DEPOLARIZATION OF SOLAR BURSTS DUE TO SCATTERING BY LOW-FREQUENCY WAVES

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Abstract. The possibility is explored that fundamental plasma emission in solar radio bursts of types I, II, and III is depolarized due to scattering off low-frequency waves. Three ways in which depolarization might occur are identified: (1) one or several large-angle scatters, (2) several small-angle scatters close to the plasma level, and (3) many small-angle scatters well above the plasma level. It is pointed out that the degree p of polarization ($p = 1$ initially) may be approximated by $p(\chi) = \cos\chi$ after one large-angle scatter through an angle χ, and that for backscatter ($\chi > \pi/2$) the sense of polarization changes (from o-mode to x-mode senses). Possibility (2) involves coupling between the o- and x-mode components through their longitudinal parts, and is explored in some detail. The wave vectors $\mathbf{k}''$ required for the scatterings are identified, and it is suggested that ion-sound waves are suitable for possibility (1) and whistlers for possibility (2). The whistlers may be generated by the streaming electrons themselves.

Large-angle scattering is favourable for depolarizing type I emission, as proposed by Wentzel, Zlobec, and Messerotti (1986). Scattering by whistlers near the plasma level is favourable for depolarizing type III bursts. Several predictions are made based on these possibilities.

1. Introduction

It is accepted that metre-wave solar radio emission is often depolarized as it propagates through the solar corona. Simple theory predicts that emission at the fundamental (F) of the plasma frequency ω_p should be 100% polarized in the sense of the ordinary mode (o-mode) of magnetoionic theory (Kai, 1970; Melrose and Sy, 1972; Melrose, 1985). Type I emission from sources near the central meridian is $\cong 100\%$ polarized in this sense, seemingly confirming the theoretical prediction. However, the F components in type II and type III bursts are only partially polarized, with degree of polarization varying between 0 and $\cong 70\%$ (Suzuki and Sheridan, 1977; Dulk and Suzuki, 1980; Suzuki and Dulk, 1985). Also the polarization of type I emission decreases for sources more than about three days from central meridian passage, and approaches zero for sources near the limb (Zlobec, 1975).

In this context depolarization means a partial conversion of an initial flux of o-mode radiation into a mixture of o- and x-mode radiation. Unpolarized emission corresponds to an equal mixture of the two modes (typically to within $\lesssim 5\%$) and its explanation provides a severe test for any proposed depolarization mechanism. Particular depolarizing mechanisms that have been considered include mode coupling due to local gradients in the plasma parameters (Cohen, 1960; Zheleznyakov and Zlotnik, 1964; Melrose, 1980), reflection off steep density gradients (Hayes, 1985) and scattering off a low-frequency wave (Wentzel, 1984; Wentzel, Zlobec, and Messerotti, 1986).

In this paper the conditions under which scattering off low-frequency waves can cause

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the inferred depolarizations are explored. Three cases are identified. One case is large-angle scattering in which the polarization changes substantially in a single scattering. A special example of this case was considered by Wentzel, Zlobec, and Messerotti (1986) in connection with partially polarized type I emission. The other two cases involve small-angle scattering. One applies close to the plasma frequency ($\omega \lesssim 1.5\omega_p$) where the longitudinal parts of the polarizations of the o- and x-modes are significant and allow $o - x$ scattering even when there is no change in the direction of propagation; this was pointed out by Wentzel (1984). The remaining case involves a small coupling term that is usually neglected, and which allows weak small-angle $o - x$ scattering even at high frequencies ($\omega \gg \omega_p$).

It is particularly difficult to account for unpolarized emission. Most depolarizing mechanisms allow partial depolarization in a single scattering, refraction, or reflection process, but only under quite exceptional circumstances is the resulting radiation an equal mixture of the o- and x-modes. In order to account for unpolarized emission, it seems that one must invoke multiple scattering, in the form of either several large-polarization-changing scatters or many small-polarization-changing scatters.

In Section 2 the scattering amplitude is defined and evaluated in relevant cases. In Section 3 the wavevectors $\mathbf{k}''$ required for the various scatterings are estimated, and in Section 4 the possible waves that could provide these $\mathbf{k}''$ are discussed. The application to F emission is discussed in Section 5.

2. The Scattering Amplitude

The scattering probability for a three-wave interaction involving two magnetoionic waves σ and σ' and a low-frequency wave F has been written down for whistlers by Melrose (1975) and for ion sound waves by Melrose (1986). It is important here to retain a factor $\tau_{ij}(\omega)$ in the probability; this factor takes account of the magnetic effects in the nonlinear response, e.g., compare Equations (6.54) and (10.24) of Melrose (1986). With this factor included, the probability is

$$w_{\sigma\sigma' F}(\mathbf{k}, \mathbf{k}', \mathbf{k}'') = \frac{\hbar e^2 \omega_p^2}{2\varepsilon_0 m_e^2 V_e^2}\, \eta_F\, \frac{\omega''}{\omega\omega'}\, R_\sigma R_{\sigma'}\, |a^{\sigma\sigma'}|^2 \times \\ \times (2\pi)^4 \delta(\omega - \omega' \pm \omega'')\delta^3(\mathbf{k} - \mathbf{k}' \pm \mathbf{k}''), \tag{1}$$

