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Comments on the photospheric dynamo model of Hénoux and Somov

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Summary. A photospheric dynamo model for the generation of coronal currents is defined to be one in which (a) the current is driven by flow of gas across field lines in the vicinity of the photosphere, by which we mean the region of weakly ionized plasma over a range of heights that includes the photosphere, and (b) the coronal current closes by flowing across the photospheric magnetic field lines due to the Pedersen or Hall conductivities of the weakly ionized plasma there. A detailed model for a photospheric dynamo has been presented by Hénoux and Somov (1987), who used the three-fluid model to treat the properties of the weakly ionized plasma. Only the equations for the two ionized components were solved. We consider the equation for the neutral component and argue that the model is unacceptable because of an implied impossibly large unbalanced stress on the neutral gas. We argue more generally that all existing photospheric dynamo models are untenable.

Key words: the Sun: flares – the Sun: atmosphere of – plasmas

1. Introduction

Hénoux and Somov (1987) have developed a model for the generation of a coronal current due to motion of the neutral gas in the photosphere across magnetic field lines. They likened their model to one developed earlier by Sen and White (1972); other related models have been proposed by Heyvaerts (1974), Obayashi (1975), and Kan et al. (1983). Such models may be referred to as *photospheric dynamo models* for solar flares. The basic idea in a photospheric dynamo model involves a supposed analogy between the photosphere-corona system and the ionosphere-magnetosphere system of the Earth; Melrose and McClymont (1987) have argued that this supposed analogy is ill-founded, and that all existing dynamo models for solar flares are untenable. This conclusion is incompatible with Hénoux and Somov's model. Our purpose here is to show that the source of this incompatibility lies in an inconsistency in the assumptions made by Hénoux and Somov.

The arguments given in this paper are specific to one class of models for coronal current generation. To avoid confusion in terminology, let us define this class of *photospheric dynamo models* as models for coronal current generation to have the following properties:

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1) The current generation mechanism relies on the electrical properties (the Pedersen or Hall conductivities) of the *weakly ionized plasma* in the vicinity of the photosphere. The current is driven by cross-field plasma flows, that is, the generation is assumed to involve the same physical processes as in an MHD generator in the laboratory (Mitchner and Kruger, 1973; Krall and Trivelpiece, 1973).

2) The electric circuit is as follows: the current flows across the field lines at the "active" footpoint, where the dynamo action is postulated to occur, upwards through the corona along one set of field lines, across the field lines at the other "passive" footpoint, and back through the corona along another set of field lines. Following Melrose and McClymont (1987), we maintain that existing photospheric dynamo models are untenable, and we demonstrate this explicitly for the model of Hénoux and Somov (1987).

An important point in the underlying physics is that the current is generated only when the neutral component flows across the field lines relative to the ionized component. We refer to this relative motion as the *slippage* of the neutral and ionized components. We follow Hénoux and Somov in using the three-fluid model for a weakly ionized plasma (Alfvén and Fälthammar, 1963) to describe the relative motions and the current generation. Our criticism of the treatment given by Hénoux and Somov is that they did not solve these equations consistently. Specifically they postulated the flow of the neutral component and did not solve the equation of motion of the neutral component. We insert the solutions obtained by Hénoux and Somov for the ionized components into the equation of motion for the neutral component and argue that this equation cannot be satisfied under relevant conditions. We conclude that the postulated flow of the neutral component relative to the ionized component is inconsistent: there is no available stress that is large enough to set up this slippage.

The arguments given here apply directly only to photospheric dynamo models. The photosphere is often regarded as a highly resistive lower boundary to the highly conducting corona; this is the case, for example, in general discussions of coronal current closure (Spicer, 1982), and in mathematical models for coronal magnetic loops (Low, 1985). The assumption that the photosphere is highly resistive needs to be reconsidered.

2. The stresses in the model

The photospheric plasma is modeled in terms of three components: electrons (e), singly charged ions (i), and neutrals (n).

