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On the elliptical polarization of Jupiter's decametric radio emission

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Abstract. We consider the origin of the nearly-unique, 100% elliptical polarization of Jupiter's decametric radio emission. After investigating the transfer of polarized radiation when coupling of the Stokes parameters is important, we concur with earlier authors that the density in and near the source region must be so low that the polarization remains fixed along the ray path; this requires $n_e \lesssim 1$ to 5 cm^{-3} for $f = 30$ MHz and proportionally lower at lower frequencies.

We determine i) the polarization of cyclotron maser radiation in these circumstances, ii) that the dispersion relation of the rarified plasma composed of energetic, anisotropic electrons is like that in vacuo, iii) that the growth rate is sufficient to saturate the maser and account for the observed brightness temperature.

We consider possible sources of plasma in and near the source region in Jupiter's inner, polar magnetosphere. The principal source, the ionosphere, has been observed by the Pioneer and Voyager spacecraft to be tightly confined by gravity, with the electron density decreasing exponentially with scale height $H \lesssim 1000$ km. A modest extrapolation of observed densities indicates that $n_e = 5$ cm^{-3} occurs at $R = 1.17$ R$_J$, a radial distance that is quite plausibly below the source region of the highest frequency decametric radiation, produced where the gyrofrequency f_c is approximately equal to 39 MHz.

For the jovian radiation at $f \lesssim 1$ MHz and similar radiation from the other giant planets and the Earth, the required density is $n_e \lesssim 0.01$ cm^{-3}; this exceedingly low density is probably rare or non-existent, which may explain why those emissions are mainly or entirely circularly polarized. Similarly, one would not expect cyclotron maser radiation from the Sun and stars to be elliptically polarized because the density in and near the source region is too high.

The theory implies that the axial ratio of the polarization ellipse is determined uniquely by the angle of emission. A puzzling feature is that while there is good agreement for the Io-A source and fair agreement for Io-B, the observed polarization does not vary with time as expected.

Key words: Jupiter – radio emission – polarization – elliptical polarization

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1. Introduction

The planet Jupiter emits very intense radio radiation at decametric wavelengths that has one very unusual characteristic: it is 100% polarized and the polarization is elliptical. To our knowledge only Jupiter and certain pulsars are observed to have elliptical polarization. The planets Earth, Saturn, Uranus and Neptune seem to emit radiation that is essentially 100% circularly polarized, although the observations to date are not completely unambiguous because there have been very few observations of all four Stokes parameters; generally only right and left circular polarization have been measured. Lecacheux *et al.* (1991) give a summary of the observations of the polarization of planetary radio emissions made by various investigators through 1990.

Lecacheux *et al.* (1991) also report on observations of the complete polarization state of Jupiter's decametric radiation over the frequency range of about 15 to 38 MHz. They describe two "storms" of radiation that were observed with the highly sensitive array and spectro-polarimeter at Nançay, France. Both storms were associated with the position of Jupiter's moon Io, lasted two to three hours, and were relatively intense so that the signal to noise was good. One was a so-called "Io–B" event that occurred when the central meridian longitude was near $CML \approx 130°$ and Io's phase from superior geocentric conjunction was near $\phi_{Io} \approx 90°$. The other was an "Io–A" event that occurred near $CML = 240°$ and $\phi_{Io} = 240°$. They found that the radiation of both events was essentially 100% elliptically polarized, so that $I = (Q^2+U^2+V^2)^{1/2}$ where I, Q, U and V are the Stokes parameters, and so that the degree of polarization $r = (Q^2+U^2+V^2)^{1/2}/I$ is equal to unity. The relative amounts of linear polarization $r_\ell = (Q^2+U^2)^{1/2}/I$ and circular polarization $r_c = V/I$ were different for the two events. The Io–B event was more linear than circular, $r_\ell \approx 0.85$ and $r_c \approx -0.52$, where the minus sign denotes right-hand circular polarization. In contrast, the Io–A event was more circular than linear, $r_\ell \approx 0.66$ and $r_c \approx -0.76$. Except for certain spectral features mentioned below, the polarization of both events was approximately constant over the frequency range of $\gtrsim 2:1$ and time interval of 2–3 hours when Jupiter rotated by more than 70° and Io revolved by more than 15° in its orbit.

An exceptional feature whose polarization differs significantly from the averages quoted above is the "great arc" that is characteristically superimposed on Io–A events (Riddle 1983). In the Io–A event of Lecacheux *et al.* (1991) its polarization was $\gtrsim 95\%$ circular, $\approx 20\%$ linear.

Boudjada and Lecacheux (1991) analyzed eleven events, six Io–B and five Io–A, that were also observed with the Nançay spectro-polarimeter. Their results differed somewhat from those of Lecacheux *et al.* (1991) in that their Figure 10 indicates that the emission at some frequencies and times to be significantly less than 100% polarized; however, this may be an artifact of the way the data were analyzed and needs confirmation. On the other hand, they too found that the radiation of Io–B events is elliptical with a higher degree of linear than circular polarization, and that Io–A events have a higher degree of circular than linear polarization.

In this paper we address the following questions: 1) What is the origin of the elliptical polarization of Jupiter? Why is Jupiter nearly unique in exhibiting elliptical polarization? 2) What are the propagation conditions that permit the polarization to remain elliptical from the source to the observer, and not to change polarization to circular in the intervening magneto-ionic plasma? Simple ideas suggest that this can occur only if the plasma along the ray path, particularly near the source, is of very low density, $\lesssim 5\ \mathrm{cm}^{-3}$. 3) How can the plasma density be so low as seems to be required? 4) For the electron cyclotron maser instability, believed to be the mechanism for producing Jupiter's decametric radiation, what are the characteristics of growth and polarization transfer when the Faraday rotation is so small that polarization mixing occurs less rapidly than wave growth? 5) Are there significant changes in the dispersion relation, and hence in the properties of the radiation, when there is no cold plasma present but only hot, anisotropic electrons? 6) Is it possible to understand quantitatively the different ratios of linear to circular polarization in the two Io-related sources?