where the 'scattering amplitude' $a^{\sigma\sigma'}$ involves the tensor $\tau_{ij} = \tau_{ij}(\omega)$ (with $\omega' \cong \omega$ for $\omega'' \ll \omega$):

$$a^{\sigma\sigma'} := e_i^{\sigma *}\, \tau_{ij} e_j^{\sigma'}, \\ \tau_{ij}(\omega) = (\omega^2 \delta_{ij} - \Omega_e^2 b_i b_j - i\omega\Omega_e \varepsilon_{ijk} b_k)/(\omega^2 - \Omega_e^2). \tag{2}$$

In (1) and (2), Ω_e is the electron cyclotron frequency, V_e is the thermal speed of electrons, and $\mathbf{b}$ is a unit vector along the direction of the magnetic field. The factor η_F is different for the different possible low-frequency wave modes F:

$$\eta_F \cong \begin{cases} 1 & \text{for ion sound waves}\,, \\[2ex] \dfrac{V_e^2}{c^2}\dfrac{\omega_p^2}{\Omega_e^2}\sin^2\theta'' & \text{for whistlers}\,, \\[2ex] \dfrac{V_e^2}{c^2}\dfrac{\omega_p^2}{\Omega_e^2} & \text{for magnetoacoustic waves}\,. \end{cases} \tag{3}$$

2.1. Detailed form of $a^{\sigma\sigma'}$

The polarization vectors may be written in the form

$$\mathbf{e}^\sigma = \frac{(K_\sigma \boldsymbol{\kappa} + T_\sigma \mathbf{t} + i\mathbf{a})}{(K_\sigma^2 + T_\sigma^2 + 1)^{1/2}} \tag{4}$$

with

$$\begin{aligned} \boldsymbol{\kappa} &= (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)\,, \\ \mathbf{t} &= (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta)\,, \\ \mathbf{a} &= (-\sin\phi, \cos\phi, 0)\,, \\ \mathbf{b} &= (0, 0, 1)\,. \end{aligned} \tag{5}$$

The parameter K_σ describes the longitudinal component of the polarization. The polarization vector $e^{\sigma'}$ is given by (4) with σ replaced by σ', and $\boldsymbol{\kappa}$, $\mathbf{t}$, $\mathbf{a}$ by $\boldsymbol{\kappa}'$, $\mathbf{t}'$, $\mathbf{a}'$ defined as in (4) in terms of θ', ϕ'.

Explicit evaluation gives the following expression for the scattering amplitude:

$$\begin{aligned} a^{\sigma\sigma'} = {} & (K_\sigma^2 + T_\sigma^2 + 1)^{-1/2}(K_{\sigma'}^2 + T_{\sigma'}^2 + 1)^{-1/2}(1 - Y^2)^{-1} \times \\ & \times \{K_\sigma K_{\sigma'}[\sin\theta\sin\theta'(\cos(\phi - \phi') + iY\sin(\phi - \phi')) + \cos\theta\cos\theta'] + \\ & + T_\sigma T_{\sigma'}[\cos\theta\cos\theta'(\cos(\phi - \phi') + iY\sin(\phi - \phi')) + \sin\theta\sin\theta'] + \\ & + \cos(\phi - \phi') + iY\sin(\phi - \phi') + \\ & + K_\sigma T_{\sigma'}[\sin\theta\cos\theta'(\cos(\phi - \phi') + iY\sin(\phi - \phi')) - \cos\theta\sin\theta'] + \\ & + T_\sigma K_{\sigma'}[\cos\theta\sin\theta'(\cos(\phi - \phi') + iY\sin(\phi - \phi')) - \sin\theta\cos\theta'] + \\ & + (K_\sigma\sin\theta + K_{\sigma'}\sin\theta' + T_\sigma\cos\theta + T_{\sigma'}\cos\theta')(Y\cos(\phi - \phi') + i\sin(\phi - \phi'))\}\,. \end{aligned} \tag{6}$$

2.2. Explicit evaluation for the magnetoionic modes

For the magnetoionic modes ($\sigma = +1$ for the o-mode, $\sigma = -1$ for the x-mode) one has (e.g., Melrose, 1980, p. 258)

$$T = \frac{Y(1-X)\cos\theta}{\frac{1}{2}Y^2\sin^2\theta - \sigma\Delta} = \frac{-\frac{1}{2}Y^2\sin^2\theta - \sigma\Delta}{Y(1-X)\cos\theta},$$

$$\Delta = [\tfrac{1}{2}Y^4\sin^4\theta + (1-X)^2Y^2\cos^2\theta]^{1/2}, \tag{7}$$

$$K = \frac{XY\sin\theta}{1-X}\,\frac{T}{T - Y\cos\theta} = \frac{XY\sin\theta(1+YT\cos\theta)}{1 - X - Y^2 + XY^2\cos^2\theta},$$

$$n^2 = 1 - \frac{XT}{T - Y\cos\theta} = 1 - \frac{X(1-X)(1+YT\cos\theta)}{1 - X - Y^2 + XY^2\cos^2\theta}.$$

