Two Models for the Radio Spectra of Stellar Flares

R. J. Beardwood and D. B. Melrose, *School of Physics, University of Sydney, NSW 2006.*

Abstract: There are two current models for the typical radio spectra of flare stars, which have a broad peak around a frequency typically 1–10 GHz. One model involves both optically thick and optically thin emission, with the turnover attributed to the source becoming self-absorbed. The other model involves only emission at optical depth unity, so that observing different frequencies corresponds to 'seeing' different regions in the source. The two models are reviewed, and new calculations for an inhomogeneous source presented.

1. Introduction

Radio emission from close binaries frequently shows a flare morphology. Mutel *et al.* (1987) discussed the spectral properties of radio flares from RS CVn's, and found that the frequency spectra have a broad maximum in the vicinity of 5 GHz (Figure 1). There is also a moderate degree ($\leq 30\%$) of circular polarisation. Observations of AE Aquarii reported by Bastien *et al.* (1988) yield spectra which, although fluctuating in time, rise on the average as $\omega^{0.3}$. No appreciable circular polarisation was detected. The emission mechanisms thought to be responsible are gyrosynchrotron and synchrotron, respectively.

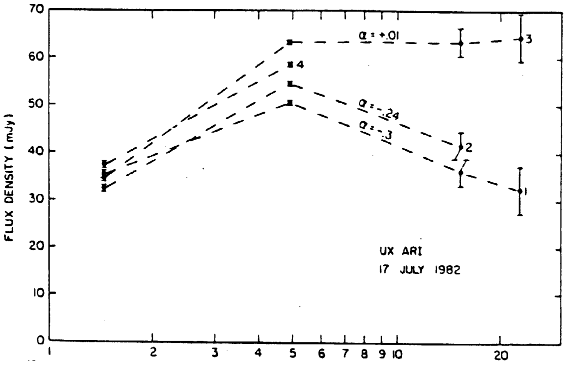

Figure 1 – Radio spectra from UX Ari. at successive times (Mutel *et al.* 1987).

We consider the case of synchrotron emission due to ultrarelativistic electrons with a power-law energy distribution

$$N(E) \propto E^{-a} \tag{1}$$

in a region permeated by a magnetic field (Melrose 1970, 1972; Pacholczyk 1970). Following Melrose (1972 – henceforth referred to as a Paper I), the equation of radiative transfer may be written

$$\frac{\partial}{\partial t}\begin{bmatrix} I \\ Q \end{bmatrix} = \varepsilon \begin{bmatrix} 1 \\ r_l \end{bmatrix} - \kappa \begin{bmatrix} 1 & \zeta \\ \zeta & 1 \end{bmatrix} \begin{bmatrix} I \\ Q \end{bmatrix}, \tag{2}$$

where I and Q are the Stokes parameters describing the total intensity and the linear polarisation, r_l and ζ depend only on power-law index a, and ε and κ are the emission and absorption coefficients, determinable from synchrotron theory. For this discussion, only the frequency dependences of ε and κ are of interest:

$$\varepsilon \propto \omega^{-(a-1)/2}, \tag{3}$$

$$\kappa \propto \omega^{(a+4)/2}. \tag{4}$$

For low frequencies, the intensity goes like ε/κ, so

$$I \propto \omega^{5/2}, \tag{5}$$

while for optically thin emission, the ε term dominates, and

$$I \propto \omega^{-(a-1)/2}. \tag{6}$$

For a homogenous source bounded front and back by parallel planes, equation (2) is readily integrated, and spectra for $a = 2$, 3 and 4 are shown in Figure 2. The peak is much sharper than those obtained by observation. In this paper we consider source models which generate broader peaks in the synchrotron spectra.

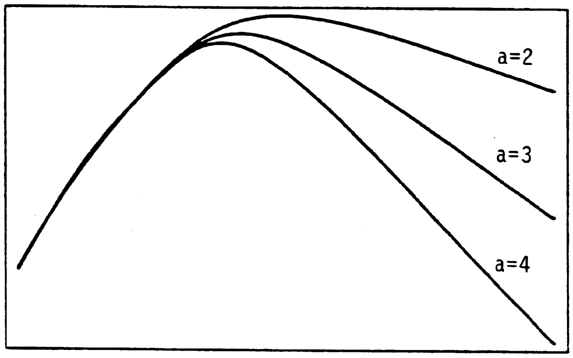

Figure 2 – Synchrotron spectra, calculated for a homogeneous source with uniform magnetic field, bounded by parallel planes. Logarithms have been taken of both intensity and frequency, so the units are arbitrary, and the axes have been omitted for clarity. The optically thick branches coincide, but the optically thin branches have different slopes determined by the power-law index.