In Section 2 we give a simplified treatment of radiation transfer when coupling of the Stokes parameters is important, and include a discussion of weak and strong mode coupling. In Section 3 we apply the foregoing ideas to derive limits on the plasma density in the source region and its vicinity. In Section 4 we discuss the dispersion relation when the plasma is hot and anisotropic. In Section 5 we discuss wave growth and consider whether or not the derived characteristics of the source region are compatible with amplification to achieve the observed brightness temperatures. In Section 6 we consider how the inferred low density in Jupiter's low magnetosphere may come about. In Section 7 we summarize our results and give our conclusions.

2. The amplification and transfer of polarized radiation

The simplest interpretation of the polarization data on DAM is that the generation of the radiation and its propagation from the source to the top of the terrestrial ionosphere occurs as though there were no cold plasma present (Lecacheux 1988; Lecacheux *et al.* 1991). This raises three questions concerning the generation and transfer of elliptically polarized gyromagnetic emission. 1) Under what conditions can the presence of a medium be ignored in the transfer of the radiation? 2) Under what conditions does a cyclotron maser amplify the fastest growing magnetoionic mode, and under what conditions does it operate as though it were in vacuo and not in a magnetized plasma? 3) What difference, if any, is there between the predicted polarization in the two cases considered in 2)? These three questions are discussed here in terms of a transfer equation for the Stokes parameters.

The transfer of polarization in a weakly anisotropic medium can be written in the form of a matrix equation for the four Stokes parameters. In the *weak anisotropy limit* the two natural wave modes are assumed to have orthogonal transverse polarization, and the difference in their refractive indices is taken into account in the evolution of the relative phase between them but ignored in determining the ray path. The transfer equation is then of the form (Sazonov and Tystovich 1968, Melrose 1980, p. 199)

$$\frac{d}{ds}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -r_V & r_U \\ 0 & r_V & 0 & -r_Q \\ 0 & -r_U & r_Q & 0 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} - \begin{pmatrix} \mu_I & \mu_Q & \mu_U & \mu_V \\ \mu_Q & \mu_I & 0 & 0 \\ \mu_U & 0 & \mu_I & 0 \\ \mu_V & 0 & 0 & \mu_I \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}, \tag{1}$$

where ds denotes an element of distance along the ray path. The Stokes parameters are defined in terms of a specific coordinate system: let this be such that the direction of wave propagation is the z axis and linearly polarized radiation with $Q = I$, $U = 0$, $V = 0$ has its electric vector along the x axis. The projection of the magnetic field on the transverse plane is assumed to be at an angle ψ to the x axis and the polarization ellipses of the two modes to have axial ratios T_+ and $T_- = -1/T_+$ and refractive indices $n_\pm^2$, respectively. Then first square matrix in (1), which describes the effect of generalized Faraday rotation, has components

$$r_Q = -\Delta k\,\frac{T_+^2-1}{T_+^2+1}\cos 2\psi, \quad r_U = -\Delta k\,\frac{T_+^2-1}{T_+^2+1}\sin 2\psi,$$
$$r_V = -\Delta k\,\frac{2T_+}{T_+^2+1}, \quad \Delta k = \frac{\omega}{c}(n_+ - n_-). \tag{2}$$

The other square matrix in (1) describes the effect of polarization-dependent absorption, here assumed to be negative, leading to maser emission. A given absorption mechanism defines two orthogonal transverse polarizations, with axial ratios t_+ and $t_- = -1/t_+$ say, by

$$\mu_Q : \mu_U : \mu_V = \frac{t_+^2-1}{t_+^2+1}\cos 2\psi : \frac{t_+^2-1}{t_+^2+1}\sin 2\psi : \frac{2t_+}{t_+^2+1}. \tag{3}$$

The matrix equation (1) may be solved by finding the eigenvalues of the sum of the square matrices on the right hand side and constructing the corresponding eigenvectors (Zheleznyakov, Sovorov and Shaposhnikov 1974; Melrose 1980, P. 207). This procedure has simple physical interpretations in two opposite limiting cases in each of which only one of the two terms on the right hand side is retained to a first approximation.

Suppose that the first term on the right hand side of (1) has a dominant effect. This is the case for $r_Q^2 + r_U^2 + r_V^2 \gg \mu_Q^2 + \mu_U^2 + \mu_V^2$. (The diagonal terms μ_I have no effect on the polarization and are not relevant in the following discussion.) Then to a first approximation the absorption term is negelected. Two of the four eigenvalues of the remaining square matrix are equal to zero and the other two are $\pm i\Delta k$. The eigenvectors corresponding to the two degenerate eigenvalues may be chosen and interpreted as the intensities in the two natural modes with axial ratios T_+ and $T_- = -1/T_+$, and the remaining two eigenvectors are proportional to the cross-correlations between the amplitudes of the two modes. The eigenvalues $\pm i\Delta k$ are then interpreted as the rate of change (with distance) of the phase factors of these cross-correlations. The effect of the (negative) absorption is then taken into account as a perturbation. The approximation $r_Q^2 + r_U^2 + r_V^2 \gg \mu_Q^2 + \mu_U^2 + \mu_V^2$ implies that the components get out of phase much faster than the polarization-dependent wave absorption or growth changes the state of polarization. Hence, the cross-correlation terms remain small (which is attributed to *differential Faraday rotation* of the modes along the ray path in the growth or absorption region), and the wave growth or damping is described in terms of the effect on the intensities of the two natural modes. Thus the absorption is described in terms of separate absorption coefficients for the two modes, and these are constructed by projecting the absorption matrix in (1) onto the eigenvectors corresponding to the two natural modes.

