THE RESISTANCES OF THE PHOTOSPHERE AND OF A FLARING CORONAL LOOP

D. B. MELROSE* and A. N. McCLYMONT

Institute for Astronomy, University of Hawaii, Honolulu, HI 96822, U.S.A.

ABSTRACT. We consider two aspects of solar flares from the point of view of circuit theory. First, we show that the so-called "dynamo models", which invoke an analogy between the Earth's magnetosphere-ionosphere circuit and the solar corona-photosphere circuit, are ill-founded. Second, we consider the rate of coronal energy release in the impulsive phase of a modest flare, and show that, if the energy going into mass motion can be neglected, the corona must present a resistance of about $10^{-3}\ \Omega$. Classical resistivity, even in a highly filamented circuit, cannot provide so high a resistance. Anomalous resistivity due to ion sound turbulence *can* provide the required resistance in this case, but is insufficient to explain the very high power levels inferred in some fast spikes.

1. INTRODUCTION

In circuit models for solar flares (e.g., Alfven, 1977; Spicer, 1977, 1982) the complex three-dimensional structure of a current-carrying magnetic loop is stripped to its bare essentials, and is described in terms of a one-dimensional system involving a current I, an inductance $L \approx \mu_0 l$ (where l is the length of the loop), and series resistances R in the corona and R_p in the photosphere. The stored magnetic energy is then $\frac{1}{2}LI^2$ and the power released in the flare is RI^2. The circuit approach, which provides a complementary viewpoint to the more conventional one based on the magnetic structure, has two main advantages. First, it emphasizes the role of currents, the fact that the currents must close, and the conditions necessary for their generation. Current closure is often obscured in the magnetic field approach, *e.g.*, by implicit closure involving surface currents on artificially introduced rigid conducting boundaries. The other main advantage of the circuit approach is that it facilitates discussion of global constraints. One example is well known: the current I is limited by the requirement that the self-field not exceed the observed magnetic field, which we take to be 0.1 T in a magnetic knot of area 10^{13} m^2 at the photosphere. In cylindrical geometry this requires $l \leq 2\pi RB/\mu_0$ with $\pi R^2 = 10^{13}$ m^2, giving a maximum current of a few times 10^{11} A, similar to that estimated observationally (Moreton and Severny, 1968). Another relevant example is the requirement that the power dissipated increase suddenly in a flare. The prevailing view (Sturrock, 1980) is that magnetic energy is stored over a time much longer than the energy release time in a flare (10^2 to 10^3 s), so that the triggering of a flare then requires a rapid increase in the coronal resistance R. Although we shall assume in §§ 3 and 4 that this is an actual resistance resulting in Joule dissipation, in general the coronal "resistance" could be merely an impedance describing the conversion of magnetic energy into another form such as kinetic energy.

"Dynamo" models (Heyvaerts, 1974; Kan, Akasofu and Lee, 1983; Henoux, 1986) offer an alternative to the "prevailing view" that avoids the need for energy storage in the corona. In a dynamo model, the current which is dissipated to heat the flare is generated synchronously

* Permanent address: School of Physics, University of Sydney, Sydney, Australia.

Solar Physics **113** (1987) 241–248.

beneath the photosphere, so that no coronal storage of the energy is required. Thus the flare is "driven" directly by events at or below the photospheric level, where mechanical stresses distort the magnetic field, generating currents which are immediately dissipated by coronal resistance.

Here we consider a *specific class* of dynamo models: those based on a supposed analogy between solar flares and magnetospheric substorms (*e.g.* Kan, Akasofu and Lee, 1983). In these models, the cross-field resistivity of weakly ionized plasma, (which is important in the ionosphere) plays a central role, as it is postulated that the current is generated as a result of stresses when weakly ionized plasma is driven across the field lines, as in an MHD generator (*e.g.*, Mitchner and Kruger, 1973). Unfortunately the resistivity of the solar photosphere is in fact entirely negligible.

