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ON THE CONTROVERSY CONCERNING TURBULENT BREMSSTRAHLUNG

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ABSTRACT

It is shown that the derivation of a nonzero growth rate for turbulent bremsstrahlung involves an incomplete summation of the nonlinear responses. When the classically correct expression is used, the growth rate is identically zero. Next we show that the radiative correction to the resonant emission of ion sound waves in the presence of nonresonant Langmuir waves is derived explicitly. Finally, we reply to criticisms by Nambu (1986) of our earlier paper and show that the inclusion of a background magnetic field gives a zero growth rate for this proposed kind of turbulent bremsstrahlung.

Subject headings: hydromagnetics — plasmas — radiation mechanisms

I. INTRODUCTION

Turbulent bremsstrahlung (hereafter TB) was suggested originally by Tsytovich, Stenflo, and Wilhelmsson (1975) (hereafter TWS) as a process for upconversion of ion sound waves into Langmuir waves in a current-carrying plasma. Three separate controversies have arisen concerning TB. The first (Vlahos and Papadopoulos 1979; Kuijpers 1980*a*; Tsytovich, Stenflo, and Wilhemsson 1981) involved uncertainty in the relation between TB and other, more familiar second-order processes in weak turbulence theory. This controversy resulted in a clearer understanding that TB is independent of other upconversion and nonlinear damping processes, but there remained doubts concerning its efficacy in relevant applications (Vlahos and Papadopoulos 1982). The second controversy concerned an alternative form of TB proposed by Nambu (1981), who called TB "induced bremsstrahlung" and, more recently (e.g., Nambu 1986), "the plasma maser effect." Several authors had noted independently that the growth rate for Nambu's form of TB is identically zero (Kuijpers 1980*b*; Tsytovich 1980; Melrose 1982). The nonexistence of Nambu's form of TB was pointed out explicitly by Melrose and Kuijpers (1984), Hirose (1984), and DuBois and Pesme (1984); Kuijpers and Melrose (1985) (hereafter KM) identified the specific error in Nambu's analysis and showed that when this error is corrected his analysis implies an identically zero growth rate. However, these analyses are strictly valid for a plasma without a background magnetic field. Recently, Nambu (1986) has argued that the inclusion of a background magnetic field allows TB to exist. Below, we prove that in this case also a correct analysis implies an identically zero growth rate.

The third controversy concerns the claim by KM, and also by Melrose and Kuijpers (1984) (hereafter MK), that the original form of TB as proposed by TSW also does not exist. The arguments in KM and MK centered around a symmetry property which implies that the growth rate for TB should be identically zero. Tsytovich (1985) claimed that an error was made in MK in connection with the symmetry properties of the real parts of the nonlinear responses. Therefore, our main purpose is to identify the specific step where the differences in the analyses of TSW and KM arise and hence to identify the particular point underlying this third controversy.

In § II a general expression for the nonlinear growth rate is written down and the analysis for TB is carried through following TSW to the point where one needs to take a resonant part. It is shown, purely classically, that TSW obtained a nonzero growth rate for TB only because they did not use the complete expression. We show that in the classically correct expression terms cancel exactly and the growth rate for TB is identically zero. In § III we reply to some criticisms of KM made by Nambu (1986 and private communication) and show that a correct evaluation of his form of TB also leads to a zero growth rate in the case of a background magnetic field. Our results are discussed in § IV.

II. SYMMETRIES AND RESONANCES

For present purposes TB may be defined as any second-order process which leads to growth of high-frequency waves due to particles resonating with low-frequency waves. This definition covers both the forms proposed by TSW and by Nambu (1981). Here we outline a derivation of the growth rate using the notation of KM.

a) Comparison with TSW

For simplicity, all fields are assumed longitudinal, and the nonlinear responses are described in terms of the susceptibilities $\chi(k, k_1, k_2)$ and $\chi(k, k_1, k_2, k_3)$, with k denoting ω, $\boldsymbol{k}$, and similarly for k_1, k_2, and k_3. Let two modes be labeled A and B, with dispersion relations $\omega = \omega_A(\boldsymbol{k})$ and $\omega' = \omega_B(\boldsymbol{k}')$ and ratios of electric to total energy $R_A(\boldsymbol{k})$ and $R_B(\boldsymbol{k}')$. Let $W_B(\boldsymbol{k}')d^3\boldsymbol{k}'/(2\pi)^3$ be the energy density in the indicated range in the waves in mode B. A general expression for the nonlinear (second-order) growth rate for the waves in mode A is (SI units) (KM, eq. [A6])

$$\gamma_A^{\mathrm{NL}}(\boldsymbol{k}) = \frac{R_A(\boldsymbol{k})}{\epsilon_0^2|\boldsymbol{k}|^2} \int \frac{d^3\boldsymbol{k}'}{(2\pi)^3} \frac{R_B(\boldsymbol{k}')W_B(\boldsymbol{k}')}{|\boldsymbol{k}'|^2} \, \mathrm{Im}\, [\omega\bar{\chi}(k, k', k, -k') - \omega\bar{\chi}(-k, k', -k, -k')]_{\omega=\omega_A(\boldsymbol{k}),\, \omega'=\omega_B(\boldsymbol{k}')} , \tag{1}$$

where Im denotes the imaginary part, $R_A(\boldsymbol{k}) = W_A^E(\boldsymbol{k})/W_A(\boldsymbol{k})$, and $W_A^E(\boldsymbol{k}) = \epsilon_0|\boldsymbol{k}|^2\,|\phi_A(\boldsymbol{k})|^2$. To treat TB using equation (1), one is to retain only those contributions to the imaginary part that arise from $\omega_B(\boldsymbol{k}') - \boldsymbol{k}' \cdot \boldsymbol{v} = 0$. There are other resonances in Im χ, but these describe other second-order processes (e.g., Melrose 1982).