Now consider the opposite limit $\mu_Q^2 + \mu_U^2 + \mu_V^2 \gg r_Q^2 + r_U^2 + r_V^2$. In this case the generalized Faraday rotation may be neglected to a first approximation. Two of the four eigenvalues of the absorption matrix in (1) are equal to μ_I and the other two are $\mu_I \pm [\mu_Q^2 + \mu_U^2 + \mu_V^2]^{1/2}$. By analogy with the previous case, the eigenvectors corresponding to the two degenerate eigenvalues may be chosen and interpreted as the intensities in the two "modes" with axial ratios t_+ and $t_- = -1/t_+$, and the remaining two eigenvectors are proportional to the cross-correlations between the amplitudes of the two "modes". For a completely polarized absorption process ($\mu_I^2 = \mu_Q^2 + \mu_U^2 + \mu_V^2$, as is the case for gyro-radiation) only one of these "modes" has a nonzero eigenvalue and so is affected by the (negative) absorption. Thus when generalized Faraday rotation is neglected, the maser produces completely polarized maser emission, and this polarization is determined by the properties of the maser process. The inclusion of Faraday rotation in the case $\mu_Q^2 + \mu_U^2 + \mu_V^2 \gg r_Q^2 + r_U^2 + r_V^2$ has only a small effect.

The intrinsic polarization of gyroemission and the polarization of the x mode of magnetoionic theory both reduce to $T = \cos\theta$ in appropriate limiting cases. For gyroemission this limit has a simple physical interpretation: it corresponds to the shape of the ellipse formed by the projection on the plane of the sky of an electron with pitch angle $\alpha = \pi/2$ in circular motion about the magnetic field in the source. The polarization characteristics of the x mode are determined by two competing effects, the currents associated with the gyrating electrons, and the charge density oscillation associated with the plasma frequency. Near the cylcotron frequency ($\omega \approx \Omega_e$) in a low density plasma ($\omega_p^2 \ll \Omega_e^2$) the former effect dominates.

The axial ratio for gyroemission differs from that of the x mode in the next order of small parameters. Gyromagnetic emission at the sth harmonic due to an electron with speed βc and pitch angle α has axial ratio

$$T = \frac{(\cos\theta - \beta\cos\alpha)J_s(sx)}{\beta\sin\alpha\sin\theta J_s'(sx)}, \quad x = \frac{\beta\sin\alpha\sin\theta}{1-\beta\cos\alpha\cos\theta}. \tag{4}$$

This reduces to $T = \cos\theta$ in the nonrelativistic limit $\beta \ll 1$ for emission at the gyrofrequency $s = 1$. A better approximation to (4) for $\beta\sin\alpha\sin\theta \ll 1$ is $T \approx \cos\theta - \beta\cos\alpha$.

For the x mode the corrections are found by expanding in the small parameters $Y - 1$ and X, where the magnetoionic parameters are defined by $Y = \Omega_e/\omega$ and $X = \omega_p^2/\omega^2$. The exact expression for the axial ratio is

$$T = \frac{-\frac{1}{2}Y^2\sin^2\theta + \Delta}{Y(1-X)\cos\theta}, \tag{5a}$$

$$\text{where} \quad \Delta^2 = \tfrac{1}{4}Y^4\sin^4\theta + (1-X)^2Y^2\cos^2\theta. \tag{5b}$$

Retaining the first order terms in the expansions gives $T \approx \cos\theta[1 - (X + Y - 1)\sin^2\theta/(1+\cos^2\theta)]$.

Thus, while the axial ratio for both gyroemission and for the x mode reduce to $T = \cos\theta$ to lowest order, the next order terms are quite different. The most important difference from $T = \cos\theta$ is likely to be from the correction $-\beta\cos\alpha$ in the axial ratio of gyroemission.

The concepts of weak and strong mode coupling are often used as an alternate means to describe the transfer of polarized radiation. The polarization of the radiation is assumed to be known at a given location and its propagation is examined in terms of the two characteristic magnetoionic modes and the changes of these modes along the ray path. When the density, field strength and scale length for changes are sufficiently large the energy in each mode remains in that mode as the radiation propagates into regions of different characteristic polarization, e.g. from where the modes are elliptical to where they are circular. There is little or no coupling of energy from one characteristic mode to the other and so is known as "weak mode coupling". On the other hand, when the density and/or field strength are sufficiently small, the energy in the two modes transfers back and forth in such a way to retain the polarization of the radiation. This "strong mode coupling" applies, for example, in vacuo (but some plasma must be present for the characteristic modes to be defined).

Now we return to the three questions posed at the beginning of this section. 1) The medium can be ignored if the change in relative phase between the components in the two modes is unimportant, which can be the case for either of two reasons. First, if the radiation is purely in one mode and the shape of the polarization ellipse does not change significantly with distance, then the relative phase difference between the two modes is irrelevant. For example, this is the case for circularly polarized radiation in a medium where the modes are circularly polarized. As discussed below, it is probably also the case near the source of Jupiter's decametric radiation. Second, the medium can be ignored in the limit of strong mode coupling when the characteristic distance over which the polarization ellipse changes shape or orientation is shorter than the distance over which the modes get out of phase. Separation into two modes is then irrelevant because the inhomogeneity of the medium prevents the difference between the two modes (of a locally homogeneous medium) becoming manifest. In this case the radiation propagates, to a first approximation, as though there were no difference between the two modes. 2) The cyclotron maser amplifies the fastest growing magnetoionic mode only if the modes get out of phase faster than they grow. When the growth rate exceeds the rate at which the modes get out of phase, the resulting polarization is characteristic of gyroemission. 3) The difference between the polarization in these two cases is negligible in the limit $\omega_p^2 \ll \Omega_e^2$ for nonrelativistic electrons when the axial ratio is $T \approx \cos\theta$. The most important correction is likely to be due to the finite speed of the electrons, giving $T \approx \cos\theta - \beta\cos\alpha$.