Ignoring the gravitational acceleration, the equations of motion of the three components are (Alfvén and Fälthammar, 1963)

$$nm_e\frac{d\boldsymbol{v}_e}{dt} = -ne[\boldsymbol{E}+\boldsymbol{v}_e\times\boldsymbol{B}] - \text{grad}\,P_e + nf_e, \tag{1}$$

$$nm_i\frac{d\boldsymbol{v}_i}{dt} = ne[\boldsymbol{E}+\boldsymbol{v}_i\times\boldsymbol{B}] - \text{grad}\,P_i + nf_i, \tag{2}$$

$$n_n m_n\frac{d\boldsymbol{v}_n}{dt} = -\text{grad}\,P_n + nf_n, \tag{3}$$

where $n_e = n_i = n$ is the number density of electrons or ions, and where m_α, P_α and f_α denote the mass, pressure and frictional force for species $\alpha = e,\ i,\ n$. The frictional forces involve the collision frequencies $\nu_{\alpha\beta}$ between particles of species α and β:

$$\boldsymbol{f}_e = -m_e(\boldsymbol{v}_e-\boldsymbol{v}_n)\nu_{ne} - m_e(\boldsymbol{v}_e-\boldsymbol{v}_i)\nu_{ei}, \tag{4}$$

$$\boldsymbol{f}_i = -m_i(\boldsymbol{v}_i-\boldsymbol{v}_n)\nu_{ni} + m_e(\boldsymbol{v}_e-\boldsymbol{v}_i)\nu_{ei}, \tag{5}$$

$$\boldsymbol{f}_n = m_e(\boldsymbol{v}_e-\boldsymbol{v}_n)\nu_{ne} + m_i(\boldsymbol{v}_i-\boldsymbol{v}_n)\nu_{ni}. \tag{6}$$

The current density is given by

$$\boldsymbol{J} = -en(\boldsymbol{v}_e-\boldsymbol{v}_i). \tag{7}$$

Hénoux and Somov (1987) considered a model that allowed them to calculate the cross-field current. It is not explicit in their model how the current closes, but this is explicit in other dynamo models (e.g., Heyvaerts, 1974; Kan et al., 1983). The calculated current across the field lines in the photosphere at one footpoint of a coronal flux tube is assumed to close by flowing along field lines through the corona, across the field lines at the other footpoint, and back along other coronal field lines. Hénoux and Somov (1987) argued that the coronal current may be calculated by first solving the foregoing equations to find $\boldsymbol{J}$. Their procedure for solving the equations is as follows:

1) Identitfy the electric field $\boldsymbol{E}$ as that associated with the cross-field flow in the convective zone; if $\boldsymbol{v}^c$ is the cross-field flow velocity in the convective zone, their argument gives $\boldsymbol{E} = -\boldsymbol{v}^c\times\boldsymbol{B}$.

2) Assume stationary flow so that the left hand sides of both (1) and (2) are zero.

3) Solve (1) and (2) for $\boldsymbol{v}_e$ and $\boldsymbol{v}_i$ in terms of $\boldsymbol{v}^c$ (only the cross-field components are relevant).

4) Insert the resulting solutions in (7) to find $\boldsymbol{J}$.

Our main comment on Hénoux and Somov's treatment is that their procedure ignores equation (3). The solution obtained for $\boldsymbol{v}_e$ and $\boldsymbol{v}_i$ allows one to determine f_n explicitly. Their static solution is physically acceptable only if the implied frictional force in (3) can be balanced by the cross-field pressure gradient in (3). Alternatively the frictional force could be balanced by the inertial term, implying a timescale for acceleration of the neutral component due to it being dragged along by the ionized component.

3. Detailed analysis

We evaluate f_n, and equate $nf_n/n_n m_n$ to $d\boldsymbol{v}_n/dt$, for some models of the solar atmosphere. Hénoux and Somov (1987) separated into radial (r) and polar (θ) components in the photosphere. Their solution leads to the following expression for $d\boldsymbol{v}_n/dt$:

$$\frac{dv_{r,n}}{dt} = \sum_{\alpha=e,i} \nu_{n\alpha}\frac{nm_\alpha}{n_n m_n}\frac{1}{\Omega_\alpha^2+\nu_{n\alpha}^2} \times[\Omega_\alpha^2(v_r^c - v_{r,n}) - \eta\varepsilon\Omega_\alpha\nu_{n\alpha}(v_\theta^c - v_{\theta,n})], \tag{8}$$

$$\frac{dv_{\theta,n}}{dt} = \sum_{\alpha=e,i} \nu_{n\alpha}\frac{nm_\alpha}{n_n m_n}\frac{1}{\Omega_\alpha^2+\nu_{n\alpha}^2} \times[\Omega_\alpha^2(v_\theta^c - v_{\theta,n}) + \eta\varepsilon\Omega_\alpha\nu_{n\alpha}(v_r^c - v_{r,n})], \tag{9}$$

where η is the sign of the charge and ε is a sign determined by the polarity of the magnetic field.