2.3. The quasi-circular approximation

The simplest useful approximation to the properties of the magnetoionic waves corresponds to circular transverse polarizations ($|T_\sigma| \cong 1$, $|K_\sigma| \ll 1$). This approximation applies for $(1-X)^2\cos^2\theta \gg \frac{1}{4}Y^2$, and gives

$$T_\sigma = -\sigma\,\frac{\cos\theta}{|\cos\theta|} \tag{8}$$

for $X < 1$. Ignoring terms of order Y, (6) then reduces to

$$a^{\sigma\sigma'} \cong \tfrac{1}{2}[\sigma\sigma'(\cos\theta\cos\theta'\,\cos(\phi-\phi') + \sin\theta\sin\theta') + \cos(\phi-\phi')]. \tag{9}$$

This approximation breaks down for $1 - X \ll 1$ and for θ close to $\pi/2$. It is adequate for a semi-quantitative discussion of large-angle scattering except in this case.

2.4. Large-angle scattering and backscattering

In the limit of small-angle scattering ($\theta' = \theta$, $\phi' = \phi$), (9) implies that there is zero transfer from one mode to the other, i.e., the scattering amplitude for $\sigma' = \sigma$ is unity and that for $\sigma' = -\sigma$ is zero. (This case is discussed more carefully below.) As the angle between the scattered wave ($\boldsymbol{\kappa}'$) and the unscattered wave ($\boldsymbol{\kappa}$) increases the magnitude of the scattering amplitude for $\sigma' = \sigma$ decreases and that for $\sigma' = -\sigma$ increases. This is seen most easily for the case where $\boldsymbol{\kappa}'$, $\boldsymbol{\kappa}$ and $\mathbf{b}$ are coplanar ($\phi' = \phi$). Then (9) reduces to $a^{\sigma\sigma'} = \frac{1}{2}[1 + \sigma\sigma'\cos(\theta - \theta')]$; for $\sigma\sigma' = 1$ this is a monotonically decreasing function of $\theta - \theta'$ and for $\sigma'\sigma = -1$ it is a monotonically increasing function of $\theta - \theta'$.

The maximum transfer from one mode to the other is for backscatter ($\boldsymbol{\kappa}' = -\boldsymbol{\kappa}$, $\mathbf{t}' = -\mathbf{t}$, $\mathbf{a}' = \mathbf{a}$). In the approximation (9) the backscattering amplitude is zero for $\sigma' = \sigma$ and unity for $\sigma' = -\sigma$. This is the most favourable case for converting o-mode radiation into x-mode radiation in a single scattering.

2.5. Small-angle scattering

In the limit of small-angle scattering ($\theta' = \theta$, $\phi' = \phi$), (6) reduces to

$$a^{\sigma\sigma'} = (K_\sigma^2 + T_\sigma^2 + 1)^{-1/2} (K_{\sigma'}^2 + T_{\sigma'}^2 + 1)^{-1/2} (1 - Y^2)^{-1} \times \times \{K_\sigma K_{\sigma'} + T_\sigma T_{\sigma'} + 1 + Y(K_\sigma + K_{\sigma'}) \sin\theta + Y(T_\sigma + T_{\sigma'}) \cos\theta\} . \tag{10}$$

Quite generally one has $T_+ T_- = -1$. It follows that the $o - x$ small-angle scattering amplitude is approximated by zero for $|K_\sigma| \ll 1$, $Y \ll 1$ even if the quasi-circular approximation does not apply. However, the terms involving K_σ and Y are not necessarily negligible. They have little effect on the scattering for $\sigma' = \sigma$, for which (10) reduces to

$$a^{\sigma\sigma} = \frac{1}{1 - Y^2} \left\{1 + \frac{2Y\cos\theta(K_\sigma \sin\theta + T_\sigma \cos\theta)}{K_\sigma^2 + T_\sigma^2 + 1}\right\},$$

but they can be important for $o - x$ scattering, for which (10) reduces to

$$a^{\sigma\sigma'} = (K_+^2 + T_+^2 + 1)^{-1/2} (K_-^2 + T_-^2 + 1)^{-1/2} (1 - Y^2)^{-1} \times \times \left\{\frac{Y \sin^2\theta[X - (1 - X)(1 - X - Y^2)]}{(1 - X)(1 - X - Y^2 + XY^2 \cos^2\theta)}\right\} . \tag{11}$$

The minimum frequency at which $o - x$ scattering is possible is the cutoff frequency $\omega = \omega_x$ for the x-mode at $1 - X = Y$. The forward scattering amplitude (11) has its maximum allowed value at $1 - X = Y$:

$$a^{\sigma\sigma'} = \frac{(1 - Y - Y^2 + Y^3) \sin^2\theta}{1 - Y\sin^2\theta - Y^2 \cos^2\theta} \cong \sin^2\theta . \tag{12}$$

It then decreases with increasing ω and changes sign at the frequency at which the quantity in square brackets in (11) is zero. Let this be $\omega = \omega_0$; one has

$$\omega_0 = [\omega_p^2 + \tfrac{1}{2}\omega_{UH}^2 + \tfrac{1}{2}(4\omega_p^4 + \omega_{UH}^4)^{1/2}]^{1/2} \cong \frac{1 + \sqrt{5}}{2}\, \omega_p \cong 1.6\omega_p , \tag{13}$$

where the approximation applies for $\Omega_e^2 \ll \omega_p^2$, with $\omega_{UH}^2 = \omega_p^2 + \Omega_e^2$. Thus for $\omega_x < \omega \lesssim 1.5\omega_p$ the longitudinal part of the polarization is important, as noted by Wentzel (1984).

