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Electron cyclotron maser emission at oblique angles

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Abstract. Observations of elliptically polarized bursts of decametric radio emission from Jupiter with axial ratios of the polarization ellipse $0.2 \lesssim T \lesssim 0.7$ imply emission at angles $45° \lesssim \theta \lesssim 80°$ to the magnetic field. Emission at oblique angles $\lesssim 60°$ is not expected in the conventional theory of electron cyclotron maser emission. It is argued that electron cyclotron emission at a given θ requires electrons with parallel velocity concentrated around $v_\parallel = v\cos\alpha = c\cos\theta$. A mechanism is proposed that might produce an appropriate "spiraling beam" distribution, with a peak in velocity space at speed $v = v_0$ and pitch angle $\alpha = \alpha_0 \neq 0$.

1. Introduction

Recent data on the Jovian decametric emission (Lecacheux *et al.*, 1991; Dulk *et al.*, 1991) show it to be intrinsically elliptically polarized, with different axial ratios, T, in different subsources. One subsource, "Io–B", has a higher degree of linear polarization than circular $(Q^2+U^2)^{1/2}/I \approx 0.85$ and $V/I \approx -0.5$, where I, Q, U and V are the Stokes parameters and $V < 0$ corresponds to right-hand polarization. For this source, if one assumes $T = \cos\theta$ (Melrose and Dulk, 1991), where θ is the angle of emission, then one has $\cos\theta = 0.25$, corresponding to emission at $\theta = 76°$ from a source in the Northern Hemisphere. Another subsource "Io–A" has $V/I \approx -0.76$, corresponding to $T \approx 0.46$ and $\theta \approx 63°$. A further subsource "Io–D" (from Jupiter's Southern Hemisphere) has left-hand polarization (e.g. Carr *et al.*, 1983; Boudjada and Genova, 1991); the extension of this source into the Io–B region was found by Dulk *et al.* (1991) to have $V/I \approx +0.85$, corresponding to $T \approx 0.55$ and hence to $\theta \approx 56°$. With the exception of the "Io–B" subsource, these inferred angles of emission are not readily compatible with the standard version of electron cyclotron maser emission (ECME). Our purpose in this paper is to discuss the implications of this.

The favored version of ECME, due originally to Wu and Lee (1979), implies emission at angles close to but different from $\theta = 90°$. The predicted emission is confined to the surface of a thin hollow cone (e.g. Hewitt *et al.*, 1981, 1982), as seems to be the case for the Jovian decametric emission (e.g. Dulk, 1967; Gurnett and Goertz, 1981). The angle, θ, of this cone is determined by $\cos\theta = \beta_{\parallel 0}$ (Ladreiter, 1991) where the typical parallel velocity, $v_\parallel$, of the electrons that supply the free energy is written $v_\parallel = \beta_{\parallel 0}c$. An oblique angle of emission of 60° would require $\beta_{\parallel 0} = 0.5$. Thus, the electrons must be quite relativistic, outside the range of validity of the usual ECME theory. [We note that the approximate relation $T = \cos\theta$ needs to be modified for emission by mildly relativistic particles. A better approximation is $T = \cos\theta - (v/c)\cos\alpha$, so that an axial ratio $T = 0.5$ implies an angle $\theta < 60°$ when this relativistic correction is taken into account. This correction is ignored in the qualitative discussion below.]

Two different versions of ECME have been considered for the auroral kilometric radiation (AKR), which is a terrestrial counterpart of the Jovian decametric radiation. The version of Wu and Lee (1979) involves free energy in a loss-cone distribution of reflected electrons and leads to emission at angles θ close to, but not equal to 90°. An earlier version of ECME (Melrose, 1973, 1976) involves free energy in a streaming distribution and leads to emission at $\theta \lesssim 60°$, favoring $\theta \approx 0$. These are $p_\perp$-driven and $p_\parallel$-driven, respectively, in the notation of Melrose (1986, p. 190). A further alternative source of free energy, suggested by Louarn *et al.* (1990), is a distribution of electrons trapped by the magnetic mirror and an upward directed electric field, as observed by the *Viking* spacecraft. ECME due to such a trapped distribution, which is $p_\perp$-driven, favors emission at $\theta = 90°$, but such emission in the X-mode cannot escape directly from the plasma (e.g. Louarn *et al.*, 1990). There are two other aspects of the theory of ECME as applied to AKR that warrant comment. One is the role of emission in other modes and harmonics (e.g. Winglee, 1985), and the other is the effect of the distribution of energetic electrons, required to drive the maser, on the wave properties (e.g. Pritchett, 1984;

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Strangeway, 1985, 1986; Pritchett and Winglee, 1989; Winglee, 1989). Such effects are ignored here, in the belief that the plasma density in the source region is sufficiently low for them to be unimportant. The emission is assumed to be at the fundamental in the *X*-mode. For a sufficiently low-density plasma this assumption is equivalent to assuming that the wave properties are those of a vacuum.