3. Plasma density in and near the source region

The simplest interpretation of the observed elliptical polarization is that it is characteristic of gyroemission, and that there is negligible Faraday rotation in and near the source at Jupiter. (There is, of course, Faraday rotation in the terrestrial ionosphere, and perhaps even in Jupiter's magnetosphere far from the emission region, i.e. where $\Omega_e \ll \omega$, but this changes only the orientation of the ellipse, not the ellipticity.) In this section we explore the implications of this assumption of the plasma density in the source region and in the jovian magnetosphere along the ray path. Here it is assumed for the present that the polarization resulting from the maser growth is indistinguishable from that of the x mode. The validity of this assumption is discussed in Section 5. Then, by hypothesis, at the source the radiation is 100% in the x mode.

As discussed above, the relative phase difference between the two modes affects the polarization of the escaping radiation only if (a) there are components in both modes and (b) the rate that the two components get out of phase is larger than the rate at which the shape or orientation of the polarization ellipse changes. By hypothesis, sufficiently near to the source there is only an x mode component; an o mode component builds up as the shape or orientation of the polarization ellipse (relative to the ambient magnetic field) change from their initial values. To discuss the effect of the plasma on the polarization we need to estimate the rate at which the modes get out of phase and the rates at which the polarization ellipse changes shape and orientation.

The phase difference $\Delta\psi$ between components in the two modes varies with distance s along the ray path according to

$$\frac{d\Delta\psi}{ds} = \frac{\omega}{c}\left(N_o - N_x\right). \tag{6}$$

The refractive index for either magnetoionic mode is given by

$$N^2 = 1 - \frac{XT}{T - Y\cos\theta}, \tag{7}$$

with T_x given by (5) and with T_o determined by the orthogonality of the modes, which requires $T_x T_o = -1$. Two approximations to $N_x - N_o$ are needed. Near the source the frequency is close to the cyclotron frequency, and the magnetoionic wave properties may be approximated by expanding in $X \approx \omega_p^2/\Omega_e^2$ and $1 - Y \approx (\Omega_e - \omega)/\Omega_e$. The x mode has a cutoff at $X = 1 - Y$. At frequencies just above the cutoff for $X \ll 1$, $X < 1 - Y \ll 1$ the following approximations the refractive indices $N_{o,x}$ apply:

$$N_x^2 \approx \frac{1 - Y - X}{1 - Y - \frac{1}{2}X\sin^2\theta}, \quad N_o^2 \approx 1 - \frac{X}{1 + \cos^2\theta}. \tag{8}$$

Further from the source when one has $X \ll 1$ and $Y \ll 1$, an expansion in X and Y gives

$$N_{o,x} \approx 1 - \tfrac{1}{2}X(1 - Y T_{x,o}\cos\theta). \tag{9}$$

(Note that the change in order of the subscripts on the RHS is required.) Thus relatively close to the source and further from it we have, respectively,

$$\frac{d\Delta\psi}{ds} = \frac{\omega}{c}\frac{X(1+\cos^2\theta)}{4(1-Y)}$$

$$\text{for} \quad X \ll 1 \quad \text{and} \quad \tfrac{1}{2}X\sin^2\theta \ll 1 - Y \ll 1, \tag{10a}$$

and

$$\frac{d\Delta\psi}{ds} = \frac{\omega}{c}X\Delta \qquad \text{for} \quad X \ll 1 \quad \text{and} \quad Y \ll 1. \tag{10b}$$

The case (10b) includes the familiar case of Faraday rotation in the limit, cf. (5), $\Delta \approx Y|\cos\theta|$.

Now let us consider how the shape and orientation of the polarization ellipse vary along the ray path (see also Daigne and Ortega-Molina 1984). The shape and orientation change as the polar angles θ and ϕ of the ray relative to the magnetic field vary, and as Y varies due to the change in the magnetic field strength. We now argue that within a few jovian radii of the source, the changes in all these quantities occur over characteristic distances of order a jovian radius. To see this consider the idealized case of a pure dipole field. (Our arguments are modified only slightly for more realistic fields.) Variations in ϕ may be determined by writing down the equation for a straight line (the ray path) in spherical polar coordinates centered on the location of the dipole. The geometric details are not important here, and it suffices to note that ϕ changes with distance along the ray path at a rate given by α_1/r, where α_1 is a geometric factor of order unity and r is the radial coordinate. Near the source the axial ratio is $T \approx \cos\theta$, and along the ray path this changes due to the change in θ. The rate of change may be written as α_2/r, where α_2 is another geometric factor of order unity. The change in shape of the polarization ellipse due to the change in magnetic field strength and hence in Y follows from (Melrose 1980, p. 261)

$$\frac{\omega}{T}\frac{\partial T}{\partial\omega} = \left(\frac{1+X}{1-X}\right)\left(\frac{1-T^2}{1+T^2}\right). \tag{11}$$

With $X \ll 1$ and with Y changing with distance at a rate of order $3/r$, where the factor three arises from the dipolar dependence of B on r, the rate of change of shape of the polarization ellipse due to this effect may be written as α_3/r, where α_3 is a third geometric factor. Hence the rate of change of the shape or orientation of the polarization ellipse near Jupiter may be written as α/R_J, where α is a geometric factor. A detailed evaluation of α would involve some tedious spherical geometry, and for present purposes it suffices to note that α is of order unity.