2. An Optically Thick Model

Nelson and Stewart (1979) suggested a model to explain the microwave spectra of extended solar bursts. In these spectra, the intensity rises on the average according to $I \propto \omega$, and reaches a maximum at a frequency ≥ 10 GHz. The proposed source model consists of a hemispherical region of homogeneous plasma, permeated by a dipolar magnetic field which extends inwards to a maximum field B_m at the centre of the source. Gyrosynchrotron radiation is emitted by power-law, mildly relativistic electrons. Nelson and Stewart further assumed that, to a good approximation, all the emission comes from a region in which the optical depth is unity; in other words, at a given frequency, only a thin shell within the source – at which the intensity is a maximum – contributes to the emission (Figure 3). For lower frequencies, the shells lie nearer the surface and are larger, so contain more electrons. Thus, on the optically thick side of the spectrum, although the intensity falls away for lower frequencies, the decrease is countered by the extra numbers of electrons contributing. The slope is then less than that for a source in which the shells all have the same number of electrons (as in a cubic model). The calculations presented give $I \propto \omega^{0.73}$, which is in satisfactory agreement with the observed frequency dependence.

The turnover in the microwave spectra is attributed to the emission becoming optically thin when, at high frequencies, the contributing plasma lies at the centre of the source where the

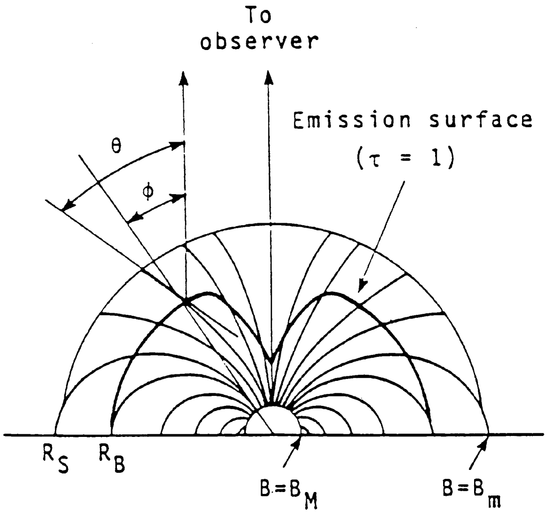

Figure 3—Nelson and Stewart's hemispherical source, showing a thin shell in which, for some particular frequency, the optical depth is unity.

field attains its maximum value. However, details of the calculation which generates the turnover are not presented. Slee *et al.* (1987) calculated spectra for a source in which both the magnetic field and the electron density fall off as some power of the distance to the source centre; again, these spectra rise slowly with frequency, without turning over.

3. Inhomogeneous Sources

The new work presented in this paper follows on from Paper I in calculating synchrotron spectra from power-law electrons in sources more complicated than the plane-parallel slab model. Contributions at all frequencies at each point are included, and the radiative transfer equation is integrated through the source. A series of spherical models is considered, in which the electron density varies as

$$n(r) \propto r^{-m}. \qquad (7)$$

It is desirable that as much of the calculation as possible be performed analytically, so the same, uniform magnetic field is used for each model; also, only values of $m=0$, 1 and 2 are considered here. Figure 4 shows a sequence of spectra, overlaid for comparison, showing the transition from a cubic to a spherical model with the electron density increasingly concentrated at the centre. The power-law index is the same for each curve, so the optically thin asymptotes are the same, and are overlaid; the peak clearly becomes increasingly broad with increasing m. As the frequency decreases, the intensity tends to $I \propto \omega^{2.5}$, and the spectra become parallel.

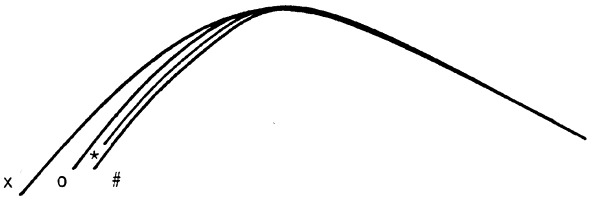

Figure 4—Intensity spectra for cubic and spherical sources with the electron density falling away from the centre of the sphere. Curves labelled x, o, and * are spherical sources with $m=2$, 1 and 0 respectively; # is a cubic model. The value of a is 3 for each curve.