At higher frequencies the final term in (10) (for $\sigma = -\sigma'$) becomes the dominant one. The leading correction to (9) for $\theta' = \theta$, $\phi' = \phi$ is (for $\sigma' = -\sigma$)

$$a^{\sigma\sigma'} \cong \frac{1}{2}\left(- \frac{Y^2 \sin^2\theta}{(1 - X)(1 - Y^2)}\right) . \tag{14}$$

This is to be compared with the contribution $a^{\sigma\sigma'} \cong -\frac{1}{4}(\theta - \theta')^2$, due to the term (9) for $\theta' \cong \theta$, $\phi = \phi''$, and $\sigma' = -\sigma$. The term (14) is the dominant polarization-changing term at high frequencies ($\omega \gg \omega_0$) for scattering angles $\chi \lesssim Y \sin\theta$.

3. The Required $\mathbf{k}''$

The three-wave matching conditions are implied by the δ-functions in (1):

$$\mathbf{k} - \mathbf{k}' \pm \mathbf{k}'' = 0\,, \qquad \omega - \omega' \pm \omega'' = 0\,. \tag{15}$$

Usually the frequency ω'' of the low-frequency wave is sufficiently small to be neglected ($\omega'' \ll \omega$). A specific scattering process involves given σ, ω, θ, ϕ and σ', ω', θ', ϕ', with $\omega' \cong \omega$ for $\omega'' \ll \omega$. For any specific scattering process the value of $\mathbf{k}''$ is determined (to within a sign) through these given values together with the dispersion relations in (15).

3.1. Large-angle scattering

Large-angle scattering requires $\mathbf{k}''$ comparable to $\mathbf{k}$ in magnitude and at large angle to $\mathbf{k}$. Thus one requires

$$k'' \cong \{X(1-X)\}^{1/2}\,\frac{\omega_p}{c} \tag{16}$$

and $1 \gtrsim |\boldsymbol{\kappa} \times \boldsymbol{\kappa}''| \gg 0$. Backscatter requires $\mathbf{k}'' = \pm 2\mathbf{k}$.

3.2. Small angle $o - o$ scattering

Small-angle scattering corresponds to $k'' \ll k$. The scattering angle, χ, which is the angle between $\mathbf{k}'$ and $\mathbf{k}$, is then given by

$$\chi \cong k''/k\,. \tag{17}$$

The direction $\boldsymbol{\kappa}''$ of $\mathbf{k}''$ is required to be nearly orthogonal to $\boldsymbol{\kappa}$:

$$\boldsymbol{\kappa}'' \cdot \boldsymbol{\kappa} \cong 0\,. \tag{18}$$

3.3. $o - x$ scattering

There are two notably different requirements for small-angle $o - x$ scattering compared with $o - o$ or $x - x$ scattering. The first is that for a given ω and θ there is a minimum value Δk of k''. The second is that for $k'' \cong \Delta k$ the direction $\boldsymbol{\kappa}''$ is required to be nearly parallel (or anti-parallel) to $\boldsymbol{\kappa}$.

The minimum value of k'' is due to the difference between the refractive indices for the o-mode and x-modes at given ω and θ. One has

$$\begin{aligned}\Delta k = k - k' &= \frac{\omega}{c}\,\{n_+(\omega,\theta) - n_-(\omega,\theta)\} \cong \\ &\cong \frac{\omega_p}{2c}\left(\frac{X}{1-X}\right)^{1/2}\{n_+^2(\omega,\theta) - n_-^2(\omega,\theta)\} = \\ &= \frac{\omega_p}{c}\left(\frac{X}{1-X}\right)^{1/2}\frac{X\Delta}{1 - X - Y^2 + XY^2\cos^2\theta}\,.\end{aligned} \tag{19}$$

Let us evaluate (19) in two extreme limits. Firstly, at the cutoff frequency for the x-mode at $1 - X - Y = 0$ (with $Y \ll 1$) we have $n_-(\omega, \theta) = 0$ and

$$\Delta k \cong \frac{\omega}{c} \left\{ \frac{Y(1 - Y)\cos^2\theta}{1 - Y\sin^2\theta - Y^2\cos^2\theta} \right\} \cong \frac{(\omega_p \Omega_e)^{1/2}}{c} |\cos\theta| . \tag{20}$$

Secondly, at high frequencies ($X, Y \ll 1$) in the quasi-circular limit, i.e., except for $|\theta - \pi/2| \lesssim \frac{1}{2}Y$, one has $n_+^2 - n_-^2 \cong 2XY|\cos\theta|$ and, hence,

$$\Delta k \cong \frac{\omega_p}{c} \left(\frac{X}{1 - X} \right)^{1/2} XY|\cos\theta| . \tag{21}$$

By inspection (20) is a special case of (21) (for $1 - X = Y$, $Y \ll 1$) and we need consider only (21).

