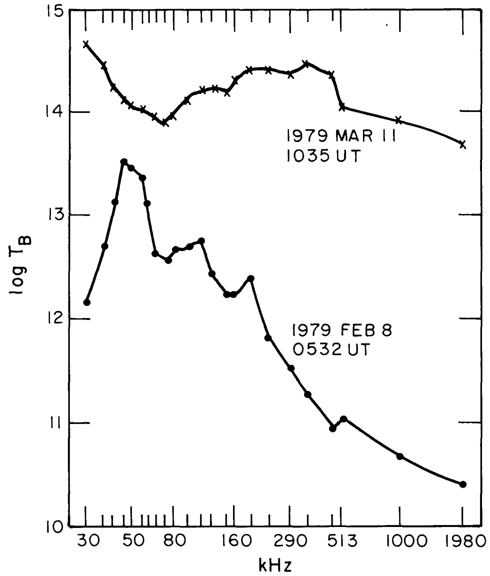

Fig. 2. Spectra of brightness temperature for two type III bursts that were accompanied by Langmuir wave events. The data were recorded by the radio astronomy experiment on ISEE-3 and processed by S. Hoang (personal communication) using the method described by Manning and Fainberg (1980) and Fainberg et al. (1985)

While there is structure on the shortest timescale so far recorded (50 ms), the more prominent structure is on a timescale of about 1 s, with a tendency for the 1 s clumps to congregate together. One statistical study (Gurnett et al., 1980) relates the peak electric field E to its variation with heliocentric distance. There is a wide scatter (by a factor $\gtrsim 100$ in E) at any one distance and possibly a decrease with distance ($E \propto R^{-1.4}$) for the average peak field.

In summary, the data suggest that typical size of a clump is such that it is convected past the spacecraft in ≈ 1 s, implying scales of 300 to 500 km for a solar wind speed 300 to 500 km s^{-1}. Peak fields are around 1 mV m^{-1}, with a substantial intrinsic variation, and mean fields are a factor of about 30 lower, again with a substantial variability.

2.2. *Correlation with type III electrons*

Lin et al. (1981) compared the occurrence of unstable electron distributions with the appearance of Langmuir waves. They found a satisfactory qualitative agreement between the onsets of the unstable distributions and of Langmuir waves. However the level of the Langmuir waves and the lack of evidence of plateau formation in the electron distribution do not fit with simple theory.

Lin et al. (1981) used their data to calculate $\partial f/\partial v$, where v denotes the velocity component along the magnetic field and f denotes the electron distribution function integrated over the two

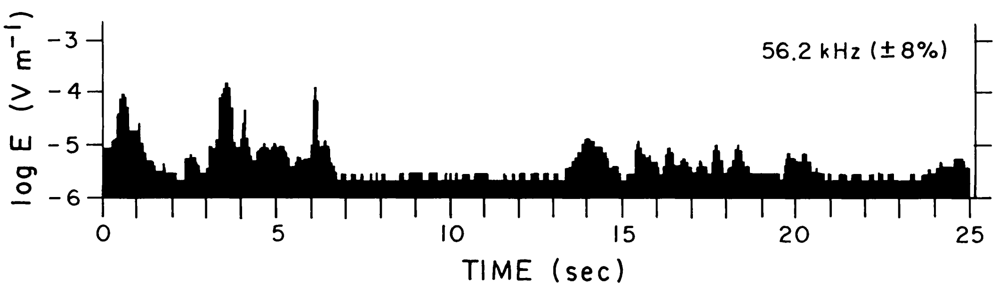

Fig. 3. An example of Langmuir waves observed with higher time resolution than in Fig. 1. Note the nearly constant saturation electric field strength of $\sim 10^{-4}$ V m^{-2} for the three strongest bursts. These data were recorded by Helios 1 on 1975 Oct 6, 0414:50.75 UT when the spacecraft was at 0.432 AU. (Gurnett and Anderson, 1977)

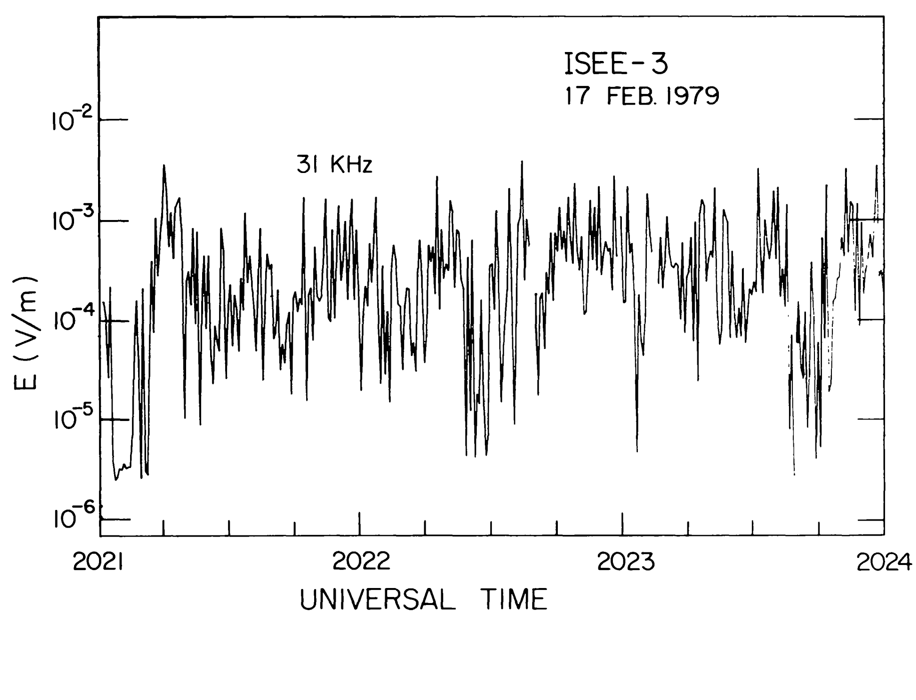

Fig. 4. An expanded plot with 0.5 s resolution of the Langmuir waves in a particular event. (Lin et al., 1981)

perpendicular velocity components. This parameter is related to the absorption coefficient for Langmuir waves propagating along the magnetic field by

$$\gamma = -\frac{\pi e^2 v^2}{\varepsilon_0 m \omega_p} \frac{\partial f}{\partial v}, \tag{1}$$

where the waves are assumed to be resonant with the electrons (i.e. $v_\phi = v$ and $k = \omega_p/v$. They found (converting to SI units) $\partial f/\partial v \approx 10^{-15}\,\mathrm{m^{-5}\,s^{-1}}$ at $v \approx 3\ 10^7\,\mathrm{m\,s^{-1}}$ persisting for ≈ 10 min. With $\omega_p/2\pi \approx 20$ kHz, these numbers imply a growth rate γ of order $0.1\,\mathrm{s^{-1}}$. As the instability should saturate in ten to twenty growth times, there would appear to be ample time for the Langmuir waves to grow to saturation.