Clearly, the fall off in electron density away from the centre of a synchrotron-emitting region of plasma contributes to the broadness of the peak in intensity. Presumably a fall off in the magnetic field would have a similar effect; unfortunately, this problem is not amenable to an analytic approach. Robinson (1974), for instance, carried out a numerical treatment of a source with a cylindrically symmetric magnetic field which decays exponentially away from the source centre. Robinson showed that the optical depth depends on the polarisation mode, but did not present any spectra in his paper.

Solving equation (2) also yields the degree of linear polarisation

$$\pi_l = Q/I. \qquad (8)$$

Figure 5 shows, for a homogeneous, spherical source, the dependence of π_l on frequency, for $a=2$, 3 and 4. As noted in Paper I, the degree of linear polarisation changes sign (corresponding to the plane of polarisation rotating through 90°) at a frequency for which the optical depth is greater than unity. As with the intensity plots, the degree of linear polarisation does become a flatter function of frequency as the electron density becomes more concentrated at the centre of the source (Figure 6). However, the value of π_l is probably not of relevance interpreting observational data; any linear polarisation generated in the source is either washed out or significantly distorted by the Faraday effect as the radiation propagates through overlying plasma. On the other hand, circular polarisation is little altered by propagation effects, and observations of RS CVns show the helicity to reverse in the broad region where the spectrum turns over. Modelling this behaviour entails the following two steps.

1. Calculate the Stokes parameter V.
2. Use gyrosynchrotron theory; the degree of circular polarisation is a first order term in an expansion in γ^{-1} (γ is the electron Lorentz factor) and so is dominated by the contribution from mildly relativistic electrons.

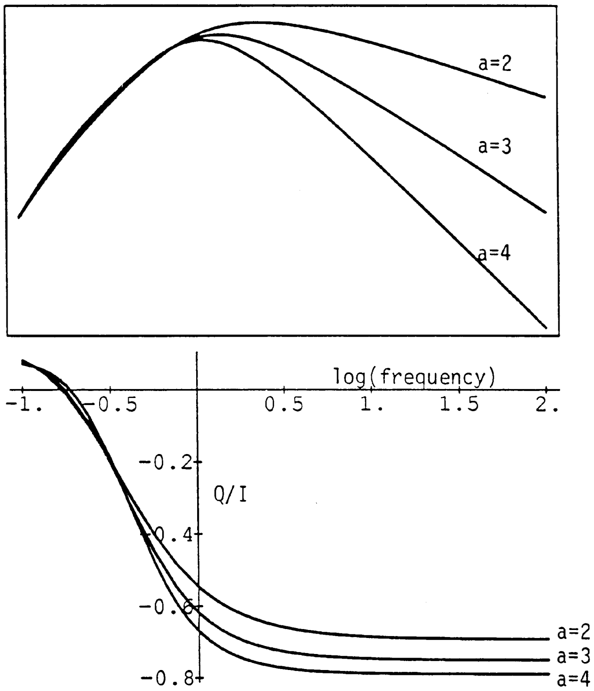

Figure 5—The intensity and linear polarisation for a homogeneous spherical source.

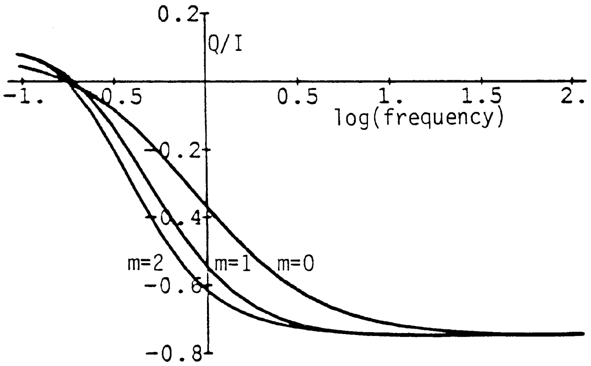

Figure 6—The degree of linear polarisation for spherical sources as a function of frequency; $a = 3$ for each curve.

The model reported here has yet to be extended to include circular polarisation.

4. Conclusion

Calculations presented in this paper show that the peak in the synchrotron frequency spectrum is broadened if the electron density in a source decreases away from its centre. The plane of linear polarisation rotates through 90° at a frequency less than that at which the intensity peaks, an effect which Faraday depolarisation makes unlikely to be observed. However, further work on modelling inhomogeneous sources using gyrosynchrotron theory may explain the reversal in the sense of circular polarisation observed in RS CVn spectra.

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