In this paper we discuss the photospheric and coronal resistances and the implications of their actual values. Three forms of resistivity are relevant: (i) the classical or Spitzer resistivity due to electron–ion collisions ($\eta = 1.3\times10^3 T^{-3/2}$ Ω m with T in Kelvin), (ii) the cross-field or Pedersen resistivity of a weakly ionized plasma, and (iii) the anomalous resistivity $\eta_{an} = \xi\mu_0 c^2/\omega_p$, due to an effective frequency $\nu_{eff} = \xi\omega_p$ of electron collisions with the plasma turbulence, where $\xi \approx 10^{-2}$ is an "efficiency factor". The effectiveness of dissipation described by a resistance R may be estimated by comparing R to the impedance $R_A = \mu_0 v_A$ associated with the generation of Alfven waves.

In § 2 we compare the solar photosphere with the terrestrial ionosphere, and point out the differences between the two which make ideas derived from magnetospheric substorms wholly inapplicable to solar flares. In § 3 we discuss the coronal flare resistance. To dissipate Ohmically the power released in the impulsive phase of a modest flare, $RI^2 \approx 10^{20}$ W (assuming that little energy goes into mass motion) with a current of 3×10^{11} A, a resistance of $\sim10^{-3}$ Ω is required. However, a simple estimate of coronal resistance based on classical Spitzer resistivity and a constant current profile yields only $\sim10^{-12}$ Ω. The resistance may be raised somewhat by filamenting the current into narrow channels, but before the required resistance is achieved, the current density becomes high enough to trigger plasma instabilities, invalidating classical resistivity. Anomalous resistivity (§ 4) due to ion sound turbulence (cf. Kuperus, 1976) in a highly filamented current system *can* provide the resistance implied in a modest flare, but this mechanism appears to be unable to account for very high power levels inferred in "kernels" and "spikes". In § 5, we summarize our conclusions.

2. COMPARISON OF THE PHOTOSPHERE AND THE IONOSPHERE

Many so-called "dynamo" models of solar flares, adapted from theories of magnetospheric substorms, have been put forward on the basis of supposed similarities between the solar photosphere and the ionosphere of Earth (*e.g.*, Kan, Akasofu and Lee, 1983). The ionosphere, which is the only part of the magnetosphere-ionosphere system "heavy" enough to support **J**×**B** stresses generated by the solar wind, and thus anchor magnetic field lines, is in fact highly resistive, so field lines can slip through it. Dynamo models have attributed this same resistive property to the solar photosphere, as if it were essential to the generation of flare power (*e.g.*, Heyvaerts, 1974; Spicer, 1982; Kan, Akasofu and Lee, 1983; Henoux, 1986).

In analogy with the ionosphere, dynamo models treat the photosphere as a thin, inertial, resistive lower boundary to the highly conducting, tenuous corona. The resistance of the photosphere (or the ionosphere) is then characterized by the height-integrated Pedersen conductivity, the inverse of which is denoted R_p here. This resistance is only significant if it is as large as, or larger than, other impedances in the circuit, of which the most important are the Alfven impedance $R_A = \mu_0 v_A$, which is of order 10^{-2}–10^{-1} Ω (see below) and the coronal resistance of 10^{-3} Ω required in a flare. Obayashi (1975) estimated the height-integrated conductivity of the photosphere, and found $R_p \approx 10^{-7}\ \Omega$. However, the weakly ionized region extends much deeper than Obayashi assumed, and a calculation for a flux tube extending down from a photospheric knot gives $R_p \approx 10^{-11}\ \Omega$ (Khan, 1987). This suggests immediately that the photosphere is unlike the ionosphere, which constitutes the main source of resistance in the ionosphere-magnetosphere path.

There are even more important differences between the ionosphere-magnetosphere and photosphere-corona systems. On the one hand, in the ionosphere-magnetosphere system

(1) the atmosphere at the base of the ionosphere is an *insulator* which completely decouples the magnetosphere from the Earth's interior, in the sense that no current or magnetic stress can be transmitted through it;

(2) the ionospheric layer is *thin*, as the Alfven propagation time across it is shorter than other relevant time scales;

(3) the stresses that generate currents are applied at the magnetosphere-solar wind interface, and the forces in this tenuous region are small compared to the inertia of the ionosphere, so that the time to accelerate the ionospheric plasma is long.