So far there is no disagreement with TSW. The point of controversy concerns the expression for $\bar{\chi}$. The proper expression is derived from the expansion for the charge density $\rho(k)$ in powers of the electrostatic potential $\phi(k)$ (cf. KM, eq. [7]):

$$\rho^{(3)}(k) \equiv \int d\lambda^{(3)}\chi(k, k_1, k_2, k_3)\phi(k_1)\phi(k_2)\phi(k_3) , \tag{2}$$

with (KM eq. [6])

$$d\lambda^{(3)} \equiv \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}(2\pi)^4\delta^4(k - k_1 - k_2 - k_3) \tag{3}$$

and (KM eq. [14])

$$\chi(k, k_1, k_2, k_3) = -e^4 \int d^3\boldsymbol{p} \frac{1}{\omega - \boldsymbol{k}\cdot\boldsymbol{v}} \boldsymbol{k}_1 \cdot \frac{\partial}{\partial \boldsymbol{p}} \left\{ \frac{1}{\omega_2 + \omega_3 - (\boldsymbol{k}_2 + \boldsymbol{k}_3)\cdot\boldsymbol{v}} \boldsymbol{k}_2 \cdot \frac{\partial}{\partial \boldsymbol{p}} \left[\frac{1}{\omega_3 - \boldsymbol{k}_3\cdot\boldsymbol{v}} \boldsymbol{k}_3 \cdot \frac{\partial f(\boldsymbol{p})}{\partial \boldsymbol{p}} \right] \right\} . \tag{4}$$

In the application to TB there are two contributions to $\phi(k_j)$ at *any* k_j:

$$\phi(k_j) = \phi_{\mathrm{L}}(k_j) + \phi_s(k_j) , \tag{5}$$

with $\phi_{\mathrm{L}}(k_j)$ describing the Langmuir waves and $\phi_s(k_j)$ the ion sound waves (cf. KM eq. [10]). On substituting equation (5) into equation (2) we retain all terms proportional to the product of two sound wave potentials and one Langmuir wave potential. There are three such terms. In view of the freedom to permute the variables k_1, k_2, and k_3 of integration in equation (2), the relevant contribution to the charge density is then

$$\rho^{(3)}(k) = \int d\lambda^{(3)} \frac{1}{2} \left[\sum_{\{k_1, k_2, k_3\}} \chi(k, k_1, k_2, k_3) \right] \phi_s(k_1)\phi_{\mathrm{L}}(k_2)\phi_s(k_3) , \tag{6}$$

where the sum is over *all* six permutations of $\{k_1, k_2, k_3\}$. Therefore, the correct expression for $\bar{\chi}$ in equation (1) is given by

$$\bar{\chi}(k, k_1, k_2, k_3) = \frac{1}{2} \sum_{\{k_1, k_2, k_3\}} \chi(k, k_1, k_2, k_3) . \tag{7}$$

There remain six different terms in $\bar{\chi}$, with these being obtained from equation (4) by permuting k_1, k_2, and k_3. However, the only part of equation (6) which is relevant to TB is the imaginary part due to resonant denominators at $\omega' - \boldsymbol{k}' \cdot \boldsymbol{v} = 0$ for $k_2 = k$ (Langmuir waves) and $k_1 = -k_3 = \pm k'$ (ion sound waves). There are four such terms. These are the four obtained from symmetrizing equation (4) as follows: the term of equation (4) itself, that obtained from it by interchanging k_1 and k_2 (both these have relevant resonances at $\omega_3 - \boldsymbol{k}_3 \cdot \boldsymbol{v} = 0$), a third term obtained from equation (4) by interchanging k_1 and k_3, and a fourth term obtained from the third by interchanging k_2 and k_3 in the third term (both these have relevant resonances at $\omega_1 - \boldsymbol{k}_1 \cdot \boldsymbol{v} = 0$). The remaining two of the six terms in $\bar{\chi}$ have no relevant resonance, i.e., do not have a resonance at $\omega' - \boldsymbol{k}' \cdot \boldsymbol{v} = 0$ for $k_1 = -k_3 = \pm k'$, and these terms are discarded in treating TB.

The essential difference between us and TSW is that in TSW (their eqs. [6a] and [7]) only two of these four relevant terms are retained. In TSW, the symmetry involved in interchanging k_1 and k_3 was imposed, but the necessary full symmetry was not. We now show that the two resonant terms at $\omega_3 - \boldsymbol{k}_3 \cdot \boldsymbol{v} = 0$ cancel each other, as do the two terms at $\omega_1 - \boldsymbol{k}_1 \cdot \boldsymbol{v} = 0$, for $k_2 = k$ and $k_1 = -k_3 = k'$. It is this exact cancellation that implies that TB does not exist. At this stage we stress that the result of equation (7) is an inevitable consequence of the classical calculation, and that one is not free *a priori* to leave out some of these contributions, that is those of the implied symmetries. Indeed, if TSW were to have substituted their equation (3) (which corresponds to our eq. [5]) in a consequent way, they would have arrived at a complete expression equivalent to our equation (7).

We now calculate explicitly Im $\bar{\chi}$. We are interested in the imaginary parts in $\bar{\chi}$ which arise from the ion sound resonance $\omega' - \boldsymbol{k}' \cdot \boldsymbol{v} = 0$ for $k_1 = -k_3 = k'$. We first consider the resonances from k_3 and later that from k_1. The two imaginary contributions from $\omega_3 - \boldsymbol{k}_3 \cdot \boldsymbol{v} = 0$ follow from equations (7) and (4):

$$\begin{aligned} \underset{3}{\mathrm{Im}}\, \bar{\chi}(k, k_1, k_2, k_3) &= \frac{1}{2}\left[\underset{3}{\mathrm{Im}}\, \chi(k, k_1, k_2, k_3) + \underset{3}{\mathrm{Im}}\, \chi(k, k_2, k_1, k_3) \right] = \frac{\pi i}{2}\frac{e^4}{m^2} \int d^3\boldsymbol{p}\delta(\omega_3 - \boldsymbol{k}_3\cdot\boldsymbol{v})\boldsymbol{k}_3 \cdot \frac{\partial f}{\partial \boldsymbol{p}} \\ &\times \frac{\boldsymbol{k}_1\cdot\boldsymbol{k}\boldsymbol{k}_2\cdot\boldsymbol{k}(\omega - \boldsymbol{k}\cdot\boldsymbol{v})^2 - \boldsymbol{k}_1\cdot\boldsymbol{k}_2\boldsymbol{k}_1\cdot\boldsymbol{k}(\omega_1 - \boldsymbol{k}_1\cdot\boldsymbol{v})^2 - \boldsymbol{k}_1\cdot\boldsymbol{k}_2\boldsymbol{k}_2\cdot\boldsymbol{k}(\omega_2 - \boldsymbol{k}_2\cdot\boldsymbol{v})^2}{(\omega - \boldsymbol{k}\cdot\boldsymbol{v})^2(\omega_1 - \boldsymbol{k}_1\cdot\boldsymbol{v})^2(\omega_2 - \boldsymbol{k}_2\cdot\boldsymbol{v})^2} \\ &= -\frac{\pi i}{2}\frac{e^4}{m^2} \int d^3\boldsymbol{p}\delta(\omega' - \boldsymbol{k}'\cdot\boldsymbol{v})\boldsymbol{k}' \cdot \frac{\partial f}{\partial \boldsymbol{p}} \frac{(\boldsymbol{k}'\cdot\boldsymbol{k})^2}{(\omega - \boldsymbol{k}\cdot\boldsymbol{v})^4} . \end{aligned} \tag{8}$$

