Energy Dissipation due to Mass Loss in a Rotating System

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Abstract: When a rotating magnetised system (angular speed Ω), such as a planet or star, loses mass there is necessarily an energy dissipation associated with the mass loss. Consider mass loss at rate $\dot{M}$, such that the matter is flung off with the orbital speed ΩR_1 at a radius $R_1 \gg R_0$, where R_0 is the radius of the planet or star. The power released is approximately equal to the power $P_{rot} = \frac{1}{2}\dot{M}\Omega^2 R^2$ carried off in rotational kinetic energy. Part of the energy released is carried off as magnetic energy in the escaping plasma, and the remainder is released through dissipation of currents. Such dissipation plausibly leads to the acceleration of particles. The power released should be important for Jupiter and for some rapidly rotating stars. For most stellar systems, the power released is small compared to that required to drive a wind.

1. Introduction

A familiar result in dynamics is that conservation of linear momentum does not imply conservation of energy, for example in an inelastic collision. The difference between the initial and final kinetic energies goes into internal energy or is dissipated as heat. A similar result applies to a rotating system that loses mass: conservation of angular momentum does not imply conservation of energy (Melrose 1979). Mass loss liberates energy, which may appear as an otherwise unsuspected source of energy internal to the system. Mass loss from a planet or star with a magnetosphere is of particular interest. In such a system, stresses are transferred by the magnetic field or, equivalently, through electric currents. The source of internal energy should be associated with the dissipation of these currents. This energy should appear in the form of anomalous heating of an ionospheric or chromospheric region or in the form of energetic particles whose source of energy is otherwise unexplained.

The central point made here is relatively subtle. It is obvious that rotation can provide energy, and it is widely recognised that in rapidly rotating systems particles can be accelerated as a result of rotation (e.g., Carbary Hill and Dessler 1976; Havnes and Goertz 1984). This acceleration may be interpreted in terms of a corotating particle in a centrifugal potential well that gets deeper with increasing distance (e.g., Melrose 1967). The energy source discussed here is related to, but is distinct from such acceleration. When any matter gains rotational energy in this way, the rotational energy of the planet or star decreases correspondingly and, in addition, a comparable amount of energy is necessarily released, but the form in which this energy release occurs is not determined and is not immediately obvious.

2. The Energy Released during Mass Loss

Consider a cylindrically symmetric, rotating system of radius R_1 that (a) is rotating with uniform angular velocity, Ω, (b) has a central mass concentration of radius $R_0 \ll R_1$, (c) can transport mass from R_0 to R_1 at constant angular velocity, and (d) loses mass from R_1. This idealised case is a reasonable approximation for planetary or stellar magnetosphere that loses mass, provided that the radial component of the velocity of the escaping material is much less than the tangential component. The case of a wind is excluded as it corresponds to this inequality being reversed.

Let the initial mass and moment of inertia of the central mass be M and I, respectively. Consider the effect of a loss of an element δM of mass at R_0 such that this mass is transported out to R_1 at constant Ω where it is thrown off with no radial component of velocity. Then I decreases by

$$\delta I = \delta M R_0^2. \quad (1)$$

The angular momentum carried off is $\delta M \Omega R_1^2$, and conservation of angular momentum implies that the angular momentum of the central mass decreases by this amount. Let the angular speed of the central mass decrease by $\delta\Omega$. Then conservation of angular momentum implies

$$\delta I\Omega + I\delta\Omega = \delta M \Omega R_1^2. \quad (2)$$

Combining (1) and (2) gives

$$I\delta\Omega = \delta M\Omega(R_1^2 - R_0^2). \quad (3)$$

Let the energy released be δE, that is, δE is the difference between the initial rotational energy of the central mass and the final rotational energy of this mass plus the kinetic energy of the element of mass thrown off.

$$\delta E = \tfrac{1}{2}I\Omega^2 - \tfrac{1}{2}(I - \delta I)(\Omega - \delta\Omega)^2 - \tfrac{1}{2}\delta M\Omega^2 R_1^2. \quad (4)$$

Assuming that the fractional changes in M, I and Ω are small, using (1) and (3) in (4) gives

$$\delta E = \tfrac{1}{2}\delta M\Omega^2(R_1^2 - R_0^2). \quad (5)$$

Hence, for $R_1^2 \gg R_0^2$, the energy released is approximately equal to the rotational kinetic energy carried off by the escaping mass.

3. Magnetic Energy Loss

The calculation leading to (5) gives no information on where the excess rotational energy goes. One may identify two obvious sinks: part of this energy is in the form of magnetic energy carried off in the escaping mass, and part of the energy must be released through dissipation of currents. The latter is clear from the theory of magnetic winds (e.g., Mestel 1968, 1990): the transfer of angular momentum to the wind is due to a torque associated with a $\mathbf{J} \times \mathbf{B}$ force, where $\mathbf{J}$ is the current density, and the dissipation of this current is another sink for the energy.