The question addressed in the present paper is how one might account for ECME at an oblique angle $\theta \approx 60^\circ$. One possibility is to appeal to the $p_\parallel$-driven version and to seek a mechanism that favors emission at $\theta \approx 60^\circ$ rather than $\theta \approx 0$. Another possibility is to look for an intermediate version of ECME that favors emission at an intermediate angle. A third possibility, which we ultimately adopt here, is to appeal to the $p_\perp$-driven version, and to assume that the electrons have the mildly relativistic energies implied by the relation $\cos\theta = \beta_{\parallel 0} \approx 0.5$. The question then arises as to how the required distribution of electrons is produced.

In Section 2 we discuss general concepts of ECME in terms of resonant ellipses, in Section 3 we consider electron distributions required to produce ECME at oblique angles, and in Section 4 we consider how the most favorable of these distributions might be produced. Our conclusions are summarized in Section 5.

2. Resonance ellipses

In discussing possible interpretations of ECME at oblique angles, a convenient tool is the concept of the resonance ellipse in velocity space, which is the Doppler condition $\omega - s\Omega - k_\parallel v_\parallel = 0$, with $\Omega = \Omega_e/\gamma$, the relativistic gyrofrequency, plotted in velocity space ($v_\parallel$–$v_\perp$ space, or v–α space in polar coordinates).

Some resonance ellipses are illustrated in Fig. 1. In this paper we use the term "strictly nonrelativistic case" to describe ellipses where the Lorentz factor γ is put equal to unity in the resonance equation and the ellipse degenerates to a vertical line (I in Fig. 1). The "semirelativistic case" is when the Lorentz factor is expanded as $\gamma^{-1} = 1 - v_\parallel^2/2c^2 - v_\perp^2/2c^2$ and the ellipse is a circle (III in Fig. 1), and the "relativistic case" is when the ellipse passes through a part of velocity space where relativistic electrons lie (II in Fig. 1).

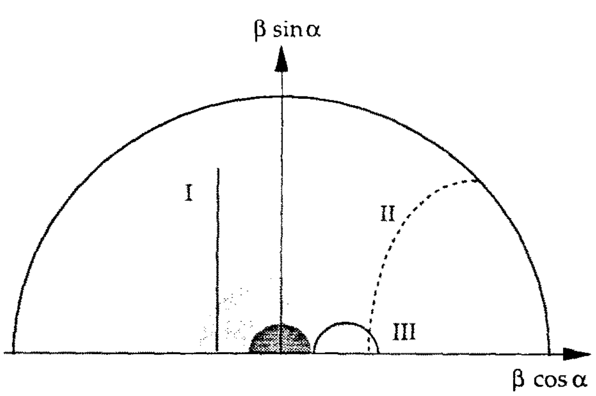

Fig. 1. Some examples of resonance ellipses are shown in the nonrelativistic region of velocity space. Thermal electrons are assumed to be confined effectively to the darkly shaded region and nonthermal electrons to the lightly shaded region. Curve I is a vertical line, curve II is an intermediate case, and curve III is an ellipse with small eccentricity and with its center and its radius comparable to the size of the shaded region. The outer semicircle corresponds to $v = c$

Quite generally, the resonance ellipse is centered on the $v_\parallel$ axis at $v_\parallel = v_c$, with semi-major axis v_R perpendicular to the $v_\parallel$ axis, and with eccentricity e. These parameters are given by:

$$\frac{v_c}{c} = \frac{\omega k_\parallel c}{s^2\Omega_e^2 + k_\parallel^2 c^2}, \quad \left(\frac{v_R}{c}\right)^2 = \frac{s^2\Omega_e^2 + k_\parallel^2 c^2 - \omega^2}{s^2\Omega_e^2 + k_\parallel^2 c^2},$$

$$e^2 = \frac{k_\parallel^2 c^2}{s^2\Omega_e^2 + k_\parallel^2 c^2}. \tag{1}$$

In the case of interest here, the emission is known to be at the fundamental, $s = 1$. The difference of the refractive index from unity is neglected; this is justified theoretically by Melrose and Dulk (1991) and observationally by Lecacheux *et al.* (1991). Then one has $k_\parallel = (\omega/c)\cos\theta$. If, in addition, the frequency is assumed to be close to the cyclotron frequency, then one has $|\omega - \Omega_e|/\Omega_e \ll |\cos\theta|$. With these assumptions the properties (1) may be approximated by:

$$\frac{v_c}{c} = \frac{\cos\theta}{1+\cos^2\theta}, \quad \left(\frac{v_R}{c}\right)^2 = \frac{\cos^2\theta - 2(\omega-\Omega_e)/\Omega_e}{1+\cos^2\theta},$$

$$e^2 = \frac{\cos^2\theta}{1+\cos^2\theta}. \tag{2}$$

The semi-minor axis is then approximated by:

$$\frac{v_R}{c}(1-e^2)^{1/2} = \frac{[\cos^2\theta - 2(\omega-\Omega_e)/\Omega_e]^{1/2}}{1+\cos^2\theta}$$

$$\approx \frac{|v_c|}{c} - \frac{(\omega-\Omega_e)/\Omega_e}{|\cos\theta|(1+\cos^2\theta)}. \tag{3}$$