To arrive at the second equality we have made use of the delta functions $\delta^4(k - k_1 - k_2 - k_3)$ in equation (3) and $\delta(\omega_3 - \boldsymbol{k}_3 \cdot \boldsymbol{v})$. In the final result of equation (8) we have used $k_2 = k$ and $k_1 = -k_3 = k'$. The expression on the right-hand side of the result in equation (8) is an even function of k, and hence one has

$$\operatorname*{Im}_{3} [\bar{\chi}(k, k', k, -k') - \bar{\chi}(-k, k', -k, -k')] = 0 \,. \tag{9a}$$

Since, according to equation (7) $\bar{\chi}$ has the same dependence on k_3 as on k_1, it immediately follows that also

$$\operatorname*{Im}_{1} [\bar{\chi}(k, k', k, -k') - \bar{\chi}(-k, k', -k, -k')] = 0 \,. \tag{9b}$$

Substitution of the results of equations (9a) and (9b) into equation (1) then directly leads to the nonexistence of TB of Tsytovich's kind.

We would like to point out that summation over the permutations $\{k_1, k_2, k_3\}$ is also present in the intermediate result in Davidson (1972, Chap. 14, eq. [26]) in the derivation of this kinetic wave equation. Further, the expression (7) is in exact agreement with the prescription in MK (eqs. [11a], [11b], [11c]).

b) *Radiative Correction*

Tsytovich (1982, 1985) has claimed that TB does exist; however, in these papers what is treated is the exchange of energy between particles and the combined field of resonant and nonresonant waves, including the effects of radiative corrections. We agree that such energy exchanges occur, but they do not imply the existence of TB. Indeed, we have pointed out in KM that TSW's form of TB is related by a crossing symmetry to a radiative correction to the resonant emission and absorption of ion sound waves. In the emission and absorption processes corresponding to this radiative correction, the Langmuir waves are emitted and reabsorbed in identical pairs so that the spectrum of Langmuir waves is strictly unaffected. It is in this sense that the radiative correction exists, but TB does not exist. That is, our arguments against the existence of TB do not imply the nonexistence of the radiative correction itself, as we now demonstrate explicitly.

The (nonlinear) radiative correction to the emission of resonant ion sound waves may be calculated from an expression similar to equation (1). We are interested in the growth rate of the ion sound waves $\gamma_B^{\rm NL}(\boldsymbol{k}')$, and this follows from equation (1) by the substitution $A \leftrightarrow B$, $k \leftrightarrow k'$. As a consequence, we need to calculates Im $[\omega_0 \bar{\chi}(k_0, k_1, k_2, k_3)]$ with $k_0 = k_2 = k'$ (ion sound) and $k_1 = -k_3 = k$ (Langmuir). Thus, in place of equation (1) we have

$$\gamma_B^{\rm NL}(\boldsymbol{k}') = \frac{R_B(\boldsymbol{k}')}{\epsilon_0^2 |\boldsymbol{k}'|^2} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{R_A(\boldsymbol{k}) W_A(\boldsymbol{k})}{|\boldsymbol{k}|^2} \,\mathrm{Im}\, [\omega' \bar{\chi}(k', k, k', -k) - \omega' \bar{\chi}(-k', k, -k', -k)]_{\omega = \omega_A(\boldsymbol{k}),\, \omega' = \omega_B(\boldsymbol{k}')} \,. \tag{10}$$

There are four terms in $\bar{\chi}(k_0, k_1, k_2, k_3)$ that have resonances at $\omega' - \boldsymbol{k}' \cdot \boldsymbol{v} = 0$; two of these are in $k_0 = k'$, and the other two are in $k_2 = k'$. The latter two may be calculated using the same procedure as in equation (8), with k_2 and k_3 interchanged. This leads to

$$\operatorname*{Im}_{2} \bar{\chi}(k_0, k_1, k_2, k_3) = -\frac{\pi i}{2} \frac{e^4}{m^2} \int d^3\boldsymbol{p}\, \delta(\omega' - \boldsymbol{k}' \cdot \boldsymbol{v}) \boldsymbol{k}' \cdot \frac{\partial f}{\partial \boldsymbol{p}} \frac{(\boldsymbol{k}' \cdot \boldsymbol{k})^2}{(\omega - \boldsymbol{k} \cdot \boldsymbol{v})^4} \,. \tag{11}$$

One of the other two terms follows from equations (4) and (7) by writing $k = k_0$ and selecting the resonance at $\omega_0 - \boldsymbol{k}_0 \cdot \boldsymbol{v} = 0$ and the remaining term follows from it by interchanging k_1 and k_3. Adding these two gives [work out the derivatives and use $\delta^4(k_0 - k_1 - k_2 - k_3)$]

$$\operatorname*{Im}_{0} \bar{\chi}(k_0, k_1, k_2, k_3) = \frac{\pi i}{2} \frac{e^4}{m^2} \int d^3\boldsymbol{p}\, \delta(\omega_0 - \boldsymbol{k}_0 \cdot \boldsymbol{v}) \boldsymbol{k}_0 \cdot \frac{\partial f}{\partial \boldsymbol{p}}$$
$$\times \frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_3 \boldsymbol{k}_2 \cdot \boldsymbol{k}_3 (\omega_3 - \boldsymbol{k}_3 \cdot \boldsymbol{v})^2 + \boldsymbol{k}_1 \cdot \boldsymbol{k}_2 \boldsymbol{k}_2 \cdot \boldsymbol{k}_3 (\omega_2 - \boldsymbol{k}_2 \cdot \boldsymbol{v})^2 + \boldsymbol{k}_1 \cdot \boldsymbol{k}_2 \boldsymbol{k}_1 \cdot \boldsymbol{k}_3 (\omega_1 - \boldsymbol{k}_1 \cdot \boldsymbol{v})^2}{(\omega_1 - \boldsymbol{k}_1 \cdot \boldsymbol{v})^2 (\omega_2 - \boldsymbol{k}_2 \cdot \boldsymbol{v})^2 (\omega_3 - \boldsymbol{k}_3 \cdot \boldsymbol{v})^2} \,. \tag{12}$$

For $k_0 = k_2 = k'$ and $k_1 = -k_3 = k$, the expressions (11) and (12) are equivalent to each other. Both contributions from equations (11) and (12) are odd functions of k', and hence give a nonzero result when inserted in equation (10). Thus, $\gamma_B^{\rm NL}(\boldsymbol{k}')$ is nonzero.