Whether or not the radiation stays in the x mode may be estimated in terms of the theory of mode coupling. The mode-coupling coefficient Q_M may be defined as the ratio of the rate of change of the relative phase to the rate of change of the shape and orientation of the polarization ellipse (e.g. Melrose 1980, p. 280). It is appropriate to consider two cases: relatively close to the source, where (10a) applies, and further from the source, where (10b) applies. Very near the source the components in the two modes would get out of phase relatively quickly, but the shape and orientation of the polarization ellipse changes only over the characterstic distance R_J/α, and hence the change in relative phase can have little effect on the polarization. The dominant contribution to any mode coupling should occur over a distance of order R_J from the source. For present purposes, a rough estimate is adequate, and we assume $\cos^2\theta \ll 1$ and suppose that the amount of Faraday rotation is of order that implied by (10a) for $Y = \frac{1}{2}$ over a distance equal to R_J. Then the mode-coupling coefficient is given approximately by

$$Q_M \approx \frac{\Omega_e}{4c}\frac{\omega_p^2}{\Omega_e^2}\bigg/\frac{\alpha}{R_\text{J}}. \tag{12}$$

Further from the source, (10b) leads to a condition that is less restrictive than (12). Hence, the condition $Q_M \ll 1$, with Q_M given by (12), is sufficient for the role of the medium to be negligible in changing the polarization.

According to the arguments leading to (12), most of the change in relative phase occurs close to the source. As a result (12) may be an overestimate of the effect for two reasons. First, near the source the polarization is almost purely x mode and hence there are not two components to get out of phase. Thus, even if the condition $Q_M \ll 1$ is not satisfied it does not necessarily follow that the effect of the medium on the polarization is significant. Second, the case (10a) is associated with the rapid increase of N_x with frequency just above the cyclotron frequency, and it is argued in the next section that the magnetoionic theory is probably not applicable, and hence that this rapid change does not occur. Nevertheless, for present purposes we assume that (12) is the appropriate condition.

Using (12) with Ω_e equated to the frequency of observation, the condition for the change of relative phase to be negligible may be reduced to a condition of the electron number density n_e. One finds

$$n_e \ll \alpha(f/25\,\mathrm{MHz}), \tag{13}$$

where f is the observed frequency. However, in view of the remarks above that this is an overestimate, we modify (13) to

$$n_e \lesssim \alpha(f/25\,\mathrm{MHz}). \tag{14}$$

Then the estimate $n_e = 5\ \mathrm{cm}^{-3}$ of Lecacheux (1988) and Lecacheux *et al.* (1991) for 30 MHz is a plausible one; we will use it in the following.

4. The dispersion relation for a hot, anisotropic plasma

The magnetoionic theory applies in a cold plasma in which only the dispersion of the electrons is important. In the source region of Jupiter's decametric radiation the electron number density appears to be so low that it is possible that the dispersion may be dominated by the energetic electrons. That is, it is possible that the only electrons present in the source region are the energetic ones that cause the cyclotron maser emission. It is then relevant to ask whether it is reasonable to use the magnetoionic theory to treat the wave growth and propagation.

Dispersion near the cyclotron frequency in a plasma which is dominated by hot electrons has been discussed in connection with cyclotron maser emission by Wu *et al.* (1981), Wong *et al.* (1982), Winglee (1983, 1985), Wu (1985), Robinson (1986, 1987), Le Quéau and Louarn (1989). Several kinds of distribution have been considered, including Maxwellian, DGH and idealized loss-cone, and it has been found that the form of the modifications to the dispersion relations are not strongly dependent on the anisotropy. (It might be remarked that a streaming motion of the electrons can be taken into account by making a Lorentz transformation, and hence that only a non-streaming anisotropy, such as a loss-cone, need be considered explicitly.) The notable effects of the nonzero temperature may be understood by considering the transition from the dispersive characteristics of a magnetoionic medium to the dispersion of the vacuum as the electron density is allowed to approach zero. In a magnetoionic medium with $\omega_p^2 \ll \Omega_e^2$ the extraordinary mode has two branches: the z mode which exists in the frequency range between a cutoff its $\omega_z \approx \omega_p^2/\Omega_e$ and a resonance just above Ω_e, and the x mode which exists in the frequency range above its cutoff at $\omega_x \approx \omega_e + \omega_p^2/\Omega_e$. These two branches are separated by a stop band. In a vacuum there is only one branch and the dispersion characteristics ($\omega = kc$) are independent of frequency. As the electron density approaches zero, near the cyclotron frequency, the resonance in the z mode, the stop band and the cutoff frequency of the x mode must disappear as the refractive index approaches the vacuum value of unity.

There are two notable effects of a nonzero temperature: a decrease in the cutoff frequency of the x mode, and a reconnection of dispersion branches. The decrease in the cutoff frequency was treated numerically by Wu *et al.* (1981) and Wong *et al.* (1982), and a simple analytic treatment of it was given by Winglee (1985). Winglee found that the cutoff frequency may be approximated by

$$\omega_x = \Omega_e \left(1 + \frac{\omega_p^2}{\Omega_e^2} - \frac{1}{2}\left\langle \frac{2v_\perp^2 + v_z^2}{c^2} \right\rangle\right), \tag{15}$$

where the angular brackets denote the average over the distribution function. This reduction is due partly (but not entirely) to the relativistic gyrofrequency Ω_e/γ decreasing with increasing Lorentz factor γ.

The spread in Lorentz factor leads to an intrinsic broadening of the cyclotron line whose width increases as the mean energy of the electrons increases. The Doppler broadening due to the parallel motion of the electrons also contributes to the line width and is more important than this relativisitc spread except near perpendicular propagation ($\cos\theta = 0$) where the parallel Doppler spread is zero. When the line width becomes comparable with the width of the stop band, the z mode and x mode branches reconnect to form a single branch (Winglee 1983, Robinson 1986, 1987). For a Maxwellian distribution, this reconnection occurs when the thermal speed V_e satisfies (Robinson 1986, 1987)

$$\frac{V_e^2}{c^2} \approx \begin{cases} \omega_p^2/\Omega_e^2 & \text{for } |\cos\theta| \gg V_e/c, \\ (\omega_p^2/\Omega_e^2)^2 & \text{for } |\cos\theta| \ll V_e/c. \end{cases} \tag{16}$$

For thermal speeds larger than this value the magnetoionic theory is inapplicable, and a better approximation is to treat the dispersion of the radiation as though the plasma were absent.