A plausible sequence of events leading to the separation of a blob of escaping plasma from the magnetosphere is illustrated in Figure 1. While the mass is confined by the magnetic field of the planet or star, the magnetic energy density must exceed the sum of the kinetic and thermal energy densities. Now suppose a blob of plasma becomes separated from the magnetosphere through formation of an X-type neutral point where magnetic reconnection occurs. In order for this to occur, the plasma motion must draw out the magnetic field, and this requires that the kinetic energy density exceed the magnetic energy density in the blob. This has two important implications. First, it implies that the magnetic energy carried off by the blob must be less than its rotational energy. Thus, although a fraction, α say, of the energy released is carried off in the form of magnetic energy within

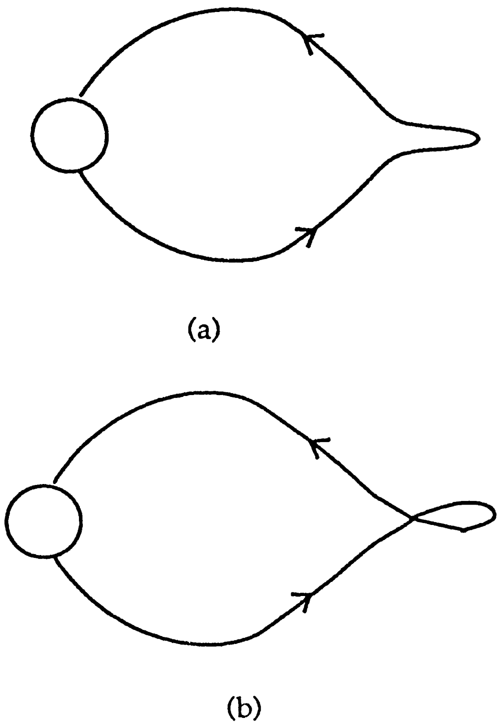

Figure 1 — The breaking off of a plasma blob in the equatoral plane of a rotating planet or star is illustrated. (a) The blob extends the magnetic field until (b) field lines reconnect allowing the blob to detach from the magnetosphere.

the blob, the separation process itself requires that this fraction be significantly less than unity. For simplicity in the following discussion, $\alpha = \frac{1}{2}$ is assumed.

A second implication of the reconnection process is that it requires dissipation of currents. Some of this dissipation occurs in the neighborhood of the X-type neutral point that forms. More generally, as the X-type neutral point forms and the reconnection proceeds, the currents associated with the distorted (e.g., from dipolar) magnetic field must adjust so that the initial magnetic configuration changes to the final magnetic configuration. The currents tend to adjust by flowing along the field lines to the ionosphere, where the Pedersen conductivity allows them to flow across field lines. Such currents may be associated with Alfvén waves, generated due to the sudden change in the magnetic stress as a blob of plasma breaks off. Dissipation of the current may occur in three regions: locally in the neighborhood of the X-type neutral point, in the ionosphere due to the Pedersen conductivity, and along the field lines due to anomalous conductivity. A detailed analysis is required to investigate these various possibilities. One plausible argument is that the dissipation is similar to that of the Alfvén waves generated by Io's motion through the magnetosphere of Jupiter, which dissipation is thought to lead to the acceleration of the electrons that produce the decametric radio bursts (e.g, Hill, Dessler and Goertz 1983).

4. Application to Jupiter

Consider the particular case of the magnetosphere of Jupiter. Corotation in a planetary magnetosphere is maintained by a viscous drag exerted by ion-neutral collisions in the ionosphere, and transferred to the magnetosphere by field-aligned currents. The details of this process were discussed by Hill (1979). Hill's model suggests that as mass drifts out through the magnetosphere, part of the rotational energy released should appear as heat due to current dissipation through ion-neutral collisions in the ionosphere, specifically, through the Pedersen conductivity. The height-integrated Pedersen conductivity is estimated to be in the range 0.02 — 10 mho (e.g., Strobel and Atreya 1983). Alternatively, the energy may be dissipated through the acceleration of fast particles in the magnetosphere. Such collisionless dissipation requires that the magnetosphere become anomalously resistive, due for example to the formation of multiple weak double layers (e.g., Smith and Goertz 1978), as occurs in the terrestrial auroral zones (e.g., Boström *et al.* 1989).