Note that by making the interchanges $k \to -k_1$ and $k_1 \to -k$, equation (8) implies

$$\operatorname*{Im}_{3} \bar{\chi}(k, k_1, k_2, k_3) = \operatorname*{Im}_{3} \bar{\chi}(-k_1, -k_2, k_3) \,, \tag{13a}$$

and by making the interchanges $k \to -k_3$, $k_3 \to -k_0$ in equation (8), equations (8) and (12) imply

$$\operatorname*{Im}_{0} \bar{\chi}(k_0, k_1, k_2, k_3) = -\operatorname*{Im}_{0} \bar{\chi}(-k_3, k_1, k_2 - k_0) \,. \tag{13b}$$

The fact that there is a positive, rather than a negative, sign on the right-hand side of equation (13a) was the basis of our proof in KM that TB does not exist. Tsytovich (1985) has stated that equation (13a) is in error, but he gave no reasons and offered no criticism of the derivation of (13a) as presented in MK. The explicit derivation of (13a) here confirms the argument given in KM that TB does not exist. The opposite sign in (13b) ensures that the growth rate for the radiative correction to the emission of resonant ion sound waves by the presence of nonresonant Langmuir waves is nonzero, as pointed out in KM and as shown explicitly above.

III. THE EFFECT OF A BACKGROUND MAGNETIC FIELD

Nambu (1986) claims that the symmetries of the response tensors in KM are wrong. Above, we have proven classically that the used symmetries in KM are correct and are fully determined by consequently working out the algebra.

a) Reply to comments by Nambu (1986)

Nambu (1986) agreed that the polarization contribution to TB vanishes in the absence of a magnetic field. Now, however, he argues that KM underestimated the importance of the polarization contribution to TB, or to the "plasma maser effect" as he now calls TB, for the case of a background magnetic field. We shall show explicitly below that his nonzero result (his eq. [6]) is wrong and also that in the magnetic case his version of TB does not exist.

In a preliminary version of his comments, Nambu criticized KM on two other grounds: (1) that KM "misidentify the plasma maser effect as the radiative correction in QED," and (2) that "such a misuse of QED to classical plasma physics leads to a misleading conclusion." Here we also reply to these two other criticisms.

The error which Nambu (1986) claimed to identify in KM's argument concerns an "additional condition" which KM failed to impose. It appears that he is critical of our lack of separation between forward and backward Langmuir waves. The importance of distinguishing between forward and backward waves was emphasized by Kuijpers (1980*b*). There is the possibility of confusion in that it is assumed that positive and negative frequencies for waves in a mode M are related by $\omega_M(-\boldsymbol{k}) = -\omega_M(\boldsymbol{k})$, and it is then not clear how one distinguishes between $\boldsymbol{k} \to -\boldsymbol{k}$ implying forward to backward waves and $\boldsymbol{k} \to -\boldsymbol{k}$ implying positive to negative frequencies. Kuijpers (1980*b*) separated mode M into a mode $M+$ and a mode $M-$. A wave with a given direction $\hat{z}$ of its phase velocity has two Fourier components (at $+\boldsymbol{k}$ in the direction $\hat{z}$ and at $-\boldsymbol{k}$ in the direction $-\hat{z}$). The given wave corresponds to the one of the two solutions of j (dependent upon whether j is positive or negative) that has $\omega_j(\boldsymbol{k} = |\boldsymbol{k}|\hat{z}) > 0$. The relation $\omega_{M\pm}(-\boldsymbol{k}) = -\omega_{M\pm}(\boldsymbol{k})$ then applies separately to these two modes $M\pm$, and the link between both modes is, in the case of reflection symmetry of the particle distributions with respect to the origin in momentum space, given by $\omega_{M+}(\boldsymbol{k}) = -\omega_{M-}(\boldsymbol{k})$. Now, if one identifies A in equation (1) as either $M+$ or $M-$, then the argument in § II carries through without modification. That is, the growth rates both for "forward" and for "backward" waves are identically zero independently of each other. There seems to be no justification for criticizing the arguments of KM on this point.

The argument relating TB to a radiative correction can be posed without reference to QED. The emission and absorption of ion sound waves by particles is modified by the presence of the Langmuir waves. The nonlinear correction (to the linear absorption coefficient) for the absorption may be treated using equation (1), and, as we have shown, it is certainly nonzero. The following points link this to the discussion of TB:

1. The matrix element, e.g., equations (11) and (8), is the same for this nonlinear correction and for TB because the same waves and the same resonance are involved in each.
2. Unlike TB, this nonlinear absorption process has a clearly identified emission counterpart. One may treat the emission directly and derive the matrix element independently. A detailed calculation shows that the nonlinear correction to the emission is proportional to $-(\boldsymbol{k} \cdot \boldsymbol{k}')^2 \delta(\omega' - \boldsymbol{k}' \cdot \boldsymbol{v})/(\omega - \boldsymbol{k} \cdot \boldsymbol{v})^4$, consistent with equation (8).
3. The absorption coefficient may be derived by appealing to detailed balancing, and the nonlinear correction so obtained reproduces that derived from equation (1) using equation (11). This supports the argument that equation (8) is correct.

Nambu's other argument against the use of QED to treat "classical" physics is unacceptable: QED is the modern theory of electrodynamics and it must be possible to treat all electrodynamic processes, including so-called classical plasma physics, using QED. Moreover, the arguments in KM do not rely in any essential way on QED.

b) The Magnetic Case

We now consider if, as claimed by Nambu (1986 and private communication), the inclusion of a background magnetic field alters the argument in KM, allowing TB to exist. This is not the case. The central argument in KM and here is that the growth rate for TB is identically zero due to an exact cancellation implied by symmetry properties of nonlinear response tensors. These symmetry properties are retained when electromagnetic effects and the effects of an ambient magnetic field are included (see Melrose 1987 for a detailed calculation of the cubic response tensor for a magnetic plasma).

In the appendix we derive the explicit expression for the "polarization" term in the effective dielectric constant ϵ_p in Nambu (1986, eq. [2]). We show that the complete expression for the imaginary part of ϵ_p for ion sound resonance in the magnetized case for ion sound waves parallel to the magnetic field and oblique Langmuir waves reduces to zero; consequently, the growth rate for turbulent bremsstrahlung due to the polarization term in the magnetized case (eq. [6] in Nambu 1986) also vanishes.

IV. DISCUSSION AND CONCLUSIONS

The controversy of interest here concerns the claim by KM that the growth rate for TB must be identically zero, contrary to the nonzero results obtained by TSW and Nambu. In § II we have identified the specific step where the difference between the results of TSW and KM arises. In brief, the nonzero result of TSW comes from an incomplete summation of the nonlinear responses. In the Appendix we show explicitly that a correct evaluation of Nambu's "polarization term" also leads to a vanishing result.

