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Coherent curvature emission and radio pulsars

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SUMMARY

Maser curvature emission in the presence of curvature drift is re-examined in detail. A physical interpretation of this mechanism is given. The maser emission is possible only when there is a beam-type distribution of outflowing electrons or positrons with Lorentz factors satisfying $\gamma \gtrsim 10^3$, which is determined by the geometry of the pulsar magnetosphere. It is shown that, for amplification to be effective in the pulsar magnetosphere, the density of the electrons or positrons in the beam must be close to a critical value beyond which the particle flow is no longer controlled by the magnetic field. It is also shown that, to obtain appreciable amplification in the range 2 to 200 MHz, the magnetic flux density in the source region must be $B > 10^8$ G. This seems to rule out the mechanism for millisecond pulsars.

Key words: masers – radiation mechanisms: miscellaneous – pulsars: general – radio continuum: stars.

1 INTRODUCTION

The high radio luminosity of pulsars implies that the emission mechanism must be coherent. In most models for a pulsar magnetosphere (e.g. Goldreich & Julian 1969; Sturrock 1971; Michel 1982), relativistic electrons and positrons in their lowest Landau orbitals propagate away from the neutron star along the curved magnetic field lines. One possible emission process associated with such a system is curvature emission. Three theories for coherent curvature emission have been suggested: coherent curvature emission by bunches (Sturrock 1971; Ruderman & Sutherland 1975; Cheng & Ruderman 1980), curvature maser via inhomogeneity due to curvature of magnetic field lines (Beskin, Gurevich & Istomin 1988), and curvature maser by curvature drift (Zheleznyakov & Shaposhnikov 1979). The theory of coherent curvature emission by bunches was criticized by Melrose (1978) on several grounds, including: (i) the lack of a satisfactory mechanism to produce bunching, and (ii) the fact that, due to velocity spread and ponderomotive force, bunching cannot be sustained over the characteristic time of emission. The relation of the version of coherent curvature emission proposed by Beskin *et al.* (1988) to either the bunching mechanism or the maser mechanism remains obscure. Moreover, the existence of such a mechanism has been questioned on fundamental grounds (Nambu 1989; Machabeli 1991) and it is not discussed further here. A maser version of curvature emission, as first suggested by Zheleznyakov & Shaposhnikov (1979), is possible despite earlier suggestions to the contrary (Blandford 1975; Melrose 1978), provided that a curvature drift is included. This drift is necessarily present, as discussed in Section 2. Although application of the maser curvature emission mechanism to pulsars has been considered (Zheleznyakov & Shaposhnikov 1979; Chugunov & Shaposhnikov 1988), its viability remains uncertain. This may be attributed partly to an error in the calculation of the spectral power, and partly to the requirements on the electrons and positrons not being clear. In this paper we discuss in detail the maser curvature emission mechanism and its possible application to pulsars.

In Section 2 the curvature drift process is described and the emissivity for curvature drift emission is written down. The requirements on the Lorentz factor of electrons and positrons for maser curvature emission to be possible are derived in Section 3. The optical depth for maser emission is derived in Section 4 and used to determine the frequency range and minimum magnetic field for the amplification to be effective. The results and their implications are discussed in Section 5.

2 COHERENT CURVATURE EMISSION BY CURVATURE DRIFT

Blandford's proof (1975) that curvature maser emission is not possible depends on his neglect of curvature drift. Zheleznyakov & Shaposhnikov (1979) argued that, when the curvature drift is included, maser emission is possible. Here, we discuss the curvature drift and explain why its inclusion is essential for maser action.

2.1 Curvature drift

The pulsar magnetosphere is assumed to be populated by a relativistic pair plasma (electrons and positrons) which is created above the polar cap and flows out along open field lines. The electrons and positrons lose their perpendicular energy rapidly due to synchrotron radiation in the strong magnetic field, and are assumed to be in their lowest Landau orbitals. Hence their motion is essentially one-dimensional along the field lines.