Resonance ellipses that are relevant to ECME must pass through a region of velocity space where the electrons have available free energy. In the semirelativistic case the source of the free energy at given $v_\parallel$, $v_\perp$ defines an ellipse with $v_c \approx v_\parallel$, $v_R \approx v_\perp$, which are both $\ll c$. However, in the strictly nonrelativistic case, only the value of $v_\parallel$ is determined by the vertical line, cf. curve I in Fig. 1. In this case, both v_c and v_R are of order unity, and such an ellipse has an arc that gives the required vertical line only if v_c and the semi-minor axis of the ellipse are nearly equal. Specifically, the value of $v_\parallel/c \ll 1$ defined by the vertical line corresponds to the point where the ellipse intersects the horizontal axis, that is, to:

$$\frac{v_\parallel}{c} = \frac{v_c}{c} - \frac{v_R}{c}(1-e^2)^{1/2} \approx \frac{(\omega-\Omega_e)/\Omega_e}{|\cos\theta|(1+\cos^2\theta)}, \tag{4}$$

where the direction of $v_\parallel$ and v_c is assumed to be positive. A further restriction on $\omega - \Omega_e$ arises from the requirement that the frequency of emission be above the cut-off frequency of the *X*-mode:

$$\frac{\omega - \Omega_e}{\Omega_e} \gtrsim 2\frac{\omega_p^2}{\Omega_e^2}, \tag{5}$$

where the refractive index is close to unity.

For the strictly nonrelativistic case, in contrast to the semirelativistic case, specifying the location of the source of free energy in velocity space does not determine both θ and ω independently, but only provides the relation (4) between them. This leads to an important difference between $p_\parallel$-driven and $p_\perp$-driven ECME, in that the former is relatively insensitive to $\theta \lesssim 60^\circ$, whereas the latter selects a small range at $\theta \approx 90^\circ$.

3. Electron distributions

The foregoing discussion involves only the kinematics of the ECME. In this section the source of the free energy for the ECME is discussed.

Absorption coefficient

The gyromagnetic absorption coefficient (which must be negative for ECME to occur) at the sth harmonic for waves in a mode M is (e.g. Melrose *et al.*, 1982; Melrose, 1986, p. 180):

$$\gamma_M(s,\omega,\theta) = -2\pi \int_0^\infty \mathrm{d}p p^2 \int_{-1}^{1} \mathrm{d}\cos\alpha A_M(s,\omega,\theta;\beta,\alpha) f_\Delta,$$

$$f_\Delta = \left[\frac{\partial}{\partial p} + \frac{\cos\alpha - n_M\beta\cos\theta}{p\sin\alpha}\frac{\partial}{\partial\alpha}\right] f$$

$$= \left[\frac{s\Omega_e}{\omega\sin\alpha}\frac{\partial}{\partial p_\perp} + \frac{n_M\cos\alpha}{mc} p \frac{\partial}{\partial p_\parallel}\right] f, \qquad (6)$$

where the detailed form of A_M in equation (6) is unimportant for present purposes. Henceforth the refractive index n_M is set equal to unity and only emission at $s = 1$ in the X-mode is considered.

The two terms in the final form in equation (6) are referred to above as the $p_\perp$-gradient and $p_\parallel$-gradient terms, respectively. ECME may be driven by either term: ECME in the semirelativistic approximation is due to the $p_\perp$-gradient (e.g. Wu and Lee, 1979; Hewitt *et al.*, 1982) and ECME in the strictly nonrelativistic limit is due to the $p_\parallel$-gradient.

The observed distribution of electrons associated with AKR is not consistent with the $p_\parallel$-driven mechanism, as described by Melrose (1976). The observed distribution has two features that could drive ECME at large angles: a loss-cone distribution in the reflected electrons and a trapped distribution (Louarn *et al.*, 1990). The trapped electrons, being centered on $\alpha = 90^\circ$ favor ECME at large angles. It is possible that a loss-cone distribution could account for emission at angles $\theta \approx 60^\circ$, but only if the electrons have speeds $v \approx c/2$.