The model adopted for the solar atmosphere is the Harvard-Smithsonian Reference Atmosphere (HSRA) (Gingerich et al., 1971) matched with the solar convection zone model of Spruit (1974). Some details of these models might be questioned. For example, Spruit's convection zone model is not consistent with more recent models such as those of Vernazza et al. (1981), specifically in the choice of atomic data. However, differences in the details of models have no effect on our conclusions.

To evaluate $|d\boldsymbol{v}_n/dt|$ from (8) and (9), we require values of n_e, n_n, m_i, m_n, $\nu_{n\alpha}$ and Ω_α as functions of height h in the atmosphere. The models give the variation of the temperature T, the electron pressure P_e and the total "gas" pressure P_g as functions of h. The number density of electrons is identified as $n_e = P_e/k_B T$, where k_B is Boltzmann's constant, and the total number density of particles n_t is similarly identified as $n_t = P_g/k_B T$. The determination of the number densities of neutral and ionized components of hydrogen and other atomic species involves using the Saha-Boltzmann equation, and including some non-LTE effects; our procedure is discussed in the Appendix. For simplicity the distribution of ionic species is replaced by an equivalent "mean" ionic species with mass m_i and number density n_i, as explained in the Appendix. Similarly, the distribution of neutral species is represented by an equivalent "mean" neutral component with mass m_n and number density n_n.

The collision frequency of a charged particle of species α with a neutral atom is given by (e.g., Krall and Trivelpiece, 1973)

$$\nu_{n\alpha} = n_n\sigma_{n\alpha}\left(\frac{k_B T_\alpha}{m_\alpha}\right)^{1/2}, \tag{10}$$

where $\sigma_{n\alpha}$ is the appropriate cross section. We assume that the temperatures T_α of all species have the same value T. The values used for the cross section are taken from data presented by Spencer and Phelps (1976) and Hunter and Kuriyan (1977), as discussed in the Appendix. The results are reduced to an effective collision frequency between the mean ionized and neutral components, and an effective collision frequency between electrons and the mean neutral component. Following Hénoux and Somov (1987) the effect of electron-ion collisions is neglected.

The value of the gyrofrequencies Ω_α depend on the strength of the magnetic field; we consider two values, $B = 0.1$ T and $B = 0.2$ T, and neglect any variation with height. In evaluating the ion gyrofrequency the mean ion mass is used.

In our numerical calculations the flow velocities are assumed to have the values $|(v_\theta^c - v_{\theta,n})| = 10^2\ \text{m s}^{-1}$, and $|(v_r^c - v_{r,n})| = 0$ and we set $\varepsilon = 1$. Equations (8) and (9) are solved over the range of heights where the degree of ionization is less than one half, i.e., $n_i/(n_n+n_i) < \frac{1}{2}$. The value of $|d\boldsymbol{v}_n/dt|$ is determined from

$$|d\boldsymbol{v}_n/dt| = [(dv_{r,n}/dt)^2 + (dv_{\theta,n}/dt)^2]^{1/2}.$$

According to (3) the value of $|dv_n/dt|$ either represents the acceleration of the neutral gas, or it represents a frictional force per unit mass that must be balanced by a pressure gradient. The results are shown in Fig. 1.

The implied acceleration of the neutral component is very large. Over a range of heights from $h \approx 0$ to $h = 700$ km, where the photospheric dynamo is usually postulated to be situated, $|dv_n/dt|$ is $\approx 5\,10^4$ to 10^5 m s^{-2}. If this is an unbalanced acceleration then it would alter the observed flows ≈ 1 km s^{-1} on the absurdly short timescale of order 10 to 20 ms.