Under these circumstances, it is appropriate to regard the ionosphere as a thin, dense layer, and to describe its resistance in terms of the height-integrated conductivity.

On the other hand, in the photosphere-corona system

(1) there is no insulating layer to isolate the current system from underlying regions, and currents can transmit stresses *through* the photosphere;

(2) thus the photosphere–convection zone has no lower limit; furthermore it cannot be regarded as thin since the Alfven propagation time scale is not short;

(3) since, in the stronger field regions, Alfven waves quickly propagate through the photosphere, it is the deeper layers of the Sun which must support **J**×**B** stresses; furthermore these stresses must be produced at the same level which provides the reaction, so the causal relation between mechanical stress and electric current is not clear-cut.

Thus the photosphere *cannot* be regarded as a thin layer, and its height-integrated conductivity is not a relevant parameter. But it is still possible that the resistance of the relatively thin current channel of an Alfven wavefront propagating through the photosphere could be an important source of dissipation. The resistance across the wavefront, $R_w \approx (\sigma_p \Delta h)^{-1}$, causes dissipation and slippage of field lines when it becomes comparable with the Alfven wave impedance $R_A \equiv \mu_0 \upsilon_A$. Since a wavefront thickness Δh is produced by an impulsive stress acting for a time $\Delta t \approx \Delta h / \upsilon_A$, motions generated on time scales $\Delta t \lesssim (\mu_0 \sigma_p \upsilon_A^2)^{-1}$ are damped. In the ionosphere, even wavefronts as thick as the ionosphere itself are damped, demonstrating that the ionosphere is highly resistive and diffusive: on the contrary, even in the most resistive levels of the photosphere only waves corresponding to time scales of less than a few seconds are damped. There can be no doubt that the photosphere is a highly conducting medium, and that the region of the solar atmosphere where the plasma is weakly ionized is of no special significance. Its electrical conductivity is high and it is strongly coupled by parallel currents to the regions both above and below. The supposed analogy to the ionosphere is not justified.

3. THE CLASSICAL RESISTIVITY OF THE CORONA

A coronal loop of length $l \approx 10^7$ m and cross-sectional area $A \approx 10^{13}$ m^2 has a classical resistance $R \approx 10^{-12}\ \Omega$ at $T \approx 10^6$ K which decreases as $T^{-3/2}$ with increasing T. This resistance is entirely negligible. If the current profile breaks up into N filaments each of cross-sectional area a_f such that the filling factor is $f_a = N a_f / A$, then both the resistance R and the current density J are increased by a factor f_a^{-1}. In order for the resistivity to remain classical, J is limited to a value $J_{\max} = n_e e \upsilon_{\max}$, where $\upsilon_{\max}$ is the threshold for a relevant current driven instability to develop. Hence there is a minimum value $f_a = I/(A n_e e \upsilon_{\max})$ that limits the enhancement in classical R (for fixed η and l) which is possible by appealing to current filamentation. Assuming that the relevant instability is ion sound turbulence, one has $\upsilon_{max} = \upsilon_s$, where υ_s is the ion sound speed, and with $I = 3\times10^{11}$ A, $n_e = 10^{17}$ m^{-3}, $A = 10^{13}$ m^2, $e = 1.6\times10^{-19}$ C, and $\upsilon_s = 10^5$ m s^{-1}, the current instability reaches threshold for $f_a \approx 10^{-5}$. Hence R can be increased by current filamentation only by a factor of ~10^5 before an instability occurs and anomalous resistivity sets in. The maximum value of classical resistance for current densities below the instability threshold is thus $R \approx 10^{-7}$ Ω, which is much smaller than the value ($R \approx 10^{-3}$ Ω) required to account for dissipation in the impulsive phase of a flare.

We conclude that—provided the bulk of the magnetic energy is released as heat or accelerated particles—some form of enhanced (non-collisional) resistivity is required. It might be remarked that an adequate power is achieved in Spicer's (1977) model only by a large current (~10^{14} A), while an even larger current (~10^{16} A) is implicit in the model of Kan, Akasofu and Lee (1983). Such a current implies a magnetic field of 10^3 T (10^7 G) in the corona. There is no evidence for such fields!

