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The Energy Release in Solar Flares: Implications of a Constant-Current Model

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Abstract: A model is explored for energy release in solar flares that involves a constant coronal current. An emerging flux tube is assumed to carry a current $I \lesssim 10^{12}$ A, and this current is assumed not to change during flare. Using a circuit model, explosive energy release is attributed to a rapid rise in the coronal resistance R_c, which must adjust to $R_c = -\dot{L}_c$, with $\dot{L}_c$ the rate of change of the coronal inductance L_c, to ensure I = constant. In this model the total energy released in the corona is twice the change in the magnetic energy stored in the corona. It is argued that this energy is inadequate to power a large flare and the implications of this conclusion are discussed.

1. Introduction

Existing models of the energy release in solar flares based either on magnetic storage in the corona or on a direct power transfer from a photospheric dynamo seem incapable of accounting for the energy released in a large flare (e.g., Spicer 1982, McClymont and Fisher 1989). An alternative type of model that seems capable of overcoming the energetic difficulties is the 'emerging flux tube model' in which the electric current that powers the flare is flowing in the flux tube when it first emerges from below the photosphere. In this paper a 'constant-current' version of this model is proposed and explored. The constant-current version is also found to be inadequate to power a large flare, and this has some interesting implications for the parameters required to account for a large flare.

The general ideas motivating the present investigation are outlined in §2. The value of the circuit parameters are discussed in §3, and the plausibility of the constant-current assumption is discussed in §4. The energy release at constant current necessarily involves a change in the inductance, as discussed in §5. The energy available is found to be inadequate for a large flare, and the implications of this are discussed in §6.

2. A Specific Circuit Model for Flares

The approach adopted here is based on an electric circuit analogue for solar flares, e.g., Alfvén and Carlqvist (1976), Alfvén (1977), Spicer (1982), Melrose (1987). In the circuit analog the energy dissipation is attributed to the power $P_c = R_c I^2$ where R_c is the resistance of the flaring coronal flux tube along which the current I is flowing. The current is restricted to $I \lesssim 10^{12}$ A in order that the magnetic field due to the current not exceed the field required to confine the current. Currents of this order are implied by the observed field in active regions (e.g., Moreton and Severny 1968, Krall *et al.* 1982, Gary *et al.* 1987). For the purpose of discussion here the value $I = 10^{12}$ A is adopted. The resistance required to account for a power $P_c \approx 10^{22}$ W released in the implusive phase of a flare is very large ($R_c \approx 10^{-2}$ ohm). One needs to invoke anomalous resistivity in thin current channels to account for such a large coronal resistance (Melrose and McClymont 1987).

The following simple idea for a model of solar flares is proposed:

(i) Under normal (non-flaring) circumstances the resistance of a coronal flux tube is small, specifically $R_c \ll 10^{-2}$ ohm.
(ii) Some trigger causes the onset of a plasma instability that leads to anomalous resistivity causing R_c to increase to $\approx 10^{-2}$ ohm in a flaring flux tube.
(iii) The power release in the coronal part of the flux tube is identified as $R_c I^2$.

This simple idea raises many questions that need to be addressed before it can form the basis of a viable model of flares.

One specific aspect of the model concerns the source (the 'dynamo') of the current I, and the circuit in which it flows. At least part of the circuit is below the photosphere and is not amenable to direct observation. Different flare models imply different forms of subphotospheric closure for the coronal current. Photospheric dynamo models (e.g., Kan, Akasofu and Lee 1983, Hénoux and Somov 1987, but cf. Melrose and Khan 1989) imply closure in the photospheric regions due to the conductivity of the partially ionized plasma there. Current closure is not usually discussed in the context of the more widely favored 'twisted flux tube model' (e.g., Sturrock 1980); presumably closure is envisaged to occur where the plasma β is of order unity. In the emerging flux tube model, which is adopted here, the current I is already flowing when the flux tube emerges from below the photosphere. In this case current closure is connected with the solar dynamo, and presumably occurs at the bottom of the convection zone and may involve a circuit around the Sun deep in the solar atmosphere.