Lin et al. also discussed quasilinear relaxation of the electrons. (Their discussion contains some confusing typographic errors.) Quasilinear relaxation may be described by (e.g. Melrose, 1980b, p. 133)

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v}\left[D(v)\frac{\partial f}{\partial v}\right] \tag{2}$$

with

$$D(v) = \frac{\pi \omega_p}{n_e m} v W(v) \tag{3}$$

where $W(v)\,\mathrm{d}v$ is the density in Langmuir waves in the range $\mathrm{d}v$ of phase speeds $v = \omega_p/k$. For waves in a range Δv we set $W(v) = \varepsilon_0 E^2/\Delta v$ and define the time τ_D for electrons to diffuse from v to $v - \Delta v$ by $(\Delta v)^2 \tau_D = D(v)$. Then (3) gives

$$\tau_D = \frac{n_e m v^2}{\pi \omega_p \varepsilon_0 E^2}\left(\frac{\Delta v}{v}\right)^3. \tag{4}$$

Using observed peak fields, Lin et al. (1981) concluded that τ_D is between 0.1 and 100 s. We discuss the estimate of τ_D and its implications further in Sect. 4(c) below.

2.3. Correlation with ion sound waves

Lin et al. (1985) discussed the correlation between Langmuir (L) waves in Tape III events and two classes of ion sound (S) waves.

One class of ion sound waves involves short wavelengths. These are interpreted as narrow band waves with Doppler-shifted frequencies of ≈ 0.5 to ≈ 5 kHz, i.e. above the ion plasma frequency (Gurnett and Frank, 1978). The Langmuir waves appear to avoid times when these ion sound waves are present. The Langmuir waves also seem to avoid times when whistlers are present.

The other class of low frequency waves occurs between approximately 50 and 300 Hz and attain electric field strengths up to $0.04\,\mathrm{mV\,m^{-1}}$ (Lin et al., 1985). They are intermittent, appearing in association with bursts of Langmuir waves. In fact, most bursts of Langmuir waves fields $\gtrsim 0.1\,\mathrm{mV\,m^{-1}}$ are found to be accompanied by these waves. Their wavenumbers are estimated to be approximately equal to those of resonant Langmuir waves, i.e. $k_S \approx \omega_p/v$. Such wavenumbers are typical of the waves participating in the processes $L \pm S \rightarrow T$ and $L \pm S \rightarrow L'$. Cairns (1984) and Lin et al. (1985) interpreted type III radiation in terms of the fundamental emission process $L \pm S \rightarrow T$ and the second harmonic emission process $L \pm S \rightarrow L'$, $L + L' \rightarrow T$. These processes are discussed below in Sects. 3 and 5.

We note that if this second class of waves are ion sound waves, then the ratio of the energy densities in these waves and the Langmuir waves is given by

$$\frac{W_S}{W_L} = \frac{1}{k_S^2 \lambda_{De}^2}\left(\frac{E_S}{E_L}\right)^2. \tag{5}$$

Lin et al. (1985) estimated all the parameters appearing on the right hand side of (5) for two bursts, and in both cases they found $W_S/W_L \approx 1$ for the maximum electric fields.

3. Suppression of the bump-in-tail instability

It is clear that the Langmuir waves do not normally grow when the necessary condition $\partial f/\partial v > 0$ for growth is satisfied. Suppression of the growth can be due either to low-frequency waves or to density inhomogeneities. We consider four specific suppression mechanisms.

3.1. Suppression due to short wavelength ion sound waves

The observation that the Langmuir waves tend to avoid times when short wavelength ion sound waves (frequencies ≈ 0.5 to 5 kHz) are present might suggest that the short wavelength waves suppress the growth. However these waves cannot satisfy the usual beat conditions for a three wave interaction.

For a Langmuir wave $(\omega_L, \boldsymbol{k}_L)$ to beat with a low-frequency wave $(\omega_F, \boldsymbol{k}_F)$ to produce another Langmuir mave $(\omega_{L'}, \boldsymbol{k}_{L'})$ the conditions

$$\omega_{L'} = \omega_L \pm \omega_F, \qquad \boldsymbol{k}_{L'} = \boldsymbol{k}_L \pm \boldsymbol{k}_F \tag{6a, b}$$

must be satisfied. For ion sound waves $(F = S)$ with $\omega_S = k_S v_S$ there is a maximum wavenumber for which (6a, b) can be satisfied (Melrose, 1982):

$$k_S \leqq 2k_L \pm k_0, \quad k_0 = \frac{2\omega_p v_S}{3V_e^2}. \tag{7}$$

One has $k_0 \approx \omega_p/65\,V_e \approx 0.3\,k_L$ for parameters reported by Lin et al. (1985). The observed short wavelength ion sound waves have k_S much larger than allowed by (7).

It is possible for the short wavelength waves to suppress the growth due to two other effects. A four-wave interaction ($L + F \rightarrow L' + F'$ schematically) is allowed (Melrose, 1963), and the three-wave interaction is allowed for $k_{L'} \approx k_S \gg k_L$ such that the L' waves are strongly Landau damped (Goldman and DuBois, 1982). These processes require particularly intense waves and particularly short wavelengths respectively. As the short wavelength waves are observed to be sporadic it seems likely that these processes could account only for sporadic suppression of the growth of the Langmuir waves. The normal suppression of the growth is probably not due to the observed short wavelength ion sound waves.

3.2. Waves with $k_S \approx k_L$

When the conditions (6a, b) are satisfied the three-wave process is quite efficient in suppressing the growth of Langmuir waves. Here we estimate the electric field strength E_S in ion sound waves with $k_S \approx 2k_L$ required to cause suppression.

The three-wave interaction can, in a single step, remove Langmuir waves from forward angles where growth occurs to backward angles where damping occurs. The rate at which the forward waves are lost due to this process is estimated in the Appendix. The effective damping rate is

$$\gamma_{NL} \approx \frac{\pi}{24} \frac{e^2 E_S^2}{m_e^2 \omega_p V_e^2} \left(\frac{v}{V_e}\right)^4. \tag{8}$$

where E_S is the electric field in the waves, and where we assume isotropic ion sound waves in a range $\Delta k_S \approx k_S \approx 2k_L = \omega_p/v$. For the parameters quoted by Lin et al. (1985), the condition $\gamma_{NL} \gtrsim 0.1\,\mathrm{s}^{-1}$ for suppression is satisfied for $E_S \gtrsim 5\,\mu\mathrm{V\,m}^{-1}$.