4. AN ION SOUND MODEL FOR THE CORONAL RESISTANCE

In one possible model for the impulsive phase of a flare, the current density in the filaments reaches the threshold for ion sound instability, so that the resistivity attains the anomalous value $\eta_{an} \approx \xi\mu_0 c^2/\omega_p$ where $\xi \equiv \nu_{eff}/\omega_p \approx 10^{-2}$. Such a model was suggested and explored briefly by Kuperus (1976). From the viewpoint adopted here, the essential requirement is that this mechanism allows one to account for a resistance $R \approx 10^{-3}$ Ω.

The ion sound instability switches on when the current density reaches approximately $n_e e \upsilon_s$. It also requires that the electron temperature greatly exceed the ion temperature: if this condition is not satisfied initially, either the ion cyclotron or Buneman instability threshold may be reached instead, or some dissipation may heat the electrons preferentially until the required condition for the ion sound instability is satisfied. Once the ion sound instability does develop it certainly heats the electrons so that υ_s and J_{max} both increase. To maintain the current density at threshold then requires that the current filaments become narrower. It is difficult to pursue further the subsequent evolution without a self-consistent model which tracks the changing plasma parameters, so we bypass these subtleties and merely estimate whether the ion sound model is viable initially.

The plasma parameters quoted in § 3 imply $f_a \approx 10^{-5}$ and hence $R_{an} = (\eta_{an} l)/(f_a A) \approx 5\times10^{-3}$ Ω. Thus an adequate resistance is possible. However, this estimate is subject to considerable uncertainty and at best one can conclude that the ion sound model is not obviously incompatible with the resistance required to account for the observed power release.

This model may be pursued further by considering the heating of the plasma and the acceleration of fast electrons. Bulk heating in a model analogous to that envisaged here has been discussed by Duijveman, Hoyng and Ionson (1981). These authors assumed, quite arbitrarily, that the energy is released in 10^{-2} of the volume. In the present model the energy release volume is less than f_a, and hence is $\lesssim 10^{-5}$ at any one instant. The total number of electrons in the filaments at any one time is $n_e f_a A l \approx 10^{32}$, and these would be heated to a few times 10^8 K in a few milliseconds. For the model to account for the observations, the current must cease to flow through each filament after no more than a few milliseconds, and then begin to form new filaments.

Acceleration of the fast electrons required to account for microwave emission has been discussed by Holman (1985) in the context of a model similar to the one discussed here.

The main point we are emphasizing concerns the resistance necessary to account for the inferred energy release in a flare. The estimated value $R \approx 10^{-3}$ Ω is for a modest flare and is averaged over the impulsive phase. It may be possible to account for a larger power in a larger flare by assuming a somewhat larger current, but one probably also requires a larger value of R. Larger values of R are also necessary to account for a higher power on shorter time scales in a flare. For example, some ''elementary bursts'' seem to require up to 10^{22} W on a time scale of a few seconds (*e.g.*, van Beek *et al.*, 1974). A power of this order is also apparently involved on a time scale of tens of milliseconds in ''sub-bursts'' (Sturrock *et al.*, 1984). A power of 10^{22} W requires a resistance of $R = 10^{-1}\ \Omega$ for $I = 3\times10^{11}$ A, and an even larger value of R if only part of the coronal current flows through the region where this power is released.

The above estimates suggest that the maximum value of the coronal resistance that is possible in the ion sound model is around $10^{-2}\ \Omega$. This is sufficiently close to the required resistance that the ion sound model is favorable, provided some modest enhancement over these estimates can be made. Conversely, if no such enhancement can be identified then one must appeal to a more efficient form of resistivity, such as that due to Buneman turbulence or to the development of a collection of double layers.

5. CONCLUSIONS

(1) We argue here (and in more detail elsewhere) that photospheric dynamo models for solar flares are untenable. Specifically, we refer only to those models that are based on an analogy with the dynamo models for magnetospheric substorms, and that involve the electrical properties of weakly ionized plasma explicitly.