An electron or a positron moving along a curved field line must be subjected to some force that causes it to follow the curved path. Zheleznyakov & Shaposhnikov (1979) argued correctly that a Lorentz force is required to cause the particle to follow the curved trajectory, and this requires a velocity component perpendicular to the plane that contains the field lines. The required Lorentz force is $(q/c)\boldsymbol{v}_d \times \boldsymbol{B}$, with $\boldsymbol{v}_d$ (called the drift velocity) given by

$$\boldsymbol{v}_d = \frac{v_\varphi^2 \gamma}{\omega_B R_B} \boldsymbol{e}_z, \tag{1}$$

where v_φ is the velocity along the field lines, γ is the Lorentz factor, $\omega_B = qB/mc$ is the non-relativistic gyrofrequency and R_B is the curvature radius of the magnetic field lines. The unit vector $\boldsymbol{e}_z$ is the binormal vector of the field lines. The expression (1) for the drift velocity may be obtained in a variety of ways. The drift may be obtained directly from orbit theory, where an average over the gyromotion about the field lines is performed. There is, however, a term called the centrifugal drift (e.g. Schmidt 1966, p. 50) that depends on the curvature of the field lines and is independent of the gyromotion; this term reproduces equation (1). Another viewpoint is that of a fluid treatment of the motion of the plasma in a pulsar magnetosphere (e.g. Mestel *et al.* 1985). The flow (velocity $\boldsymbol{v}$, Lorentz factor γ) may be separated into the rotational motion plus a flow that is along a direction

$$\boldsymbol{B}^* = \boldsymbol{B} + (mc/q)\nabla \times (\gamma \boldsymbol{v}),$$

rather than strictly along the magnetic field lines $\boldsymbol{B}$. The toroidal drift is thus superposed on the generalized isorotation term. The (non-rotational) velocity perpendicular to $\boldsymbol{B}$ reproduces equation (1). Finally, the drift was obtained by Shaposhnikov (1981) by solving Dirac's equation for a relativistic particle in its lowest Landau orbital in a curved magnetic field.

In the following discussion, it is more convenient to introduce a drift angle,

$$\theta_d = v_d / v_\varphi, \tag{2}$$

which is the angle between the momentum of the particle and the field-line direction. The sign of the drift is dependent on the sign of the charge: a negative θ_d corresponds to an electron drifting in the direction opposite to $\boldsymbol{e}_z$.

It follows from equations (1) and (2) that the drift angle is proportional to the particle energy. Hence, when the particle energy changes, for example due to emission of a photon, the drift angle also changes. This energy dependence is an essential ingredient in allowing negative absorption (maser action) to occur.

2.2 Curvature emission and absorption including curvature drift

To investigate the curvature emission and absorption including the curvature drift, one needs the emissivity with curvature drift included, which can be derived in a similar way as in Melrose's (1978, 1980) discussion of curvature emission without curvature drift. For ultrarelativistic particles, curvature radiation is concentrated in a narrow forward cone about the direction of motion of the particle, which is very nearly along $\boldsymbol{B}$. One may choose a local cylindrical coordinate system with $\boldsymbol{e}_\varphi = \boldsymbol{B}/B$, $\boldsymbol{e}_z$ the binormal vector and $\boldsymbol{e}_r = \boldsymbol{e}_\varphi \times \boldsymbol{e}_z$. In this coordinate system the wave vector $\boldsymbol{k}$ may be expressed as

$$k_r = k \sin\phi \approx k\phi,$$

$$k_\varphi = k\cos\phi\cos\theta \approx k(1 - \tfrac{1}{2}\phi^2 - \tfrac{1}{2}\theta^2)$$

and

$$k_z = k\cos\phi\sin\theta \approx k\theta,$$

where ϕ and θ are polar angles of $\boldsymbol{k}$ relative to $\boldsymbol{e}_\varphi$ and assumed to be small, $|\phi|, |\theta| \ll 1$. One first calculates current density associated with a particle moving along a drifted orbit (*cf.* equation 3, Melrose 1978; Luo & Melrose 1992). Using this current density one may obtain the total emissivity (summed over the two polarizations) in the presence of curvature drift (Luo & Melrose 1991):

$$\eta(\omega, \theta, \gamma) = \frac{q^2 \omega^2 R_B}{6\pi^3 c^2} \left\{ (\theta - \theta_d)^2 [\xi^{-1} K_{1/3}(y)]^2 + [\xi^{-2} K_{2/3}(y)]^2 \right\}, \tag{3}$$

where

$$\xi = \{2(1-n) + n[\gamma^{-2} + (\theta - \theta_d)^2]\}^{-1/2}$$

and y is defined by

$$y = \omega / (3n^{1/2} \omega_R \xi^3), \tag{4}$$

with $\omega_R = v_\varphi / R_B$. In the following discussion, the refractive index is set equal to unity, $n = 1$.