Thus the electric required for suppression by ion sound waves with $k_S \approx \omega_p/v$ is only a little above the instrumental threshold for their detection. The steady level of waves at the relevant frequencies may be estimated from Figs. 3 and 7 of Lin et al. (1985), which imply $E_S \approx 5\ 10^{-7}\,\mathrm{V\,m}^{-1}$. For comparison, Gurnett and Frank (1978) estimated that waves at a somewhat higher frequency are present a substantial fraction of the time at a level between 10 and 100 $\mu\mathrm{V\,m}^{-1}$ near the orbit of the Earth. It could be that the threshold condition for suppression by waves with $k_S \approx \omega_p/v$ is exceeded some of the time, but the most relevant existing data suggest that the actual level is slightly below the required level.

3.3. Low frequency ion sound waves

For $k_S \ll k_L$ each three-wave interaction satisfying (6a, b) causes only a small change in k_L. Many such interactions cause a diffusion of the Langmuir waves in k_L-space. Growth of the Langmuir waves tends to build up a distribution centered on a particular $\boldsymbol{k}_L$, and any diffusion drives Langmuir waves away from this peak. Suppression of the growth can occur if the rate of loss due to diffusion exceeds the rate of growth.

Muschietti et al. (1985) developed a model for suppression due to diffusion of the Langmuir waves in angle, attributed to the effects of a given spectrum of density fluctuations. Such density fluctuations would suppress the growth so effectively that they sought ways to offset the calculated effect. Muschietti et al. argued that growth is consistent with the observed spectrum of density fluctuations only if the sound waves propagate perpendicular to the magnetic field, producing density fluctuations aligned along the magnetic field.

3.4. Suppression due to refraction

In the presence of density gradients, Langmuir waves are refracted, and the associated change in k_L can remove them from the region of growth in k_L-space. This leads to a suppression of growth if the waves are refracted faster than they grow (e.g. Escande and de Genouillac, 1978; Melrose, 1980c, p. 208). Smith and Sime (1979) developed a layered model in which the density changed abruptly and randomly from one layer to the next. They found a strong suppression effect for relatively modest values of the density fluctuations.

Suppression occurs because the rays refract towards larger values of k_L and when k_L is large enough they are strongly Landau damped. The rate at which Langmuir waves are lost due to refraction can be estimated from the ray equation $\mathrm{d}\boldsymbol{k}/\mathrm{d}t = -\nabla\omega_L$. Suppose the electron density n changes by Δn over a distance L perpendicular to the ray direction (as discussed further in Sect. 4.1. below). Then the wave angle changes at the rate

$$\dot{\Theta} = \frac{\Delta n}{n} \frac{v}{2L} \tag{9}$$

for a wave with $k_L = \omega_p/v$. For $\Delta n/n \approx 10^{-2}$, $v \approx 3\ 10^7\,\mathrm{m\,s}^{-1}$ and $2L \approx 10^6$ m, (9) implies $\dot{\Theta} \approx 0.3\,\mathrm{s}^{-1}$ which is more than adequate to suppress the growth.

In summary there are three processes which can suppress the growth with parameters consistent with observational data. These involve three wave interactions with ion sound waves at $k_S \approx k_L$, scattering in angle by low frequency waves, and refraction in local density inhomogeneities. It is plausible that the last of these operates most of the time or, alternatively, throughout most of the solar wind.

4. Formation of clumps

In order for clumps to form, the suppression mechanism for wave growth must be ineffective in localized region. Here we explore the suggestion that there are well-ordered, field-aligned density structures (together with random density fluctuations) and that clumps form near the density minima.

4.1. Outline of the model

Suppose there are no random fluctuations in the electron density but there are static, field-aligned irregularities. The density profile could then be described in terms of ridges and valleys running parallel to the magnetic field. Let Δn be the density difference over a distance L between a typical ridge and the neighboring valley. Langmuir waves refract towards density minima, that is, towards the valleys. A Langmuir wave generated near the bottom of a valley is confined there in the sense that refraction prevents it from propagating in any direction other than along the valley.

This model is related to models developed to account for inferred propagation effects for radio emission from the solar corona (Bougeret and Steinberg, 1977; Duncan, 1979). Fundamental transverse waves can be trapped in density depletions and guided along them; this process is called ducting. There is evidence for strong ducting of type III radio emission in the solar corona (e.g. Suzuki and Dulk, 1985) and for ducting and scattering in the interplanetary plasma (Steinberg et al., 1984). It appears that well-ordered structures of the kind envisaged here are required to account for this ducting.

A more specific model for the static, field-aligned variations postulated here could be developed in terms of drift waves (e.g. Krall, 1968; Mikhailovskii, 1974). These waves have very low frequencies and propagate nearly across the field lines. They can be driven unstable by a gradient (e.g. in the density) across the field lines. Let L_N be the characteristic length associated with the gradient. Then the diamagnetic drift speed is $V_D \approx V_e^2/\Omega_e L_N$ (where Ω_e is the electron gyrofrequency) and the waves have frequencies $\approx kV_D$ which is much lower than the frequency of ion sound waves with the same k.

The raises the question as to whether the observed density fluctuations (Celnikier et al., 1983) are due to ion sound waves or to drift waves. In the following we assume that the density fluctuations consist of two components: a "random" component due to isotropic ion sound waves and a "regular" component which could be attributed to drift waves. The former is described by $\delta n/n$ and a mean wavenumber k_S and the latter by $\Delta n/n$ and a characteristic length L. Either of these can lead to suppression of the growth of the Langmuir waves, as discussed in Sect. 3.3. and 3.4. above. An exception is near the density minima in the "regular" component, i.e. near the bottoms of the valleys in the density profile, where the "random" variations tend to cause rays to diffuse away from parallel propagation and the "regular" component tends to refract them back to progagation along the field lines. Clumps can develop if the latter effect dominates.

4.2. Requirements on the density structures

In the model outlined above clumps develop because the suppression of Langmuir growth is inoperative in regions near density minima. This model requires that the normal suppression be due to refraction, caused either by the "random" or the "regular" components. We identify two necessary conditions for clumps to develop.

Langmuir waves must be trapped in the valleys in the density profile. Trapping occurs only if the change in plasma frequency $\Delta\omega_p = (\Delta n/n)\,\omega_p/2$ between a valley and a ridge exceeds the bandwidth of the Langmuir waves. Assuming that the waves are generated in a range Δv of phase speeds, this condition becomes

$$\frac{\Delta n}{n} > \frac{6V_e^2 \Delta v}{v^3}. \tag{10}$$

The other necessary condition is that the rate at which the waves are refracted towards the density minimum due to the "regular" variations exceed the rate at which the rays diffuse in angle due to the "random" variations. The former rate is given by (9) and the latter rate was estimated by Muschietti et al. (1984) to be

$$D_\Theta = \frac{\pi}{12} \frac{\omega_p}{(k_L \lambda_D)^2} \frac{\overline{k}_S}{k_L} \left(\frac{\delta n}{n}\right)^2. \tag{11}$$

With $k_L = \omega_p/v$ the condition $\dot{\Theta} > D_\Theta$ becomes

$$\frac{\Delta n}{n}\frac{1}{L} > \frac{\pi}{6}\left(\frac{v}{V_e}\right)^2 \left(\frac{\delta n}{n}\right)^2 \overline{k}_S. \tag{12}$$

With parameters estimated by Lin et al. (1986) (10) reduces to $\Delta n/n > 3\ 10^{-3}$, and one has $v/V_e \approx 20$ in (12).