If, alternatively, the frictional force is assumed to be balanced by a pressure gradient, then the required characteristic scale length L_n for cross-field pressure changes would be

$$L_n = P_n/|\mathrm{grad}\, P_n| = V_n^2/(nf_n/n_n m_n), \tag{11}$$

where $V_n = (k_B T/m_n)^{1/2}$ is the thermal speed of the atoms. We plot $P_g/|\mathrm{grad}\, P_n|$ versus h in Fig. 2, with $|\mathrm{grad}\, P_n|$ set equal to the implied value of the frictional force per unit volume nf_n. The scale lengths implied by Fig. 2 are absurdly short, requiring the pressure to change substantially over about kilometers to as short as a meter, depending on the height at which the dynamo is postulated to operate. Such pressure gradients could be set up only in explosive events, and could be maintained only on a timescale L_n/V_n, which is just the 10 to 20 ms already estimated.

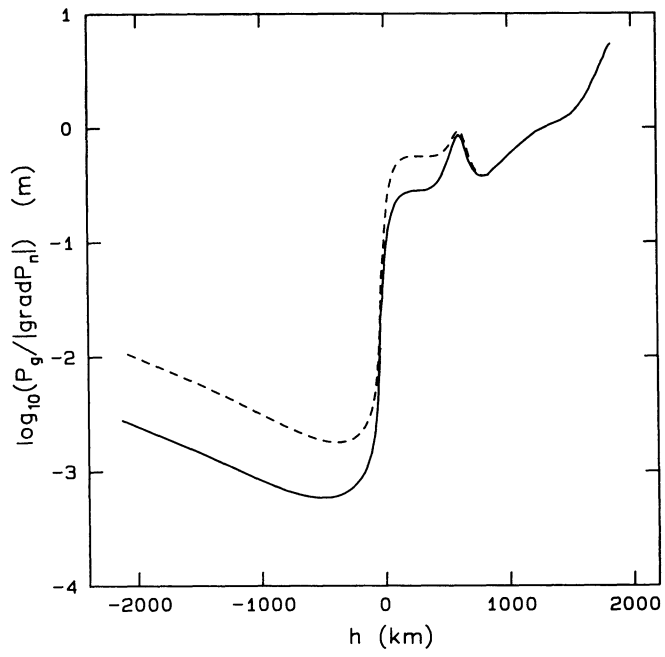

Fig. 2. The variation of $P_g/|\mathrm{grad}\, P_n|$ with height h with $|\mathrm{grad}\, P_n|$ set equal to nf_n; dashed line for $B = 0.1$ T, solid line for $B = 0.2$T

4. Conclusion

Hénoux and Somov (1987) solved the equations of motion for the ionized components on the assumption that the ionized gas is in equilibrium and that there is a specified cross-field flow of the neutral gas. We have inserted their solution into the equation of motion for the neutral gas, calculated the implied frictional force on the neutral component and found that it is extremely large. This result implies either an extremely rapid acceleration of the neutral gas due to the frictional drag by the ionized gas, or an extremely large stress on the neutral gas (an extremely large cross-field density gradient) to balance this frictional drag. The calculated acceleration would imply a timescale of tens of milliseconds for a cross-field photospheric flow velocity of ≈ 1 km s^{-1} to be stopped. The calculated pressure gradient would imply a substantial change in the pressure of the *neutral* gas in a few kilometers across the field lines. Such extreme conditions are not envisaged in Hénoux and Somov's model or in any existing photospheric dynamo models.

Photospheric dynamo models, other than that of Hénoux and Somov (1987), include those of Sen and White (1972), Heyvaerts (1974), Obayashi (1975), and Kan et al. (1983). The arguments given here may be applied to all these models. In brief, the electrodynamic stress imposed when there is any substantial slippage between the ionized and neutral components (e.g., $|v_n - v_i| \approx 1$ km s^{-1}) is enormous, and such relative flows are stopped effectively instantaneously. Thus models in which such flows are *postulated* to be the cause of dynamo action in the photosphere involve an inconsistent initial condition. Specifically, the postulated flow of neutral gas relative to ionized gas across field lines simply cannot be set up.