On inserting numerical values that are plausible for the source region of DAM, say $T_e = 10^4\,\mathrm{K}$, $n_e = 5\,\mathrm{cm}^{-3}$, $B = 10\,\mathrm{G}$, one finds $(\omega_p^2/\Omega_e^2)/(V_e^2/c^2) \approx 0.3$ and $(\omega_p^2/\Omega_e^2)^2/(V_e^2/c^2) \approx 10^{-7}$. These ratios must be greater than unity for the wave properties to be as predicted by the magnetoionic theory in the two cases in (16), respectively. One concludes that the waves should be more like the vacuum mode than like the extraordinary mode. It follows that in the treatment of the maser emission one should ignore the dispersive characteristics of the z mode and x mode; setting the refractive index equal to unity is likely to be a better approximation than calculating it from the magnetoionic theory. Furthermore, in treating the emission one should explore the possible importance of the intrinisically new modes that appear (e.g., Le Quéau and Louarn 1989). Nevertheless, for our numerical estimates below we ignore these modifications and use estimates based on quasilinear theory (e.g., Hewitt, Melrose and Rönnmark 1982). For semiquantitative purposes this should not lead to significant error. However, our use of quasilinear theory is only to facilitate numerical estimates, and is not intended to imply that the more subtle effects of the dispersion, discussed in the literature cited above, is unimportant.

5. Wave growth and compatibility with observed brightness temperatures

There are two questions that arise in connection with the low plasma density implied by the apparent absence of significant

Faraday rotation of the decametric radiation in the magnetosphere of Jupiter. One raised in Section 2 concerns the polarization of the electron cycoltron maser emission: is it characteristic of the x mode or characteristic of gyroemission? The other question is how low can the plasma density be and still be consistent with the observations: the free energy density in the source must be adequate to drive the maser. These two questions are discussed here.

The arguments given in the Section 4 suggest that the magnetoionic properties of the plasma may be inappropriate for determining the dispersion characteristics of the x mode near $\omega = \Omega_e$, but this does not necessarily imply that the difference between the two modes can be ignored in treating the maser emission. As discussed in Section 2, the polarization of the cyclotron maser emission depends on whether or not the growth rate exceeds the rate at which the magnetoionic components get out of phase. An analytic expression for the maximum growth rate was made by Melrose, Rönnmark and Hewitt (1982); the functional form that they found is

$$\frac{\Gamma_{\max}}{\Omega_e} = \zeta \frac{\omega_p^2}{\Omega_e^2} \frac{c^2}{V^2}, \tag{17}$$

where V is a characteristic speed of the electrons driving the maser, ω_p^2 is the plasma frequency associated with the number density of these electrons, and ζ is a factor of order unity. Let us estimate the factor ζ by comparing (17) with the results of detailed numerical calculations. For example, the calculations of Wong *et al.* (1981) with $\omega_p^2/\Omega_e^2 = 0.01$ and $V^2/c^2 = 1/200$ give a temporal growth rate of order $\Gamma_{\max}/\Omega_e \approx (2-10) \times 10^{-3}$. This leads to an estimate $\zeta = (1-5) \times 10^{-3}$ in (17). We wish to compare the growth rate (17) with the rate at which the modes get out of phase, as given by (10a). In (10a) we set $\cos^2\theta \ll 1$ and $1 - Y \approx V^2/c^2$, corresponding to the typical frequency where growth occurs (e.g., Hewitt, Melrose and Rönnmark 1982); at this frequency the thermal corrections to the dispersion relation discussed in Section 4 are unimportant provided that the dispersion is dominated by cold plasma. One finds that the ratio of the growth rate to the rate at which the modes get out of phase is $\approx \xi/4$, which is less than unity. Hence, it is appropriate to separate into magnetoionic modes when treating the maser emission and, to the extent that they can be distinguished, the polarization should be characteristic of the x mode rather than of cyclotron emission in vacuo.

The other question raised above is whether the low number density of electrons is consistent with the observed radiation intensity. There are two relevant conditions: 1) the amount of free energy must be adequate, and 2) the observed high brightness temperatures must be possible. These conditions are actually two alternative statements of a single one. Let ΔV_c be the coherence volume of the maser emission in the source region, and let T_B be its brightness temperature. The energy density in the radiation is $k_B T_B/\Delta V_c$, where k_B is Boltzmann's constant. Requirement 1) implies

$$k_B T_B/\Delta V_c \le \xi m_e n_e V^2, \tag{18}$$

where ξ is an efficiency factor for conversion of particle energy into cyclotron maser radiation. Requirement 2) may be stated in terms of the maximum enhancement above incoherent emission that may be produced by the maser. Incoherent emission is restricted to $k_B T_B \lesssim m_e V^2$; the maximum enhancement is by a factor equal to the number of electrons in the coherence volume $n_e \Delta V_c$. When an efficiency factor ξ for the maser is taken into account, this condition reproduces (18).

The maximum brightness temperature is poorly determined; it is believed to be in excess of 10^{17} K (Dulk 1970). If the electrons have energy around 100 keV then one requires $\xi n_e \Delta V_c \gtrsim 10^8$. The coherence volume may be estimated as the cube of the wavelength divided by the fraction of 4π sterad filled by the radiation. The radiation is thought to be confined to a thin surface of a hollow cone (Dulk 1967), suggesting that the fraction of the solid angle filled may be as small as 10^{-2}. The enhancement factor is then of order $\xi \times 10^{11}$. For $n_e = 5\,\mathrm{cm}^{-3}$ an efficiency factor $\xi \gtrsim 2 \times 10^{-4}$ is required. Thus with a modest efficiency factor the inferred electron density is consistent with the inferred brightness temperature.