The density fluctuations reported by Celnikier et al. (1983) extend over a range of wavelengths from about 10^2 to about 10^6 km, with a flattening of the spectrum at wavelengths shorter than 4000 km and a peak at wavelengths of order 600 km. At the shorter wavelengths, the integrated (over frequency) level of density fluctuations is roughly independent of the wavelength at a value $\delta n/n \approx 10^{-2}$. It is the shorter wavelengths that are relevant here. Ignoring all but the shortest wavelengths, Celnikier et al.'s data imply $\delta n/n \approx 10^{-2}$ at a wavenumber $\overline{k}_S \approx (100\ \text{km})^{-1}$ if the fluctuations are due to ion sound waves (for either $\Delta n/n \lesssim \delta n/n$ or $\overline{k}_S L \gg 1$), or they imply $\Delta n/n \approx 2\ 10^{-2}$ at $L \approx 300$ km if the fluctuations are due to static variations (for $\delta n/n \lesssim 2\ 10^{-2}$). Only the latter interpretation is compatible with (12). The peak at $L \approx 300$ km in Celnikier's et al.'s data implies that a characteristic length scale for the static density variations exists, so that refraction in the density structures may be an effective mechanism for suppressing the Langmuir wave growth. A flat spectrum of static density variations with a large range of characteristic lengths would be ineffective in causing refraction. Thus our model for clumps requires that the shortest wavelengths and peak in the spectrum of observed density fluctuations correspond to static, field-aligned variations, presumably due to drift waves.

Langmuir clumps should occur in regions with $\partial f/\partial v > 0$ provided that the two conditions implied by (10) and (12) are satisfied. Thus the amplitude $\Delta n/n$ of the "regular" variations should exceed $3\ 10^{-3}$, and the level of "random" variations should be sufficiently low that (12) is satisfied. The size of the clumps may be estimated from the inverse wavenumber of the observed density variations. The range of wavelengths from 100 to 4000 km for the flatter part of the spectrum of density fluctuations (Celnikier et al., 1983) fits well with the duration of 0.5 to 2 s of the Langmuir spikes being convected past a spacecraft at 300 to 500 km s^{-1}.

4.3. Saturation of the growth

The growth of Langmuir waves in clumps should proceed until the free energy is exhausted. The free energy arises from the positive gradient $\partial f/\partial v > 0$ in velocity space, and this is exhausted when a plateau $\partial f/\partial v = 0$ has formed. A simple model which allows us to estimate the free energy is as follows: before growth starts f increases linearly from v to $v = \Delta v$ and after saturation has occurred f is a constant between v and $v + \Delta v$, with the value of f determined by conservation of particles. The free energy density W_{fr} for $\Delta v \ll v$ is then

$$W_{fr} = \frac{1}{12} m v (\Delta v)^3 \frac{\partial f}{\partial v}. \tag{13}$$

This free energy is converted into Langmuir waves. Their electric field strength E is related to W_{fr} by the relation $\varepsilon_0 E^2 = W_{fr}$, i.e.

$$E = \left(\frac{W_{fr}}{\varepsilon_0}\right)^{1/2}. \tag{14}$$

Before comparing this simple theory with the observational data (Lin et al., 1986) it is important to note that one observational determination of $\partial f/\partial v$ takes 64 s. Our argument is that plateau formation occurs only in clumps, and the spacecraft passes through an individual clump in ≈ 1 s. Hence the observed distribution is averaged over a region very much larger than a clump, and one would not expect to observe plateau formation. Similarly if $\partial f/\partial v$ had a pronounced peak for up to ten seconds or so (in the frame of the spacecraft) this would not be readily detected because of the intrinsic time required to measure $\partial f/\partial v$.

Lin et al. (1986) estimated the parameters v, Δv and $\partial f/\partial v$ for two type III events and also estimated the peak electric fields for these events. For both events W_{fr}, as determined by (13), was between 10^{-19} and 10^{-18} J m^{-3}, implying that E from (14) is a few times 10^{-4} V m^{-1}. The observed peak fields were 1.0 and 1.5 10^{-3} V m^{-1} in the two events. In view of the uncertainties in the estimates of $\partial f/\partial v$ this agreement is reasonable. We conclude that saturation of the growth, also called quasilinear relaxation or plateau formation, does occur in the clumps of Langmuir waves.

Saturation of the instability can determine the length of a clump along the field lines. The time required for saturation to occur should be ten to twenty growth times, i.e. 10 to $20\gamma^{-1}$. The saturation time may also be estimated from the quasilinear relaxation time τ_D, cf. (4). Due to the variation in the value of E^2 in different bursts the latter estimate leads to a relatively large range of τ_D. The largest observed peak field implies a τ_D which is comparable with the estimate of $\approx 10\gamma^{-1}$. For $\gamma \approx 0.1$ s^{-1} this gives a time $\approx 10^2$ s. Electrons moving at 3 10^7 m s^{-1} propagate 3 10^6 km in 10^2 s. That is, over a distance of 3 10^6 km, or in 10^2 s, the electron distribution function is smoothed from a bump-in-tail form to a plateau. Thus clumps may be $\approx 10^4$ times as long as they are wide, but with a large variation in this length.

Once the free energy is exhausted and the growth stops in a given clump, propagation effects reinitiate the process of building up a distribution with $\partial f/\partial v > 0$. Let a plateau form over a length L and a range Δv. Then the time required for $\partial f/\partial v > 0$ to develop again is of order the time $L/\Delta v$ for the faster particles at the back of the clump region to overtake the slower particles at the front. With

$L \approx 3\ 10^6$ km and $\Delta v \approx 3\ 10^3$ km s^{-1}, this time is $\approx 10^3$ s. Thus along a given field lines we expect clumps to occur with a filling factor ≈ 0.1, an estimate that is not dependent on the very uncertain estimate of the length L of the individual clumps because the filling factor along the field lines is determined by the parameter $\Delta v/v$.

5. Radio emission from clumps

The average Langmuir wave energy density in type III events is quite low, and to account for the radio emission the nonlinear conversion of Langmuir wave energy into transverse radio waves must be quite efficient. Here we argue that the nonlinear conversion saturates at a value which provides a plausible explanation for the observed brightness temperatures of type III events.