The physical significance of the nonexistence of TB and physical arguments against the existence of TB were given in KM and will not be repeated here. However, it is relevant to pose two questions that have trivial answers if the growth rate for TB is identically zero, but that need to be answered by those who maintain that TB exists:

1. No emission process corresponding to TB (which is defined as a nonlinear absorption process) has been identified. Is there a nonlinear emission process corresponding to this supposed nonlinear absorption process? If not, does not an absorption process without a corresponding emission process violate the second law of thermodynamics?

2. From where does the energy in the Langmuir waves originate in TB? Presumably the sum of the energy densities W_s, $W_{\rm L}$, and W_p in the ion sound waves, the Langmuir waves, and the particles, respectively, is conserved. Arguments given in KM suggest that $W_{\rm L}$ cannot increase primarily at the expense of W_s: wave action (e.g., the number of wave quanta) is conserved and hence the ion sound waves can provide only a fraction $\omega_s/\omega_{\rm L}$ of the final energy in the Langmuir waves. On the other hand, a semiclassical interpretation of the resonance condition $\omega' - \boldsymbol{k}' \cdot \boldsymbol{v} = 0$ (see KM) implies that W_p changes at essentially the same rate as W_s.

The physical picture of TB seems to be the following: the resonant electrons are accelerated systematically in the wave field of the ion sound waves, and they emit bremsstrahlung as a result of this acceleration. We argued in KM that this physical picture is inconsistent with TB being a second-order process in electrodynamics. Upconversion of ion sound waves into Langmuir waves is possible as a scattering process at second order (Goldman and DuBois 1982; Melrose 1982), but we maintain that there is no other second-order process that allows such upconversion. The claims by TSW and Nambu (1981) to the contrary are based on erroneous analyses.

APPENDIX

THE POLARIZATION TERM WITH A BACKGROUND MAGNETIC FIELD

We first give the derivation of K_p. The dielectric function is related to the dielectric tensor $\mathbf{K}$ and to the electric current density of longitudinal disturbances $[\boldsymbol{E}(k) \parallel \boldsymbol{k}$, with $k \equiv (\boldsymbol{k}, \omega)]$ in the following way:

$$K(k) = \frac{\boldsymbol{k} \cdot \mathbf{K} \cdot \boldsymbol{k}}{|\boldsymbol{k}|^2} = 1 - \frac{1}{i\omega\epsilon_0} \frac{\boldsymbol{E}^*(k) \cdot \boldsymbol{j}(k)}{\boldsymbol{E}^*(k) \cdot \boldsymbol{E}(k)} . \tag{A1}$$

The zero-order part $K_0(k)$ (in $\boldsymbol{E}$) is obtained from equation (A1) by substituting (in the electric field transformation) the linear part of the induced current density,

$$j_i(k) = \sigma_{ij}(k) E_j(k) .$$

The contribution to the effective dielectric function from nonlinear currents is given by

$$K_{\rm NL}(k) = -\frac{1}{i\omega\epsilon_0} \frac{\boldsymbol{E}^*(k) \cdot \boldsymbol{j}^{\rm NL}(k)}{\boldsymbol{E}^*(k) \cdot \boldsymbol{E}(k)} . \tag{A2}$$

In particular, it follows from equation (A2) that radiation is emitted only if Im $K_{\rm NL}(k) \neq 0$. It also follows that turbulent bremsstrahlung of the kind of Nambu (1986) exists if

$$\text{Im } \tfrac{1}{2}\omega^{\rm L+}(\boldsymbol{k})\{K_p^{\rm L+}[\boldsymbol{k}, \omega^{\rm L+}(\boldsymbol{k})] - K_p^{\rm L+}[-\boldsymbol{k}, -\omega^{\rm L+}(\boldsymbol{k})]\} \neq 0 , \tag{A3}$$

and a similar expression exists for L$-$. Here, the superscript L$+$ denotes one of the solutions for Langmuir waves $\{\omega^{\rm L+}(-\boldsymbol{k}) = -\omega^{\rm L+}(\boldsymbol{k}), \boldsymbol{E}^{\rm L+}(-\boldsymbol{k}) = [\boldsymbol{E}^{\rm L+}(\boldsymbol{k})]^*\}$ and L$-$ the other solution $[\omega^{\rm L-}(-\boldsymbol{k}) = -\omega^{\rm L-}(\boldsymbol{k})$, etc.]. For particle distributions with reflection symmetry with respect to the origin in velocity space $\omega^-(\boldsymbol{k}) = -\omega^+(\boldsymbol{k})$.

The appropriate nonlinear current in equation (A3) is obtained from the expression (MK and KM)

$$j_i(k) = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} (2\pi)^4 \delta^4(k - k_2 - k_2) \cdot \sigma_{ijl}(k, k_1, k_2) E_j(k_1) E_l(k_2) \tag{A4}$$

by identifying $k \equiv [\boldsymbol{k}^{\rm L}, \omega^{\rm L+}(\boldsymbol{k}^{\rm L})]$ and picking out the contribution to E_j from sound waves at k^s and that to E_l from the beat at $k^{\rm L} - k^s$. A second contribution is obtained by interchanging the role of E_j and E_l. The beat at $k^{\rm L} - k^s$ is caused by an "external" current density and for longitudinal disturbances can be written as

$$E_l(k^{\rm L} - k^s) = -\frac{i j_l(k^{\rm L} - k^s)}{\epsilon_0[\omega^{\rm L+}(\boldsymbol{k}^{\rm L}) - \omega^s(\boldsymbol{k}^s)] K_0(k^{\rm L} - k^s)} . \tag{A5}$$

The external current density again is of the form of equation (A4) and is determined by the electric field of the Langmuir wave at $\boldsymbol{k}^{\rm L}$ and of the sound wave at $-\boldsymbol{k}^s$.

We now carry out the above procedure and substitute for every field component

$$E_r(k) = E_r^{\rm L+}(k) + E_r^{\rm L-}(k) + E_r^{s+}(k) + E_r^{s-}(k) , \tag{A6}$$

with

$$E_i^{\lambda+}(k) = e_i^{\lambda+}(\boldsymbol{k}) E^{\lambda+}(\boldsymbol{k}) 2\pi\delta[\omega - \omega^{\lambda+}(\boldsymbol{k})] ; \tag{A7}$$

we use the random phase approximation

$$\langle E_i(k) E_j(k') \rangle = \lim_{\tau V \to \infty} \frac{(2\pi)^4}{\tau V} E_i(k) E_j^*(k) \delta^3(\boldsymbol{k} + \boldsymbol{k}') \delta(\omega + \omega') , \tag{A8}$$

and, as explained above, we take only those terms which lead to a contribution to $\boldsymbol{j}^{\rm L+}(\boldsymbol{k}^{\rm L+})$.