The major source of thermal plasma within the Jovian magnetosphere is the ionosphere of Io. Corotating plasma is in a potential well at Io (Melrose 1967), and must ultimately escape by convecting outwards and being thrown off. Goertz (1980) estimated that Io produces sulfur ions at the rate $\dot{N}_{S+} = 10^{28}\ s^{-1}$ and oxygen ions at the rate $\dot{N}_{O+} = 2 \times 10^{28}\ s^{-1}$. This corresponds to a mass loss rate of $\approx 10^3$ kg s^{-1}, which is well below the maximum loss rate $\approx 10^5$ kg s^{-1} consistent with confinement by the Jovian magnetic field (Huang and Siscoe 1987). Suppose that this matter is thrown off by Jupiter at about 50 Jovian radii. Then the power released is about 2×10^{14} W. Here it is assumed that half of this is carried off as magnetic energy in the escaping mass, and half is dissipated internally in the magnetosphere.

In the case of Jupiter, a power dissipated of 10^{14} W as a result of rotational mass loss is modest in the overall energy budget of the planet. Jupiter has a power output, predominantly in the infrared, that exceeds the power absorbed from the Sun, implying an internal source of energy (Hubbard 1968). The rotational contribution cannot account for this either qualititatively or quantitatively. Qualitatively, the power released by rotational losses is dissipated either in the partially ionised regions of the ionosphere or in anomalously resistive regions of the magnetosphere, whereas the excess infrared emission arises from deeper in the neutral atmosphere. Quantitatively, the observed power excess is $\approx 3 \times 10^{17}$ W (Hanel *et al.* 1981), which is far in excess of the power available from rotational losses. However, the power released through rotational losses may be important in the energetics of the magnetosphere and in the production of fast particles.

Dissipation of a current requires an effective resistance and a potential difference across it. At least in principle, such a potential difference is available for the acceleration of particles. A current, I, and a potential drop, Φ, lead to a power $P = I\Phi$. The inferred power implies $I\Phi = 10^{14}$ W, but further arguments are required to determine I and Φ separately. Three different arguments are as follows.

First, consider limits imposed on I and Φ from the requirement that the current close in the ionosphere. Suppose that the dissipation were entirely in the ionosphere, in a region with height-integrated conductivity of Σ_P. The effective resistance is then $1/\Sigma_P$. Ohm's law implies $I = \Sigma_P\Phi$, and hence $P = \Sigma_P\Phi^2$. With $P = 10^{14}$W the highest value, $\Sigma_P = 1$ mho, of the height-integrated conductivity estimated by Strobel and Atreya (1983) gives $\Phi = 1 \times 10^7$ V, $I = 1 \times 10^7$ A and their lowest value, $\Sigma_P = 0.02$ mho, gives $\Phi = 7 \times 10^7$ V, $I = 1.4 \times 10^6$ A.

Alternatively, for the dissipation to occur predominantly in the magnetosphere, in which case the energy would appear in fast particles, the magnetosphere needs to have an anomalous resistance, R_{an}, that exceeds the ionospheric resistance, $R_{an} > 1/\Sigma_P$. It follows that for given power P, dissipation in the magnetosphere requires that Φ be higher and I be lower than the foregoing estimates based on ionospheric dissipation alone. One concludes that a potential $\Phi > 10^7$ V must be associated with a rotational energy losses by Jupiter of $\approx 10^3$ kg s^{-1} at ≈ 50 Jovian radii.

Second, a plausible limit on the current may be inferred from the current required to distort the magnetic field substantially at the L-shell where the plasma is thrown off. To distort the dipolar field substantially requires

$$I \approx \frac{2\pi\, R_J\, B_J}{\mu_0\, L^2} \tag{6}$$

This gives $I \approx 4 \times 10^{11} L^{-2}$ A. This current normally closes in the outer magnetosphere and, for L = 30-50, only a small fraction of it needs to be deflected into the ionosphere to give the required dissipation.

Third, a plausible limit on the potential drop may be inferred from the potential difference between a corotating and a nonrotating flux tube. Assuming a dipolar field and a separation $R_J L$ between the rotating and nonrotating flux tubes, one finds

$$\Phi \approx \frac{\Omega R_J^2 B_J}{L} \tag{7}$$

This gives $\Phi \approx 10^9 L^{-1}$ V. It follows that, for L = 30-50, the required potential deduced above is close to this maximum value. This is plausible: when mass is thrown off in the escaping matter and the corotating matter left behind plausibly have a relative velocity that is a significant fraction of the corotation velocity, and hence the potential should be a significant fraction of this maximum value.

It is interesting that the potential drop associated with rotational energy dissipation probably exceeds the potential drop ($\approx 10^6$ V) induced by Io's motion through the magnetosphere. The potential drop associated with Io's motion is thought to be available for the acceleration of the electrons that produce Jupiter's radio emission (e.g., Goldreich and Lynden-Bell 1969). Hence, it seems likely that the potential drop induced by rotational losses should also be important in accelerating electrons.