5.1. The maximum brightness temperature

A three wave process analogous to (6a, b) can procedure fundamental transverse (T) waves. The kinematic conditions are

$$\omega_T = \omega_L \pm \omega_F, \quad \boldsymbol{k}_T = \boldsymbol{k}_L \pm \boldsymbol{k}_F \tag{15a, b}$$

Because k_T is much smaller than k_L, (15b) requires $k_F \approx \pm k_L$. Thus the required low frequency waves are anti-parallel to the magnetic field with $k_F \approx \omega_p/v$. Semiclassical theory implies a kinetic equation for the transverse waves (e.g. Melrose, 1980b, p. 173)

$$\frac{d}{dt} N_T(\boldsymbol{k}) = \int \frac{d^3\boldsymbol{k}_L d^3\boldsymbol{k}_F}{(2\pi)^6} u_{TLF}(\boldsymbol{k}_T, \boldsymbol{k}_L, \boldsymbol{k}_F) \cdot \{N_L(\boldsymbol{k}_L) N_F(\boldsymbol{k}_F) - N_T(\boldsymbol{k}_T) [N_F(\boldsymbol{k}_F) \pm N_L(\boldsymbol{k}_L)]\}, \tag{16}$$

where the occupation numbers N_M for the waves are related to their effective temperatures T_M and their energy densities W_M by

$$T_M(\boldsymbol{k}_M) = \omega_M N_M(\boldsymbol{k}_M), \quad W_M = \int \frac{d^3\boldsymbol{k}_M}{(2\pi)^3} T_M(\boldsymbol{k}_M), \tag{17}$$

where we set Boltzmann's constant equal to unity. Outside the source $T_T(\boldsymbol{k}_T)$ is a constant along a ray and may be identified as the intrinsic brightness temperature of the source. According to (16) the maximum brightness temperature is determined by

$$N_T(\boldsymbol{k}_T) < \frac{N_L(\boldsymbol{k}_L) N_F(\boldsymbol{k}_F)}{N_F(\boldsymbol{k}_F) \pm N_L(\boldsymbol{k}_L)}, \tag{18}$$

which implies

$$T_T(\boldsymbol{k}_T) < T_L(\boldsymbol{k}_L) \tag{19}$$

for $N_F \gg N_L$. For ion sound waves ($F = S$) the condition $N_S \gg N_L$ is implied by $W_S \approx W_L$, cf. Sect. 2.3., provided that the ion sound and Langmuir waves have similar distributions in wavevector $\boldsymbol{k}$. Then $W_S \approx W_L$ implies $\omega_S N_S \approx \omega_L N_L$ and hence $N_S \gg N_L$ in view of $\omega_S \ll \omega_L$.

We may estimate T_L by equating the energy density W_L in the Langmuir waves with the free energy W_{fr} given by (13). For Langmuir waves confined to a range of angles $\Delta\theta$ and a range of phase speeds Δv and v, one has

$$W_L = \pi(\Delta\theta)^2 \left(\frac{\omega_p}{2\pi v}\right)^3 \frac{\Delta v}{v} T_L. \tag{20}$$

For the parameters quoted by Lin et al. (1986), (20) implies (for T_L now in Kelvins) $T_L \approx 10^{34} W_L$ to the nearest power of ten, where we set $\pi(\Delta\theta)^2 \approx 1$. With the estimate $W_L \approx W_{fr}$ between 10^{-18} and 10^{-19} J m^{-3} (cf. Sect. 4.3.), one infers a maximum T_L between 10^{15} and 10^{16} K.

This discussion implies that the maximum brightness temperature at the fundamental is equal to T_L for emission from a single clump. Suppose the line of sight through the source encounters many clumps, each of which would radiate at T_T in isolation, i.e. that every clump is optically thick in the sense that the conversion process reaches saturation. The observed brightness temperature is to be identified with the average over the clumps across the visible surface of the source.

As was discussed in Sect. 2(a), observed brightness temperatures are almost never greater than a few $\times 10^{14}$ K. True brightness temperatures (corrected for scattering) are thus less than a few $\times 10^{15}$ K. These values are in excellent agreement with the maximum values implied by the theory, 10^{15} to 10^{16} K.

5.2. Saturation of the conversion process in clumps

The foregoing discussion suggests that the observed brightness temperature is indeed limited by the effective temperature of the Langmuir waves in the clumps. Saturation occurs only if the path length of transverse waves through a given clump is long enough for the processes $L \pm F \rightarrow T$ to cause T_T to increase to T_L.

The rate of increase in T_L is estimated in the Appendix. Let W_F be the energy density in the low frequency waves in a range Δk_F about $k_F = \omega_p/v$. Then one finds that T_T increases roughly at the rate

$$\frac{1}{T_L}\frac{dT_T}{dt} \approx \frac{\pi}{6} \frac{e^2 v^2}{\varepsilon_0 m_e^2 V_e^4 \omega_p} \frac{k_F}{\Delta k_F} W_F. \tag{21}$$

As noted in Sect. 2(c), low frequency waves with the required properties are observed in association with the Langmuir waves (Lin et al., 1986). Assuming that they are ion sound waves, their energy density is comparable with that in the Langmuir waves, which is $W_L = 10^{-18}$ to 10^{-19} J m^{-3} according to estimates made in Sect. 4.3. With this range of $W_S = W_L$, the saturation time is between 0.1 and 1 s.

The group speed of the transverse waves generated by the processes $L \pm S \rightarrow T$ is

$$v_g \approx \sqrt{3} \frac{V_e}{v} c, \tag{22}$$

where we assume $k_L = \omega_p/v \gg k_0$, cf. (7). For parameters of relevance here, one finds $v_g \approx 0.1\,c$. The time required to propagate across a clump (≈ 300 km) is quite short ≈ 0.01 s, and saturation would not occur in one crossing. However, the identification of clumps with field-aligned density depression implies that the transverse waves are trapped in the depression and are ducted along it. The saturation time is shorter than the propagation time along a clump. Thus one expects each clump to produce collimated radiation with $T_T \approx T_L$. This radiation can escape from the density depression only after propagating sufficiently far for the decrease in the mean density to exceed the difference between the local density maxima and minima.

5.3. Radiation from a collection of clumps

Our model for a type III source consists of a collection of narrow (≈ 300 km) long ($\approx 3 \times 10^6$ km) clumps producing radiation that is intially ducted along the field lines. The observed brightness temperature is effectively an average over a range of phase space, and the density of clumps must be adequate to fill this range of phase space.