Very close to the cutoff frequency for the x-mode the scattering angle is not relevant because the direction of the x-mode waves is arbitrary. For higher frequencies the scattering angle is given approximately by

$$\chi \cong \frac{[k''^2 - (\Delta k)^2]^{1/2}}{k} . \tag{22}$$

This reduces to the same value $\chi \cong k''/k$ as for $o - o$ or $x - x$ scattering only for $k''^2 \gg (\Delta k)^2$.

4. Scattering by Waves in Specific Modes

Scattering by low-frequency waves in a particular mode is significant only if (i) waves with the required $\mathbf{k}''$ can exist in that mode, (ii) such waves are present in the plasma, with (iii) a level of excitation sufficient to scatter an o-mode wave before it escapes from the plasma. The condition (iii) may be discussed semi-quantitatively in terms of the ratio of the scattering rate and the escape rate v_g/l for waves with group speed v_g escaping from a region with linear dimension l. Multiple scattering requires this ratio to be significantly greater than unity.

4.1. Ion sound waves

The dispersion relation for ion sound waves is $\omega'' \cong k'' v_s$ with $v_s = \omega_{pi}\lambda_{\mathrm{De}}$ the ion sound speed, and with $\omega_{pi} (\cong \omega_p/43)$ the ion plasma frequency and $\lambda_{\mathrm{De}} (= V_e/\omega_p)$ the electron Debye length. The approximate dispersion relation $\omega'' \cong k'' v_s$ applies for $(k''\lambda_{\mathrm{De}})^2 \ll 1$, which corresponds to $(\omega''/\omega_{pi})^2 < 1$. The required k''s cover the range from the maximum for backscatter ($k'' = 2k$) at $k = \omega_p/2c$, cf. Equation (16) with $X = \frac{1}{2}$, through $k'' = \Delta k \cong (\omega_p/c)(\Omega_e/\omega_p)^{1/2}|\cos\theta|$, cf. Equation (20), to arbitrarily small $k'' = \chi k$ for arbitrary small scattering angles χ. Waves with all the required $\mathbf{k}''$ can exist in the ion sound mode.

Ion sound waves are observed to be present most of the time in the interplanetary mdium (Gurnett *et al.*, 1979), and it is plausible that they are also present in the solar corona. However, the mechanism by which these waves are generated is not understood and there is no basis for expecting them to be present in the corona other than by analogy with the interplanetary medium. Ion sound waves have also been observed in correlation with type III events in the interplanetary medium (Lin *et al.*, 1986) but these waves have too high a value of $k'' \lambda_{\mathrm{De}}$ to be relevant here. (Their generation mechanism is also not understood.)

The scattering rate $D_s^{\sigma\sigma'}$ for ion sound waves may be estimated by using (3) to modify a formula for scattering by whistlers given by Melrose (1975). For waves with an energy density W_s in a range $\Delta k'' \cong k''$ about k'' one finds

$$D_s^{\sigma\sigma'} \cong \frac{\pi}{8} (a^{\sigma\sigma'})^2 \frac{W_s}{n_e m_e V_e^2} \frac{\omega_p^4}{\omega^2 \omega''} . \tag{23}$$

Alternatively the level of ion sound waves may be expressed in terms of the fluctuations δn_e in the electron density at k'':

$$\frac{W_s}{n_e m_e V_e^2} = \left(\frac{\delta n_e}{n_e}\right)^2 . \tag{24}$$

Now consider the escape rate for o-mode waves with a given k being scattered by low-frequency waves. The o-mode waves escape from the localized region when k has changed by $\delta k \cong k$. This determines the characteristic distance $l \cong (1 - X)L/2$, where $L(\cong 10^8$ m) is the characteristic distance over which n_e changes. With $v_g \cong (1 - X)^{1/2} c$, the escape rate is

$$\frac{v_g}{l} \cong \frac{c}{2L} \frac{1}{(1 - X)^{1/2}} . \tag{25}$$

Multiple scattering occurs for $D_s^{\sigma\sigma'} \gg v_g/l$.

Inserting $\omega_p \cong 10^9$ s^{-1} in (23) with (24) with $|a^{\sigma\sigma'}|^2 \cong 1$, one finds that a level of fluctuations $\delta n_e/n_e \gtrsim 10^{-6}$ at $k'' \lambda_{\mathrm{De}} \cong 10^{-2}$ is required for multiple scattering. However, the required value of $\delta n_e/n_e$ is poorly determined. Also there is no obvious theoretical estimate with which it may be compared. Nevertheless, the value seems relatively modest.