We conclude that existing photospheric dynamo models are untenable. It should be emphasized that this conclusion applies to photospheric dynamo models in which the following assumption is essential (cf. the Introduction): The coronal current (involved in the flare) closes by flowing across the field lines in the vicinity of the photosphere due to the conductivity (the Pedersen or Hall conductivities) of weakly ionized plasma there. We maintain that any special electrical properties of the photospheric plasma are unimportant in relation to coronal current generation and closure. Our arguments refer only to the Pedersen or Hall conductivities, and our conclusions that the electrical resistance of the photospheric plasma is unimportant is derived only for the effects

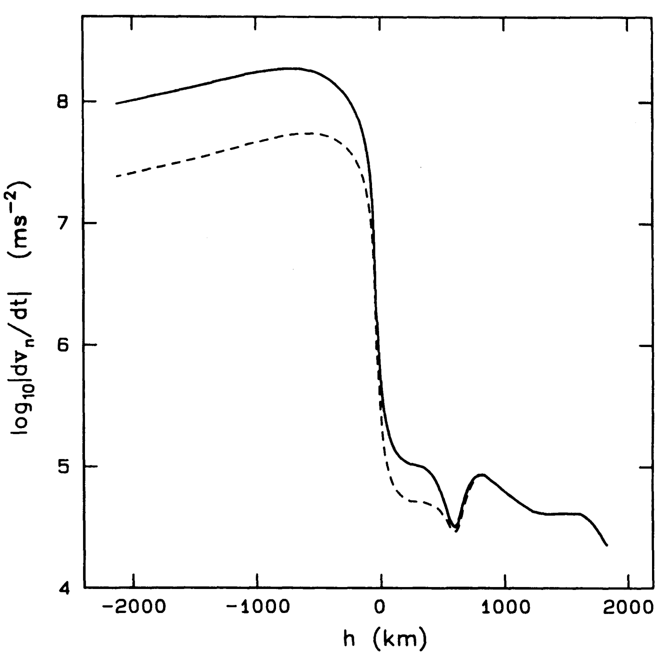

Fig. 1. The variation of $|dv_n|/dt$ with height h assuming $B = 0.1$ T (dashed line) and $B = 0.2$ T (solid line)

of these specific forms of conductivity. Hudson (1987) has discussed current closure across field line due to the Spitzer conductivity. Also a referee has pointed out that our arguments do not apply to conductivity due to inertial terms.

The detailed argument given here implies that photospheric dynamo models that invoke the Pedersen or Hall conductivity are untenable, and the present discussion supports the more general arguments against such photospheric dynamo models given by Melrose and McClymont (1987). The location of the cross-field current closure need to be considered more carefully in all models, including those that appeal to storage of energy in the corona or in a larger electrical circuit that includes the corona and regions below the photosphere. Our arguments imply that coronal currents do not close by flowing across the field lines in the vicinity of the photosphere due to the conductivity of the weakly ionized gas there. A similar conclusion is reached by Hudson (1987) for closure due to the conductivity of the ionized component of the photospheric plasma. Coronal currents, on scales of relevances to flares and stressed loops, must close deep in the solar atmosphere, and photospheric motions must reflect motions at such depths where cross-field currents are significant.

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Appendix A

Given in this Appendix are details of (a) the determination of the fractional ionization of various species of atoms in the weakly ionized part of the solar atmosphere, (b) the calculation of a "mean" ionic component to simulate the actual distribution of ions, and (c) the cross section for collisions between ionized and neutral particles.

A.1. Degrees of ionization

As explained in Sect. 3, the chosen model for the solar atmosphere gives the electron pressure P_e and the total "gas" pressure P_g, and one finds the number density of electrons from $n_e = P_e/k_B T$ and the total number density of particles from $n_t = P_g/k_B T$. The species of particle are assumed to consist of neutral atoms, electrons and singly ionized positive ions. We use the data in the HSRA model in which ten elements are retained. The total number density of hydrogen is given by

$$n_H = \frac{(n_t - n_e)}{\Sigma_{\xi=1}^{10} A_\xi}, \tag{A1}$$

where A_ξ is the abundance of element ξ relative to hydrogen. Element $\xi = 1$ is hydrogen and so one has $A_1 = 1$. The number density of other species is given by