Consider first the distribution in coordinate space. The observed brightness temperature is effectively averaged over the source, and this average gives $T_T \approx T_L$ only if at least one clump is encountered along each line of sight. (More than one clump along the line of sight does not increase T_T above T_L because the emission is optically thick at $T_T = T_L$.) The statistical distribution of clumps has not been discussed explicitly. One estimate (Sect. 2.1.) suggests that clumps fill about 1/30 of a typical path through a source. Adopting this value the mean distance between clumps is $\approx 10^4$ km if each clump is 300 km across. Along a line perpendicular to the magnetic field there would be $\approx 10^4$ clumps for a source 10^8 km across. Although these estimates are quite uncertain, it is likely that many clumps are encountered along any line of sight. Indeed the trajectory of a spacecraft through the source is an example of a path, and the observation that many clumps are encountered implies that many clumps will be encountered along any line of sight through the source.

The emission from each clump is confined to a small volume of $\boldsymbol{k}$-space. The bandwidth of local emission is restricted to the bandwidth of the Langmuir waves

$$\frac{\Delta\omega}{\omega_p} \approx 3\frac{V_e^2}{v^2}\frac{\Delta v}{v}. \tag{23}$$

Type III emission appears as a continuum in the sense that the line width is quite broad. Broad-band emission is seen if, along a particular line of sight, there are $\gtrsim \omega_p/\Delta\omega$ clumps, all radiating at slightly different frequencies. With the parameters quoted by Lin et al. (1986) one has $\omega_p/\Delta\omega \approx 10^3$. Broad-band emission is consistent with the model.

Also the strong ducting of the radiation might suggest that the transverse radiation is strongly collimated along the field lines. If the radiation is confined to a range $\Delta\Omega_i$ of solid angles in the duct and escapes into a range $\Delta\Omega_f$ of solid angles, then the maximum brightness temperature observed is $T_L \Delta\Omega_f$. However strong ducting does not necessarily imply $\Delta\Omega_i \ll \Delta\Omega_f$ because waves propagating at oblique angles can be ducted as well as waves propagating parallel to the duct. The conversion process favors emission at 90° to the field lines (for parallel Langmuir waves), and it is likely that $\Delta\Omega_i/\Delta\Omega_f$ is of order unity.

5.4. *Second harmonic emission*

There is observational evidence for second harmonic emission as well as for fundamental emission in interplanetary type III events (Dulk et al., 1984). Melrose (1982), in arguing that the observed emission must be fundamental emission on observational grounds, pointed out that there is also a theoretical argument against second harmonic emission. This argument is that in a scattering $L \pm S \rightarrow L'$ the difference $|k_{L'} - k_L|$ is limited by k_0, cf. (7), and then for coalescence $\boldsymbol{k}_{L'} + \boldsymbol{k}_L = \boldsymbol{k}_T$ to occur requires $k_T = \sqrt{3}\omega_p/c > k_0$. This condition is not satisfied. However, the argument neglects the effect of a spread in the wavenumbers of Langmuir waves (cf. Cairns and Melrose, 1985) and if this range $\Delta k = (\omega_p/v)(\Delta v/v)$ exceeds k_0 then second harmonic emission is possible by this twostep process. Estimates from the observed parameters give $\Delta k/k_L \approx \Delta v/v \approx 0.1$ to 0.2 and $k_0/k_L \approx 0.3$; this implies that the condition $\Delta k > k_0$ is not quite satisfied, but the difference is small enough that the second harmonic emission process may in fact operate.

Assuming that second harmonic emission is allowed (as seems to be required by observations), the brightness temperature T_B attained over a distance L along a ray path is roughly (e.g. Melrose, 1980b, p. 241)

$$\frac{c}{L} T_B \approx \frac{r_0 \omega_p^2}{30\, v\, T_e} T_L [T_L - 2T_B], \tag{24}$$

where the Langmuir waves are assumed to be isotropic and where $r_0 \approx 3\ 10^{-15}$ m is the classical radius of the electron. For $T_B \ll T_L$, on setting $\omega_p = 10^5\ \mathrm{s}^{-1}$ and $v = 0.1\,c$ in (24), one finds $T_B/T_L \approx 10^{-19} L_{\mathrm{km}} T_L/T_e$, with L_{km} denoting L in kilometers. Setting $T_L = 10^{16}$ K and $T_e = 10^5$ K one finds $T_B \approx 10^8 L_{\mathrm{km}}$, where T_B is in Kelvins. The maximum distance over which emission at a given frequency can arise is limited by the secular change in $f_p (\propto R^{-1})$ with radial distance R and by the bandwidth $\Delta\omega/\omega_p \approx 3(V_e/v)^2 (\Delta v/v)$ of the Langmuir waves. This maximum distance is of order $\Delta\omega/\omega_p$ times an astronomical unit, i.e. of order $3\ 10^5$ km. It follows that second harmonic emission could have a brightness temperature up to about $T_B = 3\ 10^{13}$ K from each clump. With N clumps along the line of sight, the brightness temperatures add incoherently, provided that the resulting value $T_B = N \times 3\ 10^{13}$ K does not exceed T_L. The estimate made above for the number of clumps along the line of sight, $N \approx 10^4$, suggests that the second harmonic is also likely to saturate at $T_B \approx T_L$.

As was pointed out above, an attractive feature of our theory is that it can account naturally for the brightness temperatures for the fundamental and second harmonic: these would be equal if they both saturate at $T_B \approx T_L$.

5.5. *Origin of the low frequency waves*

An important uncertainty concerns the generation of the low frequency waves. This has been discussed by Lin et al. (1986) and Cairns (1984). In brief: the threshold on W_L for the parametric decay of a Langmuir pump into a daughter Langmuir wave and an ion sound wave is not satisfied, and the observed level $W_S \approx W_L$ is not consistent with a weak turbulence process generating the ion sound waves from the Langmuir waves. (This latter point has not been made explicitly before: N_S is restricted to less than N_L, and then W_{eff}/W_L should remain less than about $\omega_S/\omega_p = v_S/v \approx 10^{-3}$.) Until the generation of the low-frequency waves is adequately understood one can have little confidence in kinematic arguments involving these waves.

6. Discussion and conclusions

We have argued that the observational data on type III events in the interplanetary medium point to a specific model, with the properties of this model outlined in the Introduction. A basic requirement of the model is that the observed density inhomogeneities be highly elongated, field-aligned structures. It is these inhomogeneities which are assumed to suppress the growth of Langmuir waves throughout most of space and allow growth to develop only near the density minima, creating elongated clumps of Langmuir waves. (In principle this assumed form for the density inhomogeneities can be tested observationally by comparing high-time-resolution data taken on a spacecraft when the magnetic field lines lie parallel vs. perpendicular to the solar wind direction.) One possibility is that the observed density fluctuations are associated with drift waves; this point requires further examination.