3. Circuit Parameters

With the dynamo region deep in the solar atmosphere, the coronal segment of the circuit in which the current flows is only a small part of a much larger circuit. The overall circuit extends at least from one footpoint to another in the convection zone, and it may close around the Sun. A simple circuit analogue is illustrated in Figure 1.

In the circuit illustrated in Figure 1 it is assumed that the current I is given by an EMF V (the dynamo). The flux tube has three sections, two corresponding to the regions below the photosphere, each with inductance $\frac{1}{2}L_o$ and resistance $\frac{1}{2}R_o$, and a coronal section with inductance L_c and resistance R_c. (A capacitance C of the corona needs to be added to allow discussion of mass motions or oscillations, but this complication is ignored here.) The circuit equation for this model is

$$\frac{d}{dt}[(L_0 + L_c)I] + (R_0 + R_c)I = V.$$

The parameters L_o, R_o, V may be estimated as follows.

For present purposes it suffices to suppress the details of the geometric configuration and to write $L = \zeta\mu_o l$ for the inductance of a current section of length l, where ζ is a parameter that depends on the geometry. Here $\zeta = 0.1$ is assumed. The inductance L_o is determined by the depth d to which the flux tubes extend below the photosphere. For a flux tube that extends to the bottom of the convection zone, one has $d \approx 2 \times 10^8$ m (e.g., Spruit 1981). This assumed depth of the EMF implies $L_o \approx 40$ H. If the circuit closes around the Sun, then the value of L_o is about a factor of 10^2 larger than this value.

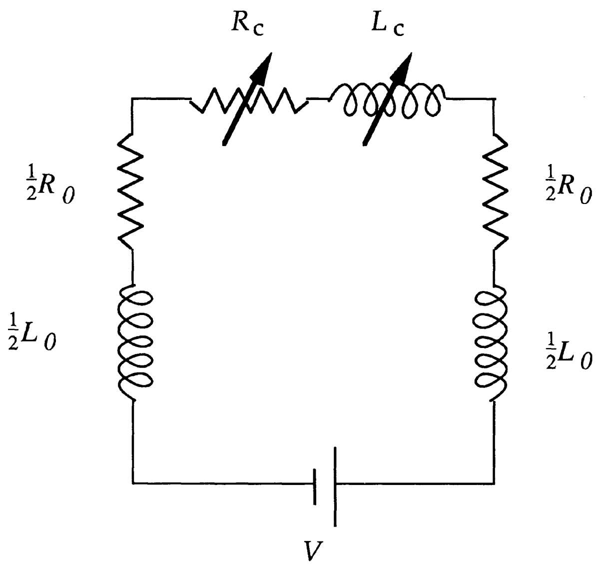

Figure 1—An idealized circuit for a magnetic flux tube that extends from the base of the convection zone into the corona and back to the base of the convection zone. The arrows indicate the assumed variability of the coronal inductance and resistance.

Consider the situation when the flux tube is about to emerge from below the photosphere. Then one has $L_c=0, R_c=0$. The current flowing is I, and Ohm's law implies that the resistance R_0 is related to the EMF V by $R_0=V/I$. An estimate of V may be made by considering the maximum potential that becomes available in the corona. The power released in a flare, which in a large flare can exceed 10^{22} W, cannot be greater than the total power VI dissipated in the circuit. This implies $V \gtrsim 10^{10}$ V. Amongst the first to recognize the need for such a large potential drop to drive a flare was Colgate (1978).

This estimate of V refers only to the voltage that appears in the corona. If R_o is much greater than the maximum value of R_c then the maximum voltage that appears in the corona is smaller than the EMF by the ratio of the maximum value of R_c to R_o. The constant-current assumption requires $R_c \ll R_o$, and hence the available voltage in the circuit needs to be much larger than 10^{10} V for the constant-current model to apply. (This does not affect the estimate of the power made above because only the potential across the coronal part of the circuit contributes to the power dissipated in the corona.)

4. The Constant-Current Assumption

The idea behind the constant-current assumption is that it has been considered plausible that the resistance of the corona during a flare remains much smaller than the resistance in the remainder of the circuit. For example, Heyvaerts (1974) suggested that for a large internal dynamo resistance compared to its external load, the dynamo behaves as a current generator, and Spicer (1982) also mentioned a constant-current dynamo. The constant-current assumption applies for $R_c \ll R_o$ in the circuit illustrated in Figure 1.