Changing dummy indices we then finally find for the "polarization" current of Nambu

$$j_i^{L+}[\boldsymbol{k}^L, \omega^{L+}(\boldsymbol{k}^L)] = \lim_{V\to\infty} \sum_{s=s\pm} -i \int \frac{d^3\boldsymbol{k}^s}{(2\pi)^3 V} \cdot \bar{\sigma}_{ijl}(k^L, k^s, k^L - k^s)\bar{\sigma}_{lrt}(k^L - k^s, -k^s, k^L) \cdot \frac{E^s(\boldsymbol{k}^s)E^s(-\boldsymbol{k}^s)E^{L+}(\boldsymbol{k}^L)e_j^s(\boldsymbol{k}^s)e_r^s(-\boldsymbol{k}^s)e_t^{L+}(\boldsymbol{k}^L)}{[\omega^{L+}(\boldsymbol{k}^L) - \omega^s(\boldsymbol{k}^s)]K_0[\boldsymbol{k}^L - \boldsymbol{k}^s, \omega^{L+}(\boldsymbol{k}^L) - \omega^s(\boldsymbol{k}^s)]}, \quad \text{(A9)}$$

where

$$\bar{\sigma}_{ijl}(k, k_1, k_2) \equiv \sigma_{ijl}(k, k_1, k_2) + \sigma_{ilj}(k, k_2, k_1), \qquad k^L \equiv [\boldsymbol{k}^L, \omega^{L+}(\boldsymbol{k}^L)], \quad \text{and} \quad k^s \equiv [\boldsymbol{k}^s, \omega^s(\boldsymbol{k}^s)] . \quad \text{(A10)}$$

Now upon substitution of equation (A9) into equation (A2) we find

$$K_p^{L+}[\boldsymbol{k}, \omega^{L+}(\boldsymbol{k})] = \lim_{V\to\infty} \sum_{s=s\pm} \frac{1}{(4\pi\epsilon_0)^2} \int \frac{d^3\boldsymbol{k}^s}{2\pi V} |E^s(\boldsymbol{k}^s)|^2 \cdot \frac{e_i^{L+}(-\boldsymbol{k}^L)e_j^s(\boldsymbol{k}^s)\bar{\sigma}_{ijl}(k^L, k^s, k^L - k^s)\bar{\sigma}_{lrt}(k^L - k^s, -k^s, k^L)e_r^s(-\boldsymbol{k}^s)e_t^{L+}(\boldsymbol{k}^L)}{\omega^{L+}(\boldsymbol{k}^L)K_0(k^L - k^s)[\omega^{L+}(\boldsymbol{k}^L) - \omega^s(\boldsymbol{k}^s)]} . \quad \text{(A11)}$$

We stress that the symmetry properties are fully determined by equations (A11) and (A3) and that no additional freedom exists to impose or leave away certain symmetries.

Below we shall calculate $\bar{\sigma}_{ijl}$ and show that the left-hand side of equation (A3) vanishes. To calculate σ_{ijl} in equation (A4), we solve the Vlasov equation for the magnetic case with a perturbation expansion for the distribution function of particles of kind j:

$$f_j(\boldsymbol{x}, \boldsymbol{v}, t) = \sum_{m=0}^{\infty} f_j^{(m)}(\boldsymbol{x}, \boldsymbol{v}, t) , \quad \text{(A12)}$$

where $f^{(m)}$ is of mth order in the wave field amplitudes. The Vlasov equation,

$$\left[\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \frac{\partial}{\partial \boldsymbol{x}} + \frac{e_j}{m_j} \boldsymbol{v} \times (B_0 \hat{z}) \cdot \frac{\partial}{\partial \boldsymbol{v}}\right] f_j^{(m)}(x, \boldsymbol{v}, \boldsymbol{t}) = -\frac{e_j}{m_j}(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot \frac{\partial}{\partial \boldsymbol{v}} f_j^{(m-1)}(\boldsymbol{x}, \boldsymbol{v}, t) , \quad \text{(A13)}$$

is solved by integration along the unperturbed orbits to give

$$f_j^{(m)}(k, \boldsymbol{v}) = \int d\lambda^{(2)} \left(-\frac{e_j}{m_j}\right) \int_{-\infty}^{t} dt' \exp[i\boldsymbol{k} \cdot (\boldsymbol{x}' - \boldsymbol{x}) - i\omega(t' - t)] \cdot [\boldsymbol{E}(k_1) + \boldsymbol{v}' \times \boldsymbol{B}(k_1)] \cdot \frac{\partial}{\partial \boldsymbol{v}'} f_j^{(m-1)}(k_2, \boldsymbol{v}') , \quad \text{(A14)}$$

where, as before, $k \equiv (\boldsymbol{k}, \omega)$,

$$d\lambda^{(2)} \equiv \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} (2\pi)^4 \delta^4(k - k_1 - k_2) ,$$

and $\boldsymbol{x}'(t')$ and $\boldsymbol{v}'(t')$ are, respectively, the unperturbed particle orbit and velocity at time t'. Proceeding in a way entirely analogous to that of Davidson (1983), and assuming $\boldsymbol{k}$, $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ to lie in the $x - z$ plane (as is done by Nambu),