We have emphasized the use of brightness temperatures in comparing theory and observation. As in a related context (Cairns and Melrose, 1985) the brightness temperature can approach a maximum value which is determined by the effective temperature of the Langmuir waves. A theoretical maximum exists for the latter, depending only on the properties of the electron stream. Data on the electrons, the Langmuir waves and the radio emission support our suggestion that the observed brightness temperature is limited by saturation effects. In particular Steinberg et al. (1984) and Dulk et al. (1984) found that many bursts have a peak brightness temperature of about 10^{14} K, which is raised to 10^{15} K if the actual source is smaller in area the apparent source by a factor ≈ 10 (Steinberg et al., 1985). The theoretical maximum estimated using observational data on the electrons is between 10^{15} K and 10^{16} K. In more detail the theory implies a brightness determined by $T_T = T_L$ (cf. (13) with (20) for $W_L = W_{fr}$),

$$T_T \propto \frac{v^5 (\Delta v)^2}{\Delta\Theta f_p^3} \frac{\partial f}{\partial v}, \tag{24}$$

where $\Delta\Theta$ is the range of angles to which the Langmuir waves are confined. The relation (24) is amenable to a statistical test of the theory. In one statistical study (Fitzenreiter et al., 1976) a relation $J_r \propto J_e^\alpha$ between the radio flux J_r and the electron flux J_e was found, with $\alpha \approx 1$ at low electron fluxes and $\alpha \approx 2.5$ at higher electron fluxes. The radio flux is proportional to T_T at a fixed frequency and area of emission, and the electron flux is proportional to $v^2 f$; these relations suggest that one can readily account for $J_r \propto J_e$ using (24). However (24) is much more specific than a relation of the form $J_r \propto J_e^\alpha$.

One point which needs further investigation is the generation of the ion sound turbulence observed in association with Langmuir waves in type III events (Lin et al., 1986; Cairns, 1984). These waves play an essential role in our theory; they are required to make the conversion process to fundamental radiation efficient enough to saturate. They also play an essential role in providing the secondary Langmuir population for second harmonic generation, which could also approach saturation and give similar brightness temperatures. The suggestion that the low frequency waves are generated by a weak turbulence process (Cairns, 1984) implies an upper limit ($W_S/W_L \leqq 10^{-3}$) which is not consistent with the data ($W_S/W_L \approx 1$). Moreover, the suggestion that these waves are generated by parametric decay instabilities (Gurnett et al., 1981; Cairns, 1984; Lin et al., 1986) is subject to some uncertainty: Lin et al. found that the Langmuir waves are below threshold for the instability $L \to L' + S$, yet waves with frequencies consistent with this process were observed. The lack of a detailed understanding of the generation mechanism for these ion sound waves leaves a major uncertainty in our model.

The present situation with the theory of type III bursts seems encouraging. The qualitative theory for type III emission has long been accepted, but all quantitative theories hitherto have failed to account for important features of the observations. The model developed here is based strongly on the data together with several relatively simple theoretical ideas. We have argued that this model provides a consistent semi-quantitative interpretation of all the important data. Such a model has not been possible previously because the relevant data on the Langmuir waver (Gurnett and Anderson, 1978), on the unstable electrons (Lin et al., 1981; Lin et al., 1986), on the density fluctuations observed in situ in the interplanetary plasma (Celnikier et al., 1983), on the detailed properties of the radiation including the important fact that it is initially at the fundamental (Dulk et al., 1984; Steinberg et al., 1985), and on the low frequency waves associated with the Langmuir waves (Lin et al., 1986), have only recently become available.

Acknowledgements. We thank J.L. Steinberg for permission to use unpublished data from ISEE-3, and thank him, M. Goldman and L. Muschietti for valuable discussions and comments. Part of this work was supported by NASA's Solar Terrestrial Theory and Solar Heliospheric Physics programs under grants NAGW-91 and NSG-7287 to the University of Colorado.

Appendix

Let L denote an initial Langmuir wave, S a sound wave and $P = T$ or L' a transverse wave or a scattered Langmuir wave. The kinetic equation for the waves P due to the process $L \pm S \to P$ is, in semiclassical form, (e.g. Melrose, 1980b, p. 175)

$$\frac{\mathrm{d}}{\mathrm{d}t} N_P(\boldsymbol{k}_P) = \sum_{\pm} \int \frac{\mathrm{d}^3 \boldsymbol{k}_L}{(2\pi)^3} \int \frac{\mathrm{d}^3 \boldsymbol{k}_S}{(2\pi)^3} u_{PLS}(\boldsymbol{k}_p, \boldsymbol{k}_L, \pm \boldsymbol{k}_S) \times [N_L(\boldsymbol{k}_L) N_S(\boldsymbol{k}_S) - N_P(\boldsymbol{k}_P)\{\pm N_L(\boldsymbol{k}_L) N_S(\boldsymbol{k}_S)\}] \tag{A1}$$

with

$$u_{PLS}(\boldsymbol{k}_P, \boldsymbol{k}_L, \pm \boldsymbol{k}_S) \approx \frac{(2\pi)^4 e^2 \hbar\omega_s(\boldsymbol{k}_S)}{8\varepsilon_0 m_e^2 V_e^2} \varrho_P(\boldsymbol{k}_L, \boldsymbol{k}_P) \cdot \delta\{\omega_P(\boldsymbol{k}_P) - \omega_L(\boldsymbol{k}_L) \pm \omega_S(\boldsymbol{k}_S)\} \times \delta^3(\boldsymbol{k}_P - \boldsymbol{k}_L \pm \boldsymbol{k}_S) \tag{A2}$$

$$\varrho_P(\boldsymbol{k}_L, \boldsymbol{k}_P) = \begin{cases} |\boldsymbol{k}_L \cdot \boldsymbol{k}_P|^2/k_L^2 k_P^2 & P = L' \\ |\boldsymbol{k}_L \cdot \boldsymbol{k}_P|^2/k_L^2 k_P^2 & P = T \end{cases} \tag{A3}$$

The dispersion relations are approximated by

$$\omega_L(\boldsymbol{k}_L) = \omega_P + \frac{3}{2}\frac{k_L^2 V_e^2}{\omega_P}, \quad \omega_S(\boldsymbol{k}_S) = k_S v_S, \tag{A4a, b}$$

$$\omega_T(\boldsymbol{k}_T) = \omega_P + \frac{k_T^2 c^2}{2\omega_P} \tag{A4c}$$

with $v_S \approx V_e/43$ the ion sound speed and V_e the thermal speed of electrons. The effective temperature $T_M(\boldsymbol{k}_M)$ of each mode is given be

$$T_M(\boldsymbol{k}_M) = \hbar\omega_M(\boldsymbol{k}_M) N_M(\boldsymbol{k}_M) \tag{A5}$$

where Boltzmann's constant is set equal to unity.