The assumption $R_c \ll R_o$ implies only that the current remains approximately constant. Here the stronger assumption that *the current remains strictly constant* is made. This stronger constant-current assumption is consistent with (1) provided that one has

$$R_c = -\dot{L}_c. \qquad (2)$$

The condition (2) requires that the coronal resistance adjust to the value determined by the rate of change of the coronal inductance. Such an adjustment is plausible if the resistance is determined by an anomalous resistivity that relies on a threshold current density being exceeded. Then the resistivity is due to a driven process, and the rate of dissipation should adjust to that required by the driver, e.g., as discussed by Duijveman, Hoyng and Ionson (1981) in a related context.

Ideally, if the condition (2) is strictly satisfied then the condition $R_c \ll R_o$ is not necessary for the current to remain constant. Conversely, for $R_c \ll R_o$ the condition (2) is not necessary for the current to remain approximately constant. However, for the constant-current assumption to be plausible, it is reasonable to expect that (2) is satisfied at least approximately, and also that R_c/R_o is small in some meaningful sense.

5. Energy Release in the Constant-Current Model

The energy released in a constant-current model for a flare is attributed to a change in the coronal part of the inductance (Spicer 1982, Zuccarello *et al.* 1987). The energy released in the corona ΔE follows by integrating the power $R_c I^2$ over time:

$$\Delta E = \int dt\, R_c I^2 = -I^2 \int dt\, \dot{L}_c = -\Delta L_c I^2,$$

where ΔL_c is the change in the coronal inductance. Energy is released if the coronal inductance decreases.

Let L_{c1} and $L_{c2}=L_{c1}+\Delta L_c$ be the coronal inductance before and after the flare. Then the magnetic energy stored changes from $\frac{1}{2}L_{c1}I^2$ to $\frac{1}{2}L_{c2}I^2$, and the change in the magnetic energy is $\frac{1}{2}\Delta L_c I^2$. Thus the energy released in the corona is equal to twice the change in the magnetic energy stored in corona. Only half the energy released comes from the stored magnetic energy. Discussion of the source of the other half of the energy released is not possible within the context of the circuit equation alone. (The circuit equation does not permit an energy integral if the inductance is a function of time.)

6. Discussion

The constant-current assumption leads to a possible energy release that is comparable with that in the twisted flux tube model, that is, comparable with the energy stored in the corona. This energy is inadequate to account for a large flare (Spicer 1982, Xue and Chen 1983, McClymont and Fisher 1989). It follows that if one is to account for the energy release in a large flare then the constant-current model is too restrictive. Specifically, an energetically viable model, at least for the most energetic flares, must release in the corona energy that is stored at subphotospheric heights. This stored energy, described here by $\frac{1}{2}L_o I^2$, can change significantly only if the current decreases significantly.

The conclusion that the constant-current assumption is not acceptable suggests that one should seek to formulate a model based on alternative assumptions. In particular, the coronal resistance R_c during a flare (i) must exceed the value $-\dot{L}_c$ given by (2), and (ii) must exceed this value by an amount at least comparable with the resistance R_o in the remainder of the circuit. Only then can the increase in the resistance R_c cause a significant back reaction on the dynamo to reduce the current. Note that there is no need for the coronal inductance to change at all during a flare; that is, one may have $\dot{L}_c=0$.

An interesting consequence of this argument is that $R_c \gtrsim R_o$ provides evidence that the resistance in the non-coronal part of the circuit is $R_o \approx 10^{-2}$ ohm. It is desirable to have a model that accounts for this value of the resistance of the subphotospheric part of the circuit. A further implication of $R_c \gtrsim R_0$ is that most of the available potential appears across the corona during a flare, and this suggests that the total potential driving the current is of order 10^{10} V, and not substantially higher than this, as the constant-current model would require. Again it is desirable to have a model that accounts for this inferred value.

These remarks indicate how a more realistic circuit model for the energy release in flares might be developed, but this is not attempted here.

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