A referee has questioned whether it is consistent to assume linear theory (that is, weak turbulence theory) in view of the high energy density in the waves implied by (18). We are not aware of any discussion of strong turbulence effects for transverse waves under conditions relevant here. Strong turbulence for Langmuir waves (e.g., Goldman 1984) is relevant when the parameter ξ in (18) exceeds a threshold value, which for the sake of discussion here is taken to be $(\Delta k \lambda_D)^2$, where Δk is the spread in wavenumber and $\lambda_D = V_e/\omega_p$ is the Debye length. Assuming that $\Delta\omega$ is the bandwidth of the radiation, one has $(\Delta k \lambda_D)^2 = (\Delta\omega/\omega_p)^2 (V_e/c)^2$. The assumed absence of cold plasma implies that $(V_e/c)^2$ is not particularly small, and could be of order 10^{-2} or higher, and provided that the bandwidth is not too narrow the condition $\xi = 2 \times 10^{-4}$ need not violate the weak turbulence requirement. Because this applies for Langmuir waves, it is instructive but not directly applicable to the present situation, Nevertheless, in view of this result and in the absence of a detailed investigation, it is reasonable to assume that weak turbulence theory is adequate for semi-quantitative estimates.

6. Low plasma density in Jupiter's magnetosphere

In the preceding sections we examined the requirements for elliptical polarization to be retained during the propagation of the radio emission and concluded that the density of the plasma in Jupiter's magnetosphere must be very low, cf. (14), particularly close to the source of radiation where the radio frequency is close to the local gyrofrequency. For the reasons discussed in Section 3 we consider an upper limit of 5 cm^{-3} near the 30 MHz source region (Lecacheux *et al.* 1991), consistent with the earlier limit of $\lesssim 10\,\mathrm{cm}^{-3}$ suggested by Parker, Dulk and Warwick (1969). In this section we consider how such a low electron density could exist in the relevant regions of Jupiter's magnetosphere.

The simplest hypothesis for the origin of the low density is that all of the ambient plasma is confined to low altitudes by gravity or expelled to high altitudes by the centrifugal force. As discussed by several authors (e.g. Melrose 1967) co-rotating particles in the region less than about 2.3 R_J from Jupiter's rotational axis have a net (gravitational plus centrifugal) acceleration that is inward toward the planet and those at greater distances have a net acceleration outward. We concentrate on the high latitude ($L \approx 6$) region at radial distance $R \lesssim 1.5$ R_J because Io-related radiation between 10 and 40 MHz is emitted there, i.e. the gyrofrequency along the Io flux tube, as calculated from the O4 model of the magnetic field (Acuña and Ness 1976), ranges from 40 MHz at $R \approx 1.0$ R_J to 10 MHz at $R \approx 1.5$ R_J. In this region of the polar magnetosphere centrifugal acceleration is negligible.

We first consider a Maxwellian distribution of electrons and ions in and above Jupiter's ionosphere. Observations by the Voyager spacecraft show that the density in the ionosphere is $n_e \lesssim 2 \times 10^5$ cm^{-3} at an altitude of $\approx$ 2000 km and that the decrease in plasma density in the region 2000 km $\lesssim h \lesssim$ 6000 km is exponential with scale height $H \approx$ 960 km (e.g. Atreya 1976). This plasma scale height is twice the neutral scale height and corresponds to a kinetic temperature of about 1200 K.

Using these numbers, we find that the density falls to 5 cm^{-3} within about 10,000 km, or at radial distance $R \approx 1.17$ R$_J$. Taking into account the uncertainty of the O4 model of the magnetic field (it is based on measurements made at $R \gtrsim 3$ R$_J$ and it is known that high-order dipole moments exist) this altitude can plausibly be associated with the 40 MHz gyrofrequency level.

We conclude that the Maxwellian plasma of Jupiter's ionosphere and thermosphere is confined to relatively low altitudes and is not likely to affect the polarization of the decametric radiation.

We now consider whether another component of plasma may exist, analogous to that in the Earth's plasmasphere.

In the case of Earth the plasmasphere exists in the closed magnetic field at radial distances up to 4 to 6 R$_\oplus$, i.e. below the zero-potential altitude of 6.5 R$_\oplus$ in the equatorial plane where the inward gravitational acceleration equals the centrifugal acceleration, or more precisely the equipotential surface of the convection and corotation electric fields (e.g. Shulz and Lanzerotti 1976). The density in the plasmasphere is $\lesssim 100 - 1000$ cm^{-3}, while just outside the plasmapause it drops to about 1 cm^{-3} (e.g. Akasofu 1977). The source of the material is the ionosphere: solar radiation produces ions of energy $\sim$1 eV that diffuse throughout the plasmasphere.

There is no direct information on the existence of a similar plasmasphere at Jupiter. If, in analogy with the Earth, it exists below the zero-potential distance of 2.3 R$_J$, then it does not affect the radiation from the radio source region at $L \approx 6$ and low altitudes (although it could refract radiation directed toward it, which is not of concern here).

Another possible source of plasma in and near the radio source region is the outer magnetosphere or the Io torus. Electrons could be trapped on the magnetic field lines, mirroring in the region of interest below $R \approx 1.5$ R$_J$ (e.g. Melrose 1967). However, the existence of the Io torus and the turbulence therein is likely to scatter those particles in pitch angle and prevent all but a small fraction from reaching these low altitudes.

In summary, the various sources of plasma we have identified seem to be incapable of supplying material of density greater than $\sim$1 cm^{-3} to the radio source region, so the low density we have inferred from the existence of elliptical polarization seems plausible.

7. Conclusions

The existence of radiation that is 100% polarized and elliptical poses a number of questions for theory, questions seldom if ever posed before because elliptical polarization is so rare in nature. The results presented here answer some of these questions. We have encountered no major difficulty in explaining the elliptical polarization, provided that the density in the source region at Jupiter and in a large volume surrounding it is very low, $\lesssim 5$ cm^{-3} where the gyrofrequency is about 30 MHz, and proportionally lower at lower frequencies, cf. (14). Then the only electrons present may be the energetic ones that produce the maser emission. Even in this extreme case, the plasma may be treated as a magnetoionic medium, and the wave polarization is characteristic of the x mode rather than of gyroemission. However, these two characteristic polarizations are similar, cf. (4) and (5a). Although the number of electrons is small, the growth rate of the cyclotron maser instability is found to be sufficiently high to saturate the maser and account for the high brightness temperatures observed, provided that its efficiency (in the sense defined in Section 5) exceeds the modest value of about 10^{-4}.