$$n_\xi = A_\xi n_H. \tag{A2}$$

Local thermodynamic equilibrium (LTE) is assumed for all species other than hydrogen. A non-LTE correction is included for hydrogen assuming a one-level atom. The fractional degree of ionization $f_\xi = n_{\xi i}/n_{\xi 0}$ is determined by the Saha-Boltzmann ionization equation, with the number densities $n_{\xi i}$ and $n_{\xi 0}$ of the ionized and neutral components, respectively, summing to n_ξ. The equation may be written in the form

$$f_\xi = \frac{n_{\xi i}}{n_{\xi 0}} = \exp\left[-\frac{\chi_\xi - \chi_{\xi\mathrm{eff}}}{k_B T}\right], \tag{A3}$$

where χ_ξ denotes the first ionization potential of atom ξ. The "effective potential" $\chi_{\xi\mathrm{eff}}$ is given by

$$\chi_{\xi\mathrm{eff}} = k_B T\left[\ln\left(2\frac{U_{\xi i}}{U_{\xi 0}}\right) + \frac{3}{2}\ln\left(\frac{2\pi m_e k_B T}{h^2}\right) - \ln n_e - \ln b_\xi\right]. \tag{A4}$$

The ratio $U_{\xi i}/U_{\xi 0}$ of the partition functions for the first ionized and neutral states is given by the HSRA model. The only non-LTE correction included is b_1, and we use the values tabulated by Gingerich et al. (1971). In this way we calculate

$$n_{\xi i} = n_\xi \frac{f_\xi}{1+f_\xi}, \quad n_{\xi 0} = \frac{n_{\xi i}}{f_\xi} \tag{A5}$$

at each height.

In the vicinity of the photosphere the dominant ions are heavy ions such as iron, magnesium and silicon. Outside this region protons dominate.

A.2. The "mean" ionic species

The actual distributions of ions and neutrals are simulated by a mean ionic component with number density n_i, mass m_i and a mean neutral component with number density n_n and mass m_n. These are determined by

$$n_i = \sum_{\xi=1}^{10} n_{\xi i}, \quad m_i = \sum_{\xi=1}^{10} \frac{n_{\xi i} m_\xi}{n_i},$$
$$n_n = \sum_{\xi=1}^{10} n_{\xi 0}, \quad m_n = \sum_{\xi=1}^{10} \frac{n_{\xi 0} m_\xi}{n_n}. \tag{A6}$$

A.3. Collisional cross sections

The collision frequencies are given by (10) in which the cross section $\sigma_{n\alpha}$ is that for momentum transfer. We assume that the neutral component is dominated by hydrogen and consider only collisions between ionized particles and neutral hydrogen.

For electron-hydrogen collisions, the data of Spencer and Phelps (1976) in the range 0.17 to 2.8 eV are approximated by the logarithmic fit:

$$\left(\frac{\sigma_{ne}}{10^{-19}\,\mathrm{m}^2}\right) = a + b\ln\left(\frac{\varepsilon}{\mathrm{eV}}\right), \tag{A7}$$

with $a = 2.48$ and $b = -0.815$, and where ε (set equal to $\frac{3}{2}k_B T$) is the energy of the electron.

For ion-neutral collisions, the cross section is determined completely by charge exchange. For resonance charge-exchange proton-hydrogen collisions the data of Hunter and Kuriyan (1977) in the range 0.1 to 2.7 eV are approximated by a power-law fit:

$$\left(\frac{\sigma_{ni}}{\pi a_0^2}\right) = p\varepsilon^q, \tag{A8}$$

with ε the proton energy (set equal to $\frac{3}{2}k_B T$) in electron volts, and with $a_0 = 5.29\,10^{-11}$ m, $p = 103$, and $q = -0.124$. We are aware of no data on the heavy ion-hydrogen cross sections for energies of relevance to this problem. For such collisions we assume that the cross section is given by $\pi(r_H+r_\xi)^2$, where r_H and r_ξ are the atomic radius of hydrogen and of the heavy atom, respectively. Data on these radii are taken from Allen (1973). The actual cross section used is the weighted mean, in the sense of (A6), of the cross sections for different species.

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Note added in proof: In Fig. 2 the label of the units of the abscissa should read km not m.