$$\boldsymbol{k} = k_\perp \hat{x} + k_z \hat{z} ,$$

eventually one finds

$$\begin{aligned} \boldsymbol{j}^{(2)}(k) = \sum_j -\frac{e_j^3}{m_j^2} \int d\lambda^{(2)} \sum_{n,p,q=-\infty}^{+\infty} \int d^3\boldsymbol{v} \left\{\frac{[(n - p + q)v_\perp/z]J_{n-p+q}(z),\ -iv_\perp J'_{n-p+q}(z),\ v_z J_{n-p+q}(z)}{\omega - n\omega_{cj} - k_z v_z}\right\} \\ \cdot \left\{E_x(k_1)\frac{nJ_n(z)}{z}\left[\left(1 - \frac{k_{1z}v_z}{\omega_1}\right)\frac{\partial}{\partial v_\perp} + \frac{k_{1z}v_\perp}{\omega_1}\frac{\partial}{\partial v_z}\right] + E_y(k_1)\frac{J'_n(z)}{i}\left[\left(1 - \frac{k_{1z}v_z}{\omega_1}\right)\frac{\partial}{\partial v_\perp} + \frac{k_{1z}v_\perp}{\omega_1}\frac{\partial}{\partial v_z}\right]\right. \\ \left. + E_z(k_1)J_n(z)\left[\frac{n\omega_{cj}k_{1\perp}v_z}{\omega_1 k_\perp v_\perp}\frac{\partial}{\partial v_\perp} + \left(1 - \frac{n\omega_{cj}k_{1\perp}}{\omega_1 k_\perp}\right)\frac{\partial}{\partial v_z}\right]\right\} \cdot \frac{J_p(z_2)}{\omega_2 - q\omega_{cj} - k_{2z}v_z} \\ \cdot \left\{E_x(k_2)\frac{qJ_q(z_2)}{z_2}\left[\left(1 - \frac{k_{2z}v_z}{\omega_2}\right)\frac{\partial}{\partial v_\perp} + \frac{k_{2z}v_\perp}{\omega_2}\frac{\partial}{\partial v_z}\right] + E_y(k_2)\frac{J'_q(z_2)}{i}\left[\left(1 - \frac{k_{2z}v_z}{\omega_2}\right)\frac{\partial}{\partial v_\perp} + \frac{k_{2z}v_\perp}{\omega_2}\frac{\partial}{\partial v_z}\right]\right. \\ \left. + E_z(k_2)J_q(z_2)\left[\frac{q\omega_{cj}v_z}{\omega_2 v_\perp}\frac{\partial}{\partial v_\perp} + \left(1 - \frac{q\omega_{cj}}{\omega_2}\right)\frac{\partial}{\partial v_z}\right]\right\} f_j^0(\boldsymbol{v}) , \end{aligned} \quad \text{(A15)}$$

where $\omega_{cj} \equiv e_j B_0/m_j$ is the electron cyclotron frequency for particles of kind j, $f_j^0(\boldsymbol{v})$ is the zero-order distribution function of kind j, and the $J_m(z)$ are Bessel functions of integer order, we use the notation $J'_m(z) \equiv dJ_m(z)/dz$, and the argument $z \equiv k_\perp v_\perp/\omega_{cj}$. Comparison of equation (A15) with equation (A4) now directly gives σ_{ijl}. Further simplification arises from the assumptions (as in Nambu)

$$\boldsymbol{k}^s = k^s \hat{z} \quad \text{and} \quad \boldsymbol{k}^L = k_\perp^L \hat{x} + k_z^L \hat{z} \quad \text{(A16)}$$

and the longitudinal wave character. For longitudinal waves we can choose the polarization vector,

$$\boldsymbol{e}(\boldsymbol{k}) = i\frac{k_z}{|\boldsymbol{k}|}\hat{z} + i\frac{k_\perp}{|\boldsymbol{k}|}\hat{x} , \quad \text{(A17)}$$

so that $e^*(k) = e(-k)$. One then finally finds

$$K_p^{L+}[k, \omega^{L+}(k)] = \lim_{V\to\infty} \sum_{s=s\pm} \frac{1}{(4\pi\epsilon_0)^2} \int d^3k^s \frac{|E^s(k^s)|^2}{2\pi V} \frac{1}{K_0(k^L - k^s)}$$
$$\cdot \Bigg[\frac{e^3}{m^2} \sum_{n,q} \int \frac{d^3v}{\omega^L |k^L|} \frac{J_{n+q}(2z^L)J_n(z^L)J_q(z^L)}{\omega^L - n\omega_{ce} - k_z^L v_z}$$
$$\cdot \Bigg\{[k_z^L v_z + (n+q)\omega_{ce}] \frac{\partial}{\partial v_z} \frac{1}{\omega^L - \omega^s - q\omega_{ce} - (k_z^L - k_z^s)v_z} \left[\frac{q\omega_{ce}}{\omega^L - \omega^s} \frac{v_z}{v_\perp} \frac{\partial}{\partial v_\perp} + \left(1 - \frac{q\omega_{ce}}{\omega^L - \omega^s}\right) \frac{\partial}{\partial v_z}\right]$$
$$\times f^0(v) + (k_z^L v_z + n\omega_{ce})\left[\frac{n\omega_{ce}}{\omega^L - \omega^s} \frac{v_z}{v_\perp} \frac{\partial}{\partial v_\perp} + \left(1 - \frac{n\omega_{ce}}{\omega^L - \omega^s}\right) \frac{\partial}{\partial v_z}\right] \frac{1}{\omega^s - k_z^s v_z} \frac{\partial}{\partial v_z} f^0(v)\Bigg\}\Bigg]$$
$$\cdot \Bigg(\frac{e^3}{m^2} \sum_{n,q} \int \frac{d^3v}{(\omega^L - \omega^s)|k^L|} \frac{J_{n+q}(2z^L)J_n(z^L)J_q(z^L)}{\omega^L - \omega^s - n\omega_{ce} - (k_z^L - k_z^s)v_z} v_z \cdot \Bigg[\frac{\partial}{\partial v_z} \frac{1}{\omega^L - q\omega_{ce} - k_z^L v_z}$$
$$\times \left\{k_z^L \left[\frac{q\omega_{ce}}{\omega^L} \frac{v_z}{v_\perp} \frac{\partial}{\partial v_\perp} + \left(1 - \frac{q\omega_{ce}}{\omega^L}\right) \frac{\partial}{\partial v_z}\right] + q\omega_{ce}\left[\left(1 - \frac{k_z^L v_z}{\omega^L}\right) \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} + \frac{k_z^L}{\omega^L} \frac{\partial}{\partial v_z}\right]\right\} f^0(v)$$
$$+ \left\{k_z^L \left[\frac{n\omega_{ce}}{\omega^L} \frac{v_z}{v_\perp} \frac{\partial}{\partial v_\perp} + \left(1 - \frac{n\omega_{ce}}{\omega^L}\right) \frac{\partial}{\partial v_z}\right] + n\omega_{ce}\left[\left(1 - \frac{k_z^L v_z}{\omega^L}\right) \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} + \frac{k_z^L}{\omega^L} \frac{\partial}{\partial v_z}\right]\right\}$$
$$\cdot \frac{1}{-\omega^s + k_z^s v_z} \frac{\partial}{\partial v_z} f^0(v)\Bigg]\Bigg) + \langle l = x \rangle + \langle l = y \rangle \equiv \lim_{V\to\infty} \sum_{s=s\pm} \int d^3k^s \frac{|E^s(k^s)|^2}{2\pi V K_0(k^L - k^s)} AB$$
$$+ \langle l = x \rangle + \langle l = y \rangle , \tag{A18}$$

where by definition $\omega^L \equiv \omega^{L+}(k^L)$ and $\omega^s = \omega^s(k^s)$ and we have made use of the addition theorems for Bessel functions

$$J_n(z) = \sum_{q=-\infty}^{+\infty} J_{n+q}(2z)J_q(z) \quad \text{and} \quad \sum_{p=-\infty}^{+\infty} J_{n-p+q}(z)J_p(z) = J_{n+q}(2z) .$$