The absorption coefficient per unit time for a one-dimensional distribution (Melrose 1978, 1980) is

$$\Gamma(\omega, \theta) = -\frac{(2\pi c)^3 N_0}{2\omega^2 mc^2} \int d\gamma \frac{df(\gamma)}{d\gamma} \eta(\omega, \theta, \gamma), \tag{5}$$

where N_0 is the particle number density, and $f(\gamma)$ is normalized to unity. There must be a domain in γ over which $df(\gamma)/d\gamma > 0$, in order that the essentially positive η may nevertheless yield a negative $\Gamma(\omega, \theta)$. Integration by parts then yields the requirement that $d\eta/d\gamma$ be negative over some domain. The condition $df(\gamma)/d\gamma > 0$ requires an inverted energy population. If $d\eta/d\gamma < 0$ is also satisfied, this inverted energy population provides the source of free energy to drive the maser emission.

One may determine when negative absorption is possible by substituting equation (3) into equation (5). However, before carrying out detailed calculations it is appropriate to give a qualitative description of how the inclusion of the drift motion allows amplification to occur. Let $\bar{\eta}(\omega, \theta, \gamma)$ denote the emissivity for curvature emission in the absence of the curvature drift. Then, following Blandford (1975) and Melrose (1978), one finds $d\bar{\eta}/d\gamma > 0$, which implies that curvature maser emission is not possible. The emissivity,

$\eta(\omega, \theta, \gamma)$, for curvature emission with the curvature drift included may be obtained by replacing the angle θ by $\theta-\theta_d$ in $\bar{\eta}(\omega, \theta, \gamma)$, that is, one has $\eta(\omega, \theta, \gamma)=\bar{\eta}(\omega, \theta-\theta_d, \gamma)$. Then, since $\theta_d \propto \gamma$, one has

$$\frac{d\eta}{d\gamma}=\frac{\partial\eta}{\partial\gamma}-\frac{\theta_d}{\gamma}\frac{\partial\eta}{\partial\theta}. \qquad (6)$$

It follows that negative absorption is possible in principle, provided that one has $\partial\eta/\partial\theta>0$ for $\theta_d>0$, or $\partial\eta/\partial\theta<0$ for $\theta_d<0$, if the second term on the right-hand side of equation (6) is greater in magnitude than the first term. A graphical description of this interpretation is shown in Fig. 1.

3 ENERGY REQUIREMENT FOR ELECTRONS AND POSITRONS

Curvature maser emission requires the curvature drift, but it is driven by the free energy in the particle beam. Hence, for maser emission to occur, the Lorentz factor of the particles must be larger than a certain value. This minimum energy requirement may be estimated in the following way. In equation (6), the first term is proportional to $1/\gamma^3$ and the second term is proportional to $\theta_d\theta/\gamma$. The condition $d\eta/d\gamma<0$ requires that the second term be comparable to the first term at a viewing angle where the emissivity is not too small (Luo & Melrose 1992). Hence one requires

$$\gamma^2|\theta_d|\Delta\theta_0>1, \qquad (7)$$

where $\Delta\theta_0$ is the angular width of the radiation pattern. Note that (7) is only a necessary condition for masering to occur. (If equation 7 is not satisfied, the maser curvature emission is not possible.) Three cases need to be considered, corresponding to high, low and intermediate frequencies (e.g., Jackson 1962, p. 485).

First, consider emission at frequencies $\omega \ll \omega_c$, where $\omega_c=\gamma^3\omega_R$ is the characteristic frequency of the curvature radiation. The angular width is then $\Delta\theta_0 \approx (3\omega_R/\omega)^{1/3}$. Since from (2),

$$|\theta_d|=\gamma v_\varphi/\omega_B R_B=\gamma\omega_R/\omega_B,$$

from (7) the Lorentz factors must satisfy the condition

$$\gamma>(\omega/\omega_R)^{1/9}(\omega_B/\omega_R)^{1/3}\approx(\omega_B/\omega_R)^{1/3}.$$

In a pulsar magnetosphere, one-dimensional motion plausibly applies only within the light cylinder. Then the magnetic flux density is typically within the range 10^6–10^{12} G. The curvature radius cannot be less than 10^4 m for a dipolar magnetic field, and in this case the necessary condition for the curvature maser to operate is that the Lorentz factor cannot be less than 10^3.