Acceleration of electrons due to such potential drops along the field lines could account for two aspects of the Jovian radio emission and for the copious output by Jupiter of mildly relativistic electrons. First, although some of the decametric radio emission (DAM) is strongly correlated with Io, there is also non-Io related emission. Assuming that acceleration due to Io's potential drop leads to the Io-related DAM, it is plausible that acceleration due to the potential drop associated with rotational losses leads to the non-Io-related DAM. Second, a potential of a few MV could account for the electrons of a few MeV that produce the decimetric radio emission (DIM) through their synchrotron emission. Perhaps more relevant is the inference that Jupiter is a copious source of electrons of a few MeV (e.g., Schardt and Goertz 1983). A possible source for such electrons is acceleration on high L shells due to the potential associated with rotational losses. This suggests that the production of such electrons should correlate with the rate of plasma escape from the Io torus, and hence with the rate of injection of plasma due to Io's volcanism. An alternative model involving acceleration in the Jovian magnetotail would imply a correlation with the properties of the solar wind (Pesses and Goertz 1976).

5. Application to Stars

The power released through rotational losses in stars is unlikely to be important in most cases. The reason is that stellar mass loss usually involves a wind, and the power required to drive the wind is larger than the power available from rotational losses. For example, consider the solar wind. The plasma in the solar wind effectively corotates with the Sun out to about 10 $R_\odot$, where the orbital speed is ≈ 20 km s^{-1}. This is less than 0.1 times the radial speed of the solar wind. It then follows from (5) that the power released due to the rotational losses is less than 0.01 times the power supplied to drive the radial outflow of the solar wind.

However, there are some interesting possible exceptions where the rotational energy may be important. One is the magnetic Ap stars, which have exceptionally strong magnetic fields allowing corotation out to large distances. Any mass loss from these stars may then lead to a relatively large power dissipation per unit mass loss rate. Another example, for which there is some evidence for mass loss is the rapidly rotating, active star AB Doradus. Collier Cameron (1988) developed models for large coronal loops, to explain the X-ray emission from ABDor, and noted that these loops extend to well beyond the radius at which the centrifugal force balances gravity. Near the tops of such loops the hot plasma can condense to form neutral clouds, for which there is some observational evidence (e.g., Collier Cameron and Robinson 1989a,b). This neutral gas cannot return to the star, because of the centrifugal potential energy barrier, and must ultimately be thrown off. The mass loss rate for this star is unknown: the rotational power released is of order 3×10^{24} W times $\dot{M}$ in units of $10^{-10} M_\odot$ yr^{-1}. This power must be associated with a current, I, flowing along field lines between the neutral clouds and the denser regions of the stellar atmosphere, and a potential drop, Φ, along the field lines, such that the power is given by $I\Phi$. The limits (6) and (7) may be applied to the stellar case. Plausible values for a star correspond to Ω similar to that of Jupiter, a radius 30 times that of Jupiter, and a surface magnetic field 100 times that of Jupiter. Then (6) and (7) give $I \lesssim 10^{15} L^{-2}$ A, and $\Phi \lesssim 10^{14} L^{-1}$ V, respectively. By analogy with the Jovian case, only a small fraction of this current is likely to be deflected into the stellar photosphere, and the actual potential drop is likely to be a significant fraction of this maximum potential drop.

The power dissipated depends on the unknown rate of mass loss. For $\dot{M}$ in units of $10^{-10} M_\odot$ yr^{-1}, a rotation period of 10 hr and escape at a distance $10^9 L$ m, the power dissipated is of order $10^{23} \dot{M} L^2$ W. This suggests that only a modest mass loss is required to explain nonthermal radio emission from AB Dor (e.g., Slee *et al.* 1986), and that there is a potential of $\lesssim 10^{13}$ V available to accelerate the electrons that produce the radio emission. A detailed model for the acceleration of the electrons is needed for this suggestion to be pursued further.

6. Conclusions

Mass loss from a corotating planet or star provides an internal energy source, with the power determined by (5). The power can be expressed in the form $P = I\Phi$, where I and Φ are a current and a potential drop induced by the stresses required to provide angular momentum to the escaping mass. The required current is probably only a small fraction of that, cf. (6), associated with the distortion of the magnetic field in the mass loss region, and the potential drop is likely to be a substantial fraction of the maximum available, cf. (7).

This mechanism may account for the copious production of relativistic electrons by Jupiter, and for the electrons that produce the non-Io-related DAM. The model is relevant to mass loss from stars only when the outflow speed is less than the corotation speed. However, when this condition is satisfied, very large potential drops are available in the stellar atmosphere, e.g., $\lesssim 10^{13}$ V in AB Dor. One would expect electrons accelerated by this potential to be observable through nonthermal radio emission.

Acknowledgements

I thank Vladimir Usov and George Dulk for helpful comments.

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