4.2. Whistlers

The dispersion relation for whistlers is

$$k''^2 = \frac{\omega_p^2}{c^2} \frac{\omega''}{\Omega_e |\cos \theta''| - \omega''} . \tag{26}$$

Near $\omega'' = \Omega_e(\cos \theta'')$ the waves have large k'' and are called resonant whistlers; at a given ω'' whistlers exist only for $|\cos \theta''| > \omega''/\Omega_e$. Lower-hybrid waves may also be included by regarding them as a limiting case of resonant whistlers at $|\cos \theta''| \lesssim \frac{1}{43}$, with

$\omega'' \cong \omega_{LH} \cong (\Omega_e \Omega_i)^{1/2}$ (where the motion of the ions becomes important). At $\omega'' \lesssim \Omega_i$ the whistler mode changes character and becomes the magnetoacoustic mode.

Whistlers can be generated by streaming electrons. Let v be the streaming speed, then the resulting whistlers have

$$k'' \cong \frac{\Omega_e}{v\,|\cos\theta''|} \,. \tag{27}$$

These whistlers have $\omega'' \ll \Omega_e\,|\cos\theta''|$ in (26) for $(v^2/c^2)\cos^2\theta'' \gg \Omega_e^2/\omega_p^2$. For $(v/c)\,|\cos\theta''| < \Omega_e/\omega_p$ only resonant whistlers can be generated by electrons with a given v.

The maximum required k'' $(= \omega_p/c)$ is for backscatter at $\omega = \sqrt{2}\,\omega_p$. Such a large k'' is possible for whistlers only in the limit $\omega'' \cong \Omega_e$, $|\cos\theta''| \cong 1$; which corresponds to parallel electron-cyclotron waves. More generally large-angle scattering requires resonant whistlers. Such waves can be generated by electrons with speeds typical of those in type III events. However, the waves are restricted to a small range of angles θ'' for given k'', and this severely restricts the possible large-angle scatters.

Other than for large-angle scattering, for scattering through an angle $\chi \lesssim 1$ the whistlers required have $\omega''/\Omega_e\,|\cos\theta''| \cong (1 - X)\chi^2$, where (21) has been used. For small angle scattering ($\chi \ll 1$) such waves can be generated only by very fast electrons with speed v satisfying $v/c \gg (\Omega_e/\omega_p)^{1/2}\,|\cos\theta\cos\theta''|^{-1}$. For $\Omega_e/\omega_p \cong 0.1$ these would be significantly faster than most electrons in a type III event. For $o - x$ scattering close to the x-mode cutoff frequency the required whistlers can be generated by electrons with $v/c \cong (\Omega_e/\omega_p)^{1/2}$, requiring electrons with $v/c \cong 0.3$ for $\Omega_e/\omega_p = 0.1$, and this is more favourable.

The scattering rate for whistlers is given roughly by (cf. Melrose, 1975)

$$D_w^{\sigma\sigma'} \cong \frac{\pi}{16}\,|a^{\sigma\sigma'}|^2\,\frac{W_w}{n_e m_e c^2}\,\frac{\omega_p^2}{\omega^2}\,\frac{\omega_p^6}{\Omega_e^3 k''^2 c^2}\,, \tag{28}$$

where the dependence on θ'' is ignored. With k'' determined by (27), the maximum value of W_w generated by the streaming electrons (with number density n_1) is $W_w \cong n_1 m_e c^2 \Omega_e^2/\omega_p^2$. Then (28) reduces to

$$D_w^{\sigma\sigma'} \cong \frac{\pi}{16}\,|a^{\sigma\sigma'}|\,\frac{n_1}{n_e}\,\frac{\omega_p^2}{\omega^2}\,\frac{\omega_p^4}{\Omega_e^3}\,\frac{v^2}{c^2}\,. \tag{28$'$}$$

For any reasonable choice of parameters one finds $D_w^{\sigma\sigma'} \gg v_g/l$, cf. Equation (25). Thus, provided the instability is effective in generating whistlers, then the scattering of o- and x-mode waves by the whistlers should be efficient.

4.3. Other Wave Modes

Other wave modes of possible interest are lower hybrid waves and MHD waves. Lower hybrid waves are restricted to nearly perpendicular propagation, and this severely restricts the range of possible scattering processes. (These restrictions have been used by Wentzel, Zlobec, and Messerotti (1986) to explain certain features of the polarization of type I bursts.) The wave numbers k'' for MHD waves are very small, and these waves are relevant only for small-angle scattering through $\chi \cong \frac{1}{43}$. Only magnetoacoustic waves are relevant because Alfvén waves cause negligible density fluctuations.

5. Application to Fundamental Plasma Emission

The foregoing discussion suggests three alternative ways in which the polarization of F plasma emission may be reduced through scattering. These are:

(1) A single large-angle scatter can change the polarization substantially; a scatter through $\cong 90°$ reduces the degree of polarization to $\cong 0$, and a backscatter reverses the sense of polarization. Roughly isotropic ion sound waves with $\omega''/\omega_{pi} \cong V_e/c$ are the most favourable scatterers. Resonant whistlers and lower hybrid waves are also possible, but severely restrict the possible angular ranges of the scatterings.