For semi-quantitative purposes only the term $N_L N_S$ is retained in the square brackets in (A1) and both $N_L(\boldsymbol{k}_L)$ and $N_S(\boldsymbol{k}_S)$ are assumed to be constant over some range of wavevectors and zero otherwise. The $\boldsymbol{k}_L$-integral in (A1) is performed over the final δ-function in (A2). The $\boldsymbol{k}_S$-integral is evaluated in spherical polar coordinates with the polar axis aligned along the vector $\boldsymbol{k}_P$. The k_S-integral is performed over the remaining δ-function in (A3), after which the angular integrals are trivial for isotropic S waves and highly collimated L waves:

$$\int dk_S \delta\{\omega_P(\boldsymbol{k}_P) - \omega_L(\boldsymbol{k}_P \pm \boldsymbol{k}_S) \pm \omega_S(\boldsymbol{k}_S)\} k^3 \approx \frac{\omega_p}{3V_e^2} k_S^2,$$

where k_S is written in terms of $k_L = \omega_p/v, k_0$, and the spherical angles. Note that the delta functions require $k_S \approx 2\omega_p/V$ for $P = L'$, and $k_S \approx \omega_p/v$ for $P = T$ (for $k_L \gg k_0$). The energy density W_S and electric field E_S in the ion sound waves in a range Δk_S at $k_S \approx \omega_p/v$ are related to T_S approximately by

$$W_S = \varepsilon_0 E_S^2 \left(\frac{v}{V_e}\right)^2 = \frac{4\pi}{3}\left(\frac{k_S}{2\pi}\right)^3 T_S \frac{\Delta k_s}{k_s}. \tag{A9}$$

After summing over the $\pm$ contributions in (A1), the result (8) follows for $P=L'$ and $\Delta k_S = k_S$ with $\gamma_{NL} = (dT_{L'}/dt)/T_{L'}$. The result (21) is obtained for $P=T$ and applies to any low frequency mode. (Only in the relation (A9) between E_S and W_S is it assumed that the low frequency waves are ion sound waves.)

References

Bougeret, J.L., Steinberg, J.J.: 1977, *Astron. Astrophys.* **61**, 777

Cairns, I.H.: 1984, Proc. Course and Workshop on Plasma Astrophysics, Varenna, Italy ESA SP-207, p. 141

Cairns, I.H., Melrose, D.B.: 1985, *J. Geophys. Res.* **90**, 6637

Celnikier, L.M., Harvey, C.C., Jegou, R., Kemp, M., Moricet, P.: 1983, *Astron. Astrophys.* **126**, 293

Dulk, G.A., Steinberg, J.L., Hoang, S.: 1984, *Astron. Astrophys.* **141**, 30

Duncan, R.A.: 1979, *Solar Phys.* **63**, 389

Escande, D.F., de Genouillac, G.V.: 1978, *Astron. Astrophys.* **68**, 405

Fainberg, J., Hoang, S., Manning, R.: 1985, *Astron. Astrophys.* **153**, 145

Fitzenreiter, R.J., Evans, J.G., Lin, R.P.: 1976, *Solar Phys.* **46**, 437

Ginzburg, V.L., Zheleznyakov, V.V.: 1958, *Soviet Astron. AJ.* **2**, 653

Goldman, M.V.: 1983, *Solar Phys.* **89**, 403

Goldman, M.V., DuBois, D.F.: 1982, *Fluids* **25**, 1062

Gurnett, D.A., Anderson, R.R.: 1976, *Science* **194**, 1159

Gurnett, D.A., Anderson, R.R.: 1977, *J. Geophys. Res.* **82**, 632

Gurnett, D.A., Frank, L.A.: 1978, *J. Geophys. Res.* **83**, 58

Gurnett, D.A., Anderson, R.R., Scarf, F.L., Kurth, W.S.: 1978, *J. Geophys. Res.* **83**, 4147

Gurnett, D.A., Anderson, R.R., Tokar, R.L.: 1980, in M.R. Kundu and T.E. Gergely (eds.) *Radio Physics of the Sun*, Reidel, Dordrecht, p. 369

Gurnett, D.A., Maggs, J.E., Gallagher, D.I., Kurth, W.S., Scarf, F.L.: 1981, *J. Geophys. Res.* **86**, 8833

Kaplan, S.A., Tsytovich, V.N.: 1968, *Soviet Astron. AJ.* **11,** 834

Kellogg, P.J.: 1976, *Solar Phys.* **46**, 543

King, J.: 1983, *Interplanetary Medium Data Book*, *Suppl. 2*, NSSDC **83-01**

Knoll, R., Epstein, G., Hoang, S., Huntzinger, G., Steinberg, J.L., Fainberg, J., Grena, F., Mosier, S.R., Stone, R.G.: 1978, *ISEE Trans. Geosci. Electronics* **GE-16**, 199

Krall, N.A.: 1968, *Adv. Plasma Phys.* **1**, 153

Lin, R.P., Potter, D.W., Gurnett, D.A., Scarf, F.L.: 1981, *Astrophys. J.* **251**, 364

Lin, R.P., Levedahl, W.K., Lotko, W., Gurnett, D.A., Scarf, F.L.: 1985, "Evidence for Non-Linear Wave-Wave Interactions in Solar Type III Radio Bursts", *Astrophys. J.* (in press)

Manning, R., Fainberg, J.: 1980, *Space Sci. Instr.* **5**, 161

Melrose, D.B.: 1980a, *Space Sci. Rev.* **26**, 3

Melrose, D.B.: 1980b, c, *Plasma Astrophysics*, Volumes I and II, Gordon and Breach, New York

Melrose, D.B.: 1982, *Solar Phys.* **79**, 173

Melrose, D.B.: 1983, *Solar Phys.* **87**, 359

Mikhailovskii, A.B.: 1974, *Theory of Plasma Instabilities*, Volume 2, Consultants Bureau, New York

Muschietti, L., Goldman, M.V., Newman, D.: 1985, *Solar Phys.* **96**, 181

Papadopoulos, K., Goldstein, M.L., Smith, R.A.: 1974, *Astrophys. J.* **190**, 175

Scarf, F.L., Fredericks, R.W., Gurnett, D.A., Smith, E.J.: 1978, *IEEE Trans. Geosci. Electronics* **GE-16**, 191

Smith, D.F., Sime, D.: 1979, *Astrophys. J.* **233**, 998

Steinberg, J.L., Dulk, G.A., Hoang, S., Lecacheux, A., Aubier, M.: 1984, *Astron. Astrophys.* **140**, 39

Steinberg, J.L., Hoang, S., Dulk, G.A.: 1985, *Astron. Astrophys.* **150**, 205

Sturrock, P.A.: 1964, in W.H. Ness (ed.) *Physics of Solar Flares*, AAS-NASA Symp., NASA Sp-50, Washington, D.C., p. 357

Suzuki, S., Dulk, G.A.: 1985, Chap. 12 in, *Solar Radiophysics*, eds. D.J. McLean and N.R. Labrum, Cambridge University Press