The terms in angular brackets correspond to contributions in equation (A11) with $l = x$ and $l = y$, respectively, while the explicit first term is for the contribution $l = z$. The first factor in curly brackets is (cf. eq. [A11])

$$A \equiv \frac{e_i^L e_j^s \bar{\sigma}_{ijz}}{4\pi\epsilon_0 \omega^L} \tag{A19}$$

and the second factor in curly brackets is

$$B \equiv \frac{\bar{\sigma}_{zrt} e_r^s e_t^L}{4\pi\epsilon_0(\omega^L - \omega^s)} . \tag{A20}$$

As for the imaginary parts of A and B from the resonance $\omega^s - k^s v_z = 0$, upon partial integration of both expressions one ends up with only first-order derivatives, and one obtains generally, without further assumptions

$$\text{Im } A = \text{Im } B . \tag{A21}$$

Now, for the real parts of A and B, if one considers only the contributions for $n = 0 = q$ (as is done by Nambu for the "string field case") one obtains

$$\text{Re } A = -\text{Re } B . \tag{A22}$$

Together with equation (A21) this results in Im $K_p = 0$, a zero left-hand side of equation (A3), and the nonexistence of his kind of turbulent bremsstrahlung. This is in contrast with his result (his eq. [6]) and shows that it is not legitimate to take only the contributions from J_0, as is also clear from the following "alternative" expression for unity:

$$1 = \left[\sum_{n=-\infty}^{+\infty} J_n(z)\right]^m ,$$

which is in general not well approximated by the contributions from $n = 0$.

However not only is equation (A22) true for the leading order in the Bessel functions, but also for the general expressions given by equation (A18). This is shown in the following. From Melrose (1972) it follows that

$$-\bar{\sigma}_{ijl}(k, k_1, k_2)\omega_1\omega_2 = \kappa_{ijl}(k, k_1, k_2) , \tag{A23}$$

where κ_{ijl} is the general expression for the relevant nonlinear response tensor in a magnetoplasma. In Melrose (1972) it is shown that for the nonresonant part, the following relation holds (eq. [32] of Melrose 1972):

$$N\kappa_{ijl}(k, k_1, k_2) = N\kappa_{lji}(-k_2, k_1, -k) . \tag{A24}$$

Also, from the reality condition, one has

$$N\kappa_{ijl}(k, k_1, k_2) = N\kappa_{ijl}(-k, -k_1, -k_2) . \tag{A25}$$

Since the real parts of A and B correspond to the nonresonant parts of κ (see eqs. [A19] and [A20]), one finds with equations (A24) and (A25) that

$$\begin{aligned} \text{Re } A &= -\frac{e_i^{\text{L}}(-\boldsymbol{k}^{\text{L}})e_j^s(\boldsymbol{k}^s)N\kappa_{ijl}(k^{\text{L}}, k^s, k^{\text{L}} - k^s)}{\omega^{\text{L}}(\boldsymbol{k}^{\text{L}})\omega^s(\boldsymbol{k}^s)[\omega^{\text{L}}(\boldsymbol{k}^{\text{L}}) - \omega^s(\boldsymbol{k}^s)]} \\ &= -\frac{e_t^{\text{L}}(-\boldsymbol{k}^{\text{L}})e_r^s(\boldsymbol{k}^s)N\kappa_{lrt}(-k^{\text{L}} + k^s, k^s, -k^{\text{L}})}{\omega^{\text{L}}(\boldsymbol{k}^{\text{L}})\omega^s(\boldsymbol{k}^s)[\omega^{\text{L}}(\boldsymbol{k}^{\text{L}}) - \omega^s(\boldsymbol{k}^s)]} \\ &= \frac{e_t^{\text{L}}(\boldsymbol{k}^{\text{L}})e_r^s(-\boldsymbol{k}^s)N\kappa_{lrt}(k^{\text{L}} - k^s, -k^s, k^{\text{L}})}{[\omega^{\text{L}}(\boldsymbol{k}^{\text{L}}) - \omega^s(\boldsymbol{k}^s)]\omega^s(-\boldsymbol{k}^s)\omega^{\text{L}}(\boldsymbol{k}^{\text{L}})} \\ &= -\text{Re } B . \end{aligned} \tag{A26}$$

Therefore, the result of equation (A22) is exact, and one finally has for equation (A11), Im $K_p^{\text{L}+}[\boldsymbol{k}, \omega^{\text{L}+}(\boldsymbol{k})] \propto$ Im A(Re A + Re B) = 0, and, *a fortiori*, also the left-hand side of equation (A3) vanishes.

We conclude that the turbulent bremsstrahlung of the kind Nambu (1986) describes does not exist in the general case of a magnetoplasma.

REFERENCES

Davidson, R. C. 1972, *Methods in Nonlinear Plasma Theory* (New York: Academic).
———. 1983, in *Handbook of Plasma Physics I*, ed. A. A. Galeev and R. N. Sudan (Amsterdam: North Holland), p. 523.
DuBois, D. F., and Pesme, D. 1984, *Phys. Fluids*, **19**, 1746.
Goldman, M. V., and DuBois, D. F. 1982, *Phys. Fluids*, **25**, 1062.
Hirose, A. 1984, *Comments Plasma Phys. Controlled Fusion*, **8**, 117.
Kuijpers, J. 1980*a*, *Ap. J.* (*Letters*), **238**, L165.
———. 1980*b*, *Astr. Ap.*, **83**, 201.
Kuijpers, J., and Melrose, D. B. 1985, *Ap. J.*, **294**, 28.
Melrose, D. B. 1972, *Plasma Phys.*, **14**, 1035.
———. 1982, *Australian J. Phys.*, **35**, 67.
Melrose, D. B. 1987, *Australian J. Phys.*, **40**, 139.
Melrose, D. B., and Kuijpers, J. 1984, *J. Plasma Phys.*, **32**, 239.
Nambu, M. 1981, *Phys. Rev. A*, **23**, 3272.
———. 1986, *Ap. J.*, **303**, 347.
Tsytovich, V. N. 1980, *Soviet, J. Plasma Phys.*, **6**, 608.
———. 1982, *Phys. Scripta*, **T2**, 54.
———. 1985, *Soviet Phys.—JETP*, **62**, 483.
Tsytovich, V. N., Stenflo, L., and Wilhelmsson, H. 1975, *Phys. Scripta*, **11**, 251.
———. 1981, *Ap. J.* (*Letters*), **246**, L89.
Vlahos, L., and Papadopoulos, K. 1979, *Ap. J.* (*Letters*), **234**, L217.
———. 1982, *Ap. J.* (*Letters*), **252**, L75.

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