(2) Several small-angle scatter with $k'' \cong \Delta k$, cf. Equation (21), can substantially reduce the polarization for $\omega \lesssim 1.5\omega_p$. Whistlers generated by electrons with $v/c \cong (\Omega_e/\omega_p)^{1/2}$ are favourable scatterers.

(3) Many small-angle scatters can reduce the degree of polarization due to each scattering causing a reduction of order Y^4, cf. Equation (14). Lower-frequency ion-sound waves ($\omega''/\omega_{pi} \cong (\Omega_e/\omega_p)^{1/2} V_e/c$) are favourable scatterers.

Suppose that for any of these, the ratio of $o - x$ to $o - o$ or $x - x$ scattering is r. Then initially completely polarized o-mode radiation (degree of polarization $p = 1$) has degree of polarization $p = (1 - r)/(1 + r)$ after one scattering, and

$$p = \left(\frac{1 - r}{1 + r}\right)^N \tag{29}$$

after N scatterings.

5.1. Type I bursts

Wentzel, Zlobec, and Messerotti (1986) have suggested that the variation of the degree of polarization of type I emission with distance of the source from the central meridian may be explained in terms of a single large-angle scatter off lower hybrid waves. This corresponds to possibility (1) above. The predicted variation of the polarization can be estimated roughly as follows. According to (9), with $\theta' = \theta$ for simplicity, the scattering amplitudes for $o - x$ and $o - o$ scatterings are in the ratio $\tan^2 \chi/2$, where $\chi = \theta' - \theta$ is the angle through which the scattering occurs. This gives $r = \tan^4 \chi/2$ and $p = \cos \chi$ after one scatter. This suggests that the degree of polarization should vary as $\cos \chi$, with χ the longitude of the type I source from the central meridian.

Note that the estimate made here of $p = \cos \chi$ after one scattering does not agree with the form $p = 2 \cos^2 \chi/(1 + \cos^2 \chi)$ suggested by Wentzel, Zlobec, and Messerotti (1986). These authors based their form on an intuitive argument involving partial conversion into linear polarization, with subsequent Faraday depolarization. The form $p = \cos\chi$ suggested here applies in the quasi-circular limit, and is reasonable for scattering at $\omega \gtrsim 1.5\omega_p$ and for angles θ not close to $\pi/2$. In general the dependence on the scattering angle χ is complicated by the dependence on the other parameters θ, θ', ϕ and ϕ' in (6) with $\cos\chi = \cos\theta\cos\theta' + \sin\theta\sin\theta\cos(\phi - \phi')$, and with K_σ, T_σ and $K_{\sigma'}$, $T_{\sigma'}$ depending on θ and θ', respectively. A detailed analysis of the general case (6) is unlikely to lead to a simple general result.

In the case of lower hybrid waves, as suggested by Wentzel, Zlobec, and Messerotti (1986), $\mathbf{k}''$ is orthogonal to the magnetic field. It is reasonable to expect the change in polarization in this case to be similar to that for reflection off a density gradient orthogonal to the magnetic field. Such analysis of the polarization changes occurring as a result of reflection has been made by Hayes (198) (for a steep density gradient orthogonal to $\mathbf{B}$); no simple general approximate result emerges from the detailed analysis.

Thus the approximate form $p = \cos\chi$ is oversimplified but it is difficult to improve upon it. The question of whether one can account for unpolarized bursts near the limb in terms of a single scattering remains open; multiple scattering is probably required.

5.2. Type II and Type III Bursts

Unlike type I bursts, fundamental type II and type III bursts are never observed to be completely polarized. Any proposed depolarization mechanism needs to explain four features: (i) the absence of completely polarized bursts, (ii) the wide range of degrees of polarization ($\cong 0$ to $\cong 70\%$), (iii) unpolarized bursts, and (iv) the absence of negative (x-mode) polarization.

Possibility (1) involves only one large-angle scattering, and then one would expect at least some unscattered radiation to escape, i.e., one would expect some completely polarized bursts. Also it requires very special conditions to produce unpolarized radiation in a single scatter, and negative polarization cannot be readily excluded. Possibility (1) seems unfavourable.

Possibility (3) requires a very large number of scatterings. The ratio r implied by (14) is $r = \eta(\Omega_e/\omega_p)^4$, with η a factor of order unity. By way of illustration, after N scatters one has $p \cong 1 - 2N\eta(\Omega_e/\omega_p)^4$ and to account for $< 70\%$ polarization requires $N > 0.15\eta^{-1}(\Omega_e/\omega_p)^{-4}$. With $\Omega_e/\omega_p = 0.1$ and $\eta \cong 1$ one required $N \gtrsim 1500$ scatterings; to produce unpolarized ($p < 5\%$ bursts) with the same parameters requires $N \gtrsim 7500$. It may well be that rays do experience such a large number of small-angle scatters.

The remaining possibility (2) requires that the depolarizing scattering occur at $\omega < \omega_0$. The $o - x$ coupling is strongest at the x-mode cutoff frequency, where (12) gives $a^{+-} \cong \sin^2\theta$, and then decreases effectively to zero approximately proportional to $\omega^2 - \omega_0^2$, $\omega_0 \cong 1.6\omega_p$.