(2) The circuit model for solar flares (Alfven, 1977; Spicer, 1977, 1982) allows one to estimate the coronal resistance required to account for the observed power release (neglecting power going into mass motion): the resulting value is $R \approx 10^{-3}\ \Omega$.

(3) It is *not* possible produce this resistance by Spitzer resistivity, even when enhanced by filamentation of the current.

(4) It *is* possible to account for this resistance in terms of anomalous resistivity due to ion sound turbulence, but even this mechanism has difficulty accounting for the higher power levels inferred in some flare ''kernels'' and ''spikes''.

ACKNOWLEDGEMENTS

This work was supported by NSF under grant ATM-8619853 and by NASA under grant NAGW-864.

REFERENCES

Alfven, H.: 1977, *Rev. Geophys. and Space Phys.* **15**, 271.
Duijveman, A., Hoyng, P., and Ionson, J. A.: 1981, *Ap. J.* **245**, 721.
Henoux, J.-C.: 1986, in V. E. Stepanov and V. N. Obridko (eds.) *Solar Maximum Analysis*, VNU Science Press, Utrecht, p. 109.
Heyvaerts, J.: 1974, *Solar Phys.* **38**, 419.
Holman, G. D.: 1985, *Ap. J.* **293**, 584.
Kan, J. R., Akasofu, S.-I., and Lee, L. C.: 1983, *Solar Phys.* **84**, 153.
Khan, J. I.: 1987, *Proc. Astron. Soc. Australia* (in press).
Kuperus, M.: 1976, *Solar Phys.* **47**, 79.
Mitchner, M., and Kruger, C. H., Jr.: 1973, *Partially Ionized Gasses*, Wiley, New York.

Moreton, G. E., and Severny, A. B.: 1968, *Solar Phys*. **3**, 282.
Obayashi, T.: 1975, *Solar Phys*. **40**, 217.
Spicer, D. S.: 1977, *Solar Phys*. **53**, 305.
Spicer, D. S.: 1982, *Space Sci. Rev*. **31**, 351.
Sturrock, P. A.: 1980, in P. A. Sturrock (ed.) *Solar Flares*, Colorado Associated University Press, Boulder, p. 411.
Sturrock, P. A., Kaufmann, P., Moore, R. L., and Smith, D. F. : 1984, *Solar Phys*. **94**, 341.
van Beek, H. F., de Feiter, L. D., and de Jager, C.: 1974, *Space Res*. **14**, 447.

DISCUSSION

UCHIDA: I didn't understand one point. I think you must be talking about that part of the current above the MHD current. For example, according to your picture, no motion is allowed, but according to my understanding no motion relative to the magnetic field movement is allowed in the case that you are considering. In the high beta region the magnetic field can be twisted and modified causing corresponding currents to flow. In that case you do not need to know the Pedersen current and everything like that, just the self-consistent current. The point I'm always wondering about the circuit model is that people choose the circuit and consider it as fixed, but in many cases that's not the case. The circuit itself can move, or be deformed, or whatever.

MELROSE: Can I make three points in reply to you. One is yes, I agree, that if you go down to where plasma beta is such that the turbulent motions can move the magnetic field, then you can indeed get currents closing down there, but that's a long way below the photosphere. In the photosphere if you take the pores the beta is very small. Plasma just simply has to flow around the magnetic pores in the photosphere, it can't do anything else. The beta is far too small.

UCHIDA: I see, you're talking about the problem that twisting out the magnetic field in very low-beta region, solely by the motion of plasma there. If you go down from the photosphere maybe a hundred km, then depending on the magnetic field that you consider, a high-beta condition may hold, and a thousand km is very tiny.

MELROSE: Let me make my second point. Yes indeed, what you need is cross-field currents. You can get cross-field currents if you just look at the inertial term **J X B** = force supplied. You can get three effects. The **J** balancing the inertial term, which is the one you're talking about, which involves Alfven waves. The other two terms there are the two terms we've talked about, the Spitzer conductivity and the Pedersen conductivity, and they won't work. Although I accept your point, there is no model in which the currents are generated in that way, that is by Alfven waves whose propagation time is of order the timescale for the storage of magnetic energy in flares. I was going to make a third point, but I'll stop there.