The two other cases (Jackson 1962, p. 485) are medium frequencies, $\omega\approx\omega_c$, when the angular width is $\Delta\theta_0\approx 1/\gamma$, and high frequencies, $\omega \gg \omega_c$, when the angular width is $\Delta\theta_0\approx(\omega_c/\omega)^{1/2}/\gamma$. One then obtains $\gamma>(\omega_B/\omega_R)^{1/2}$ for medium frequencies and $\gamma>(\omega/\omega_R)^{1/7}(\omega_B/\omega_R)^{2/7}$ for high frequencies. With $\omega\gg\omega_c=\gamma^3\omega_R$, then for high frequencies one has $\gamma\gg(\omega_B/\omega_R)^{1/2}$. Since appreciable amplification

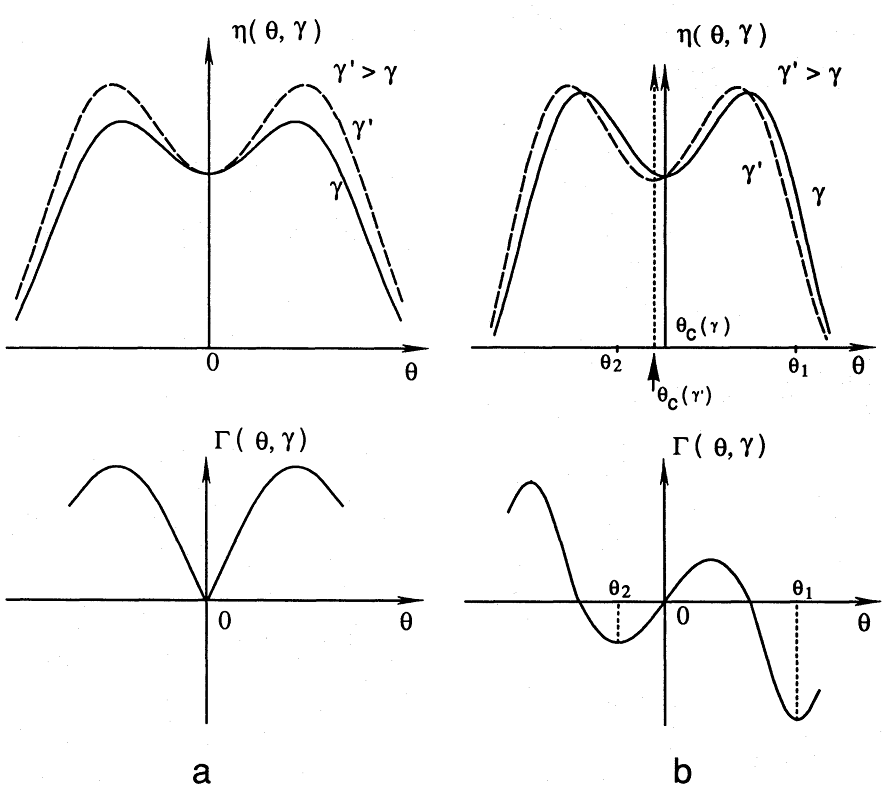

Figure 1. Curvature maser emission via curvature drift. (a) Curvature emission in which the curvature drift is neglected. (b) Curvature emission when the curvature drift is taken into account. The drift angle θ_d is negative. The centre of the emissivity curves is located at $\theta_c\approx\theta_d$. The maximum amplification occurs at $\theta_1\approx\Delta\theta_0$ and $\theta_2\approx-0.3\Delta\theta_0$. (The absorption curves are displayed below the emissivity curves corresponding to each case.)

requires that the modulus of the effective optical depth be greater than unity, in order to determine in which frequency range the amplification is important, one needs to estimate the optical depth. In the next section it is shown that, for $\omega \approx \omega_c$ and $\omega \gg \omega_c$, amplification is unimportant due to the small optical depth.

4 OPTICAL DEPTH

To estimate the optical depth, it is adequate to consider a line of sight along a bunch of magnetic field lines with curvature radius R_B. The radiation is within a small forward cone ($|\phi| \ll 1$ and $|\theta| \ll 1$). The integration along the line of sight can be obtained by multiplying the absorption coefficient in equation (5) by a characteristic emission time $\Delta\theta_0 R_B/c$, since the absorption coefficient is independent of angle ϕ in the small-angle approximation. Then, for a nearly mono-energetic distribution of radiating particles of a single species (electrons or positrons), the maximum modulus of optical depth for negative absorption is estimated to be

$$\tau_a \approx \frac{1}{8\pi}\left(\frac{\tilde{\omega}_B}{\omega}\right)\left(\frac{W_p}{W_m}\right)\Delta\theta_0, \tag{8}$$

at viewing angle $\theta \approx \pm 0.3\Delta\theta_0$, where the positive sign corresponds to the drift angle $\theta_d > 0$, and the negative sign corresponds to $\theta_d < 0$ (see Fig. 1b). Here $\tilde{\omega}_B = \omega_B/\gamma$ is the relativistic gyrofrequency, $W_p = mc^2\gamma N_0$ is the particle energy density and $W_m = B^2/8\pi$ is the magnetic energy density. The one-dimensional motion requires that $W_p/W_m \lesssim 1$. As discussed above, we have

$$\Delta\theta_0 = \begin{cases} (3\omega_R/\omega)^{1/3} & \text{for } \omega \ll \omega_c, \\ 1/\gamma & \text{for } \omega \approx \omega_c, \\ (\omega_c/\omega)^{1/2}/\gamma & \text{for } \omega \gg \omega_c. \end{cases} \tag{9}$$