The whistlers required for this type of scattering can be generated by the streaming electrons themselves. Such whistlers have the component of $\mathbf{k}''$ along $\mathbf{k}$ approximately equal to Δk, and the component of $\mathbf{k}''$ orthogonal to this direction determines the scattering angle χ through (22). With $\mathbf{k}$ necessarily oblique for effective $o - x$ scattering ($r \sim \sin^2 \theta$), whistlers confined to a forward cone (about $\theta'' = 0$), as expected in a streaming instability, are favourable for the scattering.

A possible difficulty with this mechanism is that $o - x$ scattering is not possible for $\theta = 0$, and $o - o$ and $x - x$ scattering at $\theta = 0$ require waves at $\theta'' = \pi/2$, which is not possible for whistlers. Thus one might expect waves at $\theta = 0$ to escape without scattering, and so produce 100% polarization at an appropriate viewing angle. This possibility can be argued away as follows. Fundamental plasma emission due to Langmuir waves propagating along the magnetic field (as expected) has a dipolar emission pattern $\sim \sin^2 \theta$ that favours o-mode radiation at $\theta \cong \pi/2$ and excludes o-mode radiation at $\theta = 0$. Refraction subsequently causes θ to decrease. However, scattering sufficiently close to the plasma level, as proposed here, occurs before this strong forward collimation occurs. Thus most of the scattering of the initial o-mode radiation occurs at oblique angles $1 \gg \sin^2 \theta \gg 0$.

5.3. Predictions

Suppose that scattering by whistlers with $k''/\Delta k \gtrsim 1$ is the dominant depolarization mechanism. Then several predictions may be made. From (20) and (27) one finds $k''/\Delta k \cong (\Omega_e/\omega_p)^{1/2} (v/c)^{-1}$. One can satisfy $k''/\Delta k \gtrsim 1$ only if $v/c \lesssim (\Omega_e/\omega_p)^{1/2}$ is sufficiently small. Hence, one would expect *low* polarization to correspond to large values of $k''/\Delta k$, i.e., *low* polarization should correlate with (i) strong fields and (ii) low streaming speeds.

On the other hand the depolarization could be due to ion-sound waves. Then large-angle scattering and small-angle $o - x$ scattering are both possible. They involve ion sound waves in different frequency regimes, with large-angle scattering requiring $\omega''/\omega_{pi} \cong v_e/c$, and $o - x$ scattering requiring $\omega''/\omega_{pi} \cong (\Omega_e/\omega_p)^{1/2} V_e/c$. There is no direct information on such waves in the solar corona. For type III events in the interplanetary medium, strong scattering should correlate with the presence of such ion sound waves. It seems that type III events at kilometric wavelengths are always weakly polarized (unpublished Voyager data of A. Lecacheux) and this suggests that multiple scattering is always strong for type III bursts in the solar wind.

6. Conclusions

Depolarization mechanisms for solar radio bursts can be classified as due to refraction ('mode coupling'), reflection (off steep density gradients) and scattering (due to low-frequency waves). Here only the third of these has been considered. Three possibilities for reducing the polarization substantially have been identified: (1) a single-large angle scatter, (2) several $o - x$ scatters with $k'' \cong \Delta$k at $\omega \lesssim 1.5\omega_p$, and (3) many small-angle scatters.

The polarization of fundamental type I bursts can be explained in terms of (1), as has been discussed by Wentzel, Zlobec, and Messerotti (1986). The discussion here suggests that the degree of polarization (initially 100%) after one scattering through an angle χ

is better approximated by $p(\chi) = \cos\chi$ than by the value $p(\chi) = 2\cos^2\chi/(1 + \cos^2\chi)$ suggested by these authors. An interesting qualitative point that does not appear to have been made previously is that in a backscatter ($\chi > \pi/2$) the sense of polarization reverses, i.e., o-mode radiation becomes x-mode radiation. Although refraction back into the forward direction is possible in principle, thereby allowing x-mode radiation to be produced, it seems more likely that escaping radiation would involve an even number of backscatters and, hence, would always be in the same mode as at the point of generation.

For type II and type III bursts possibility (2) seems favourable. The required low-frequency waves can be whistlers generated by the streaming electrons themselves. Two predictions have been made based on this suggestion: if the scattering is due to whistlers then one expects *low* polarization to correlate with (i) strong fields and (ii) low streaming speeds. Possibility (3) cannot be ruled out. Very many small-angle scatters are required. The scattering could be due to ion sound waves with $(\omega_p/\Omega_e)^{1/2} V_e/c$.

In conclusion one point is worth emphasizing: it is very difficult to account in principle for complete depolarization. It seems that multiple scattering with each scattering reducing the polarization by some fraction is the only realistic possibility. The fact that unpolarized bursts are common at metric wavelengths, and seemingly the rule rather than the exception at kilometric wavelengths suggests that a typical ray experiences very many scatterings close to the point where it is generated. Thus, independent of other evidence for multiple scattering of solar radio waves, the low degree of polarization for some (but not all) fundamental plasma emission implies that multiple scattering occurs for weakly polarized emission.

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