The condition (14) is a strong one. Put in another form it is $f_p/f_B \lesssim (3\alpha/f_B)^{1/2}$, which is $f_p/f_B \lesssim 10^{-3}$ to 10^{-2} for $0.3 \lesssim f_B(\mathrm{MHz}) \lesssim 30$. It is thus 10–100 times stronger than the condition $f_p/f_B \lesssim 0.1$ for electron cyclotron maser radiation to be dominantly x mode at the fundamental. This is consistent with the observed absence of o-mode and harmonic radiation in the hectometric and decametric ranges.

We suggest that the density of electrons in the large volume surrounding the source region is low because the plasma scale height in Jupiter's upper ionosphere and lower magnetosphere is small, $\lesssim$1000 km or 0.014 R$_J$, at least near the decametric source in the polar regions near Io's L shell, $L \approx 6$. This is perhaps not surprising because the thermosphere of Jupiter has nearly the same temperature, $\approx$ 1000 K, as does the Earth's but the distance scale is more than 10 times larger. Thus the conditions that occur on Earth at several radii exist on Jupiter at several tenths of a jovian radius. Furthermore, it is observed that outside the plasmasphere of the Earth the density drops to a very low value, $\sim$ 1 cm^{-3} (e.g., Akasofu 1977; Perraut *et al.* 1990). A plasmasphere on Jupiter, if it exists, would not extend to $L \approx 6$.

We return to two questions posed in Section 1: (1) Why is Jupiter nearly unique in exhibiting elliptical polarization? Before answering we remark that even Jupiter's radiation at frequencies below some ill-determined limit somewhere between 1 and 10 MHz is not elliptically polarized, but circularly (e.g. Ortega-Molina and Lecacheux 1991). Similarly, the radiation of the planets Earth, Saturn, Uranus and probably Neptune is mainly or entirely circularly polarized and is confined to frequencies $\lesssim$ 1 MHz. We note that (14) implies that the critical density to retain elliptical polarization decreases with decreasing frequency. Thus, for example, the density in the large volume surrounding a 1 MHz source must be smaller than 0.02 cm^{-3}, a rather severe requirement. Indeed, in the Earth's magnetosphere, such low densities do not exist, perhaps not even in the isolated, evacuated flux tubes in which the AKR radiation seems to be emitted. For the Sun, a class of whose microwave spike bursts at frequencies $\gtrsim$1 GHz are attributed to the cyclotron maser instability (e.g. Benz 1986), and for flare stars whose brightest emission is also usually attributed to the cyclotron maser instability, (14) implies $n_e \lesssim 40$ cm^{-3} in a stellar atmosphere where the gyrofrquency is $\gtrsim$1 GHz. Again, this is a very severe requirement, so we do not expect the radiation to be elliptically polarized, consistent with observations reported to now.

(2) Is it possible to understand quantitatively the different ratios of linear to circular polarization in the two Io-related sources, Io–B and Io–A? For an Io–B storm Lecacheux *et al.* (1991) found $\langle r_\ell \rangle \approx 0.85$ and $\langle r_c \rangle \approx -0.52$, whereas for an Io–A storm they found $\langle r_\ell \rangle \approx 0.66$ and $\langle r_c \rangle \approx -0.76$. From the relationship between the degree of circular polarization and the axial ratio, $r_c = 2T/(T^2 + 1)$, we convert the observed values of r_c into axial ratios and find $T \approx 0.28$ and 0.46 for sources Io–B and Io–A respectively. The axial ratio for gyro-radiation

is $T = \cos\theta$, whence $\theta_{obs} \approx 74°$ and 63° for sources Io–B and Io–A respectively. On the other hand, using the O4 model of Jupiter's magnetic field (Acũna and Ness 1976) one can trace the field line from Io to the 10–40 MHz source region at $R \approx 1.1$ to 1.5 R_J, and then calculate the angle between that field line and the direction of the observer on Earth. The result, for the middle of the Io–B source region, is $\theta_{mod} \approx 85°$, and for the Io–A source region it is $\theta_{mod} \approx 60°$.

For source Io–A the agreement is very good between the value $\theta_{obs} \approx 63°$ and the value $\theta_{mod} \approx 60°$ from the O4 model. For source Io–B, θ_{obs} is closer to 90° than for source Io–A, consistent with expectations, but $\theta_{obs} \approx 74°$ is smaller than $\theta_{mod} \approx 85°$. This could be because the O4 model does not accurately represent the field in the low magnetosphere (Aubier and Genova 1985; Bagenal and Leblanc 1988, 1990) or because Io affects the radiation (possibly by augmenting the precipitation of energetic electrons) not just on its instantaneous field line but, on average, somewhat downstream.

Of the questions that remain, we mention two: i) Why is the polarization so stable during a storm of radiation that may last for two or three hours? In three hours Io moves about 20° in its orbit, so the angle θ between the field in Io's flux tube and the Earth changes by a similar amount. One would thus expect the circular polarization of source Io–B to increase with time, and that of source Io–A to decrease. However, Lecacheux *et al.* (1991) find the polarization to be nearly constant, both as a function of time for 2–3 hours, and of frequency over about one octave. ii) Does the electron cyclotron maser mechanism operate over the inferred range of emission angles with respect to the magnetic field of about 60° to 90°? In particular, the nearly circular polarization of the great arc contrasts with the more elliptical polarization of the radiation surrounding it in frequency and time. Does the great arc's polarization indicate that the radiation is emitted at a small angle with respect to the magnetic field, or does weak mode coupling change the polarization?

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