There is another negative absorption peak with the maximum modulus of optical depth about $2\tau_a$ at viewing angle $\theta \approx \pm\Delta\theta_0$, where the upper sign corresponds to $\theta_d < 0$ and the lower sign to $\theta_d > 0$ (see Fig. 1b). From (8), one may conclude that amplification is important only when the frequencies satisfy $\omega \lesssim \tilde{\omega}_B$. As discussed in the previous section, for $\omega \approx \omega_c$ and $\omega \gg \omega_c$, one has

$$\tilde{\omega}_B/\omega \lesssim \omega_B/(\gamma^4\omega_R) < \omega_R/\omega_B < 1,$$

so only the case $\omega \ll \omega_c$ fulfils the requirement $\omega \lesssim \tilde{\omega}_B$.

We may draw the following conclusions from the foregoing discussion of the curvature maser.

(i) Effective amplification occurs only in the low-frequency range, that is, $\tau_a > 1$ can be satisfied only for $\omega \ll \omega_c$. One can estimate the magnetic field B required for effective maser curvature in the emission region. From equation (8), with $\Delta\theta_0 \approx (3\omega_R/\omega)^{1/3}$ for $\omega \ll \omega_c$, one finds that $B > 1.6 \times 10^{-6}\,(\omega/\omega_R)^{1/6}\omega^2/\omega_R$ G is required to obtain effective amplification $\tau_a > 1$. Therefore, for the frequency to be in the radio band, the magnetic field cannot be too small. For emission in the range 2–200 MHz one requires $B > 10^8$ G. It follows that, for a dipolar magnetic field with $B_0 = 10^{12}$ G at the stellar surface, the emission region must be within 20 stellar radii of the star.

(ii) For maser action to be possible the Lorentz factor must be high enough, typically $\gamma \gtrsim 10^3$, which is determined by the geometry of the pulsar magnetospheres (*cf.* Section 3).

(iii) The amplification favours a small curvature radius. If one takes $B \approx 10^{10}$ G, $\omega/2\pi = 200$ MHz, and $W_p/W_m = 0.1$, from (8) one has $R_B < 4 \times 10^4$ m to achieve $\tau_a > 1$, which is about the order of the stellar radius. Such a small curvature radius is possible only near the stellar surface, where the dipole approximation may not be valid. In addition to a small curvature radius, effective amplification also requires that the ratio of particle energy density to magnetic field energy density be not too small (see equation 8), but restricted by $W_p/W_m \lesssim 1$.

According to the first of these conclusions the curvature maser mechanism requires a strong magnetic field in the source region, and this seems to be excluded for millisecond pulsars. Hence, even if it can be argued that curvature maser emission is a possible mechanism for other pulsars, a different emission mechanism would seem to be required for millisecond pulsars.

5 CONCLUSIONS AND DISCUSSIONS

We have discussed curvature maser emission due to the curvature drift. The estimation of optical depth (8) is based on the absorption coefficient (5) which is smaller than Chugunov & Shaposhnikov's (1988) by a factor $2\pi/\Delta\theta_0$. The difference is due to the fact that they used the normalized time interval $\Delta\theta_0 R_B/c$ instead of $2\pi R_B/c$ (see equation 3 and equation 15 in Chugunov & Shaposhnikov 1988). The detailed analysis presented here shows that appreciable amplification can be obtained only in strong magnetic fields. This condition can be satisfied by a typical pulsar with surface magnetic flux density B_0 of order of 10^{12} G. However, it places a severe constraint on its application to millisecond pulsars, since it is believed that the magnetic flux density on the surface of such pulsars is of the order of 10^8 G.

The discussions in Section 4 are based on the assumption that the beam is of a single species of particles (only electrons or positrons). If the beam is composed of electrons and positrons and is quasi-neutral, that is, each of them has nearly the same distribution, the amplification is further reduced due to cancellation from each species. This can be shown from equations (6) and (5), in which occurrence of maser action at a certain viewing direction depends explicitly on the sign of the charge, e.g. amplification at the viewing angle $\theta \approx 0.3\Delta\theta_0$ for positrons corresponds to true absorption for electrons. If the radiating particles are composed of electrons and positrons, the absorption coefficient involves integration over both electrons and positrons. Hence the total optical depth is smaller than (8).

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