Possible sources of free energy

Three general types of distribution function could lead to ECME at $\theta \approx 60^\circ$:

(1) A positive $p_\parallel$-gradient for nonrelativistic electrons (close to the origin of velocity space), as in the model proposed by Melrose (1976).

(2) An intermediate case with both a positive $p_\perp$-gradient and a positive $p_\parallel$-gradient.

(3) A positive $p_\perp$-gradient for mildly relativistic electrons, with $v_\parallel/c \approx 0.5$, corresponding to a mildly relativistic version of the model proposed by Wu and Lee (1979).

These three possibilities, which are illustrated in Figs 2, 3 and 4, respectively, are discussed separately below.

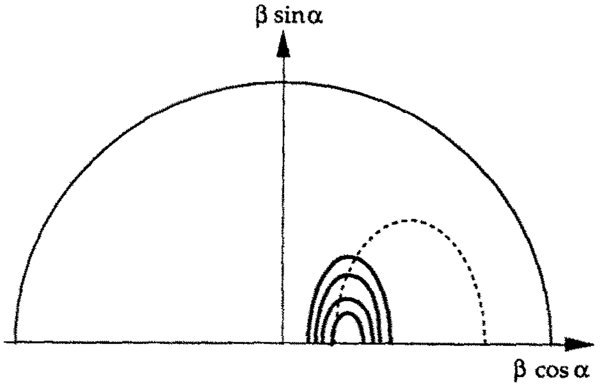

Fig. 2. Contours of an idealized distribution are indicated by dark lines in velocity space. The contours correspond to ECME driven by a $p_\parallel$-gradient. A resonance ellipse corresponding to $\cos\theta \approx 0.5$ is indicated by the dashed curve

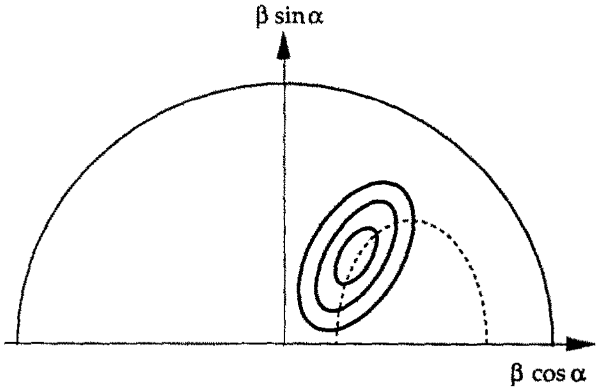

Fig. 3. As in Fig. 2, with the contours oriented at 60° to the horizontal

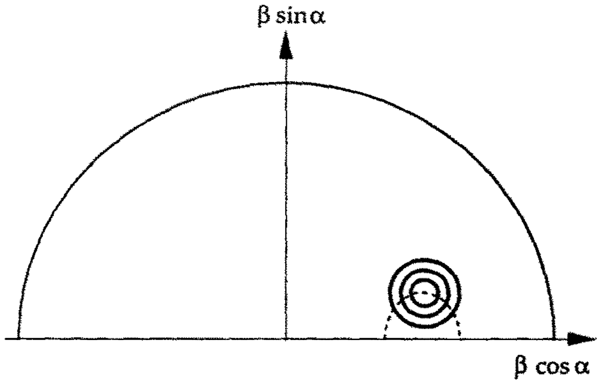

Fig. 4. As in Fig. 2 for a spiraling-beam distribution

Parallel-driven ECME at oblique angles

In the strictly nonrelativistic case the ellipse is approximated by a vertical line, cf. Fig. 1. The integral of the $p_\perp$-gradient term over $p_\perp$ is then proportional to:

$$\int_0^\infty \mathrm{d}p_\perp \frac{\partial}{\partial p_\perp} f(p_\perp, p_\parallel) = -f(0, p_\parallel), \tag{7}$$

which always contributes to positive damping and not to growth. As a consequence, in this strictly nonrelativistic case any growth must be driven by the $p_\parallel$-gradient term.

There is a range of ellipses, with semi-minor axis $v_\mathrm{R}(1-e^2)^{1/2}$ approximately equal to v_c, that all have nearly vertical sections near the peak in the distribution function in Fig. 2. These ellipses have centers over a wide range, and equation (2) implies that this corresponds to a range of $\cos\theta \gtrsim 0.5$, and hence to the range $0 \leqslant \theta \lesssim 60^\circ$ in which the $p_\parallel$-driven ECME can operate. To account for preferential emission at $\theta \approx 60^\circ$ some additional effect is required. There seems no reason why the maser itself should favor this angle, and hence one needs to look for an independent reason for favoring oblique emission. One possibility is to appeal to the geometric structure of the region where the maser operates. Effective growth depends not only on the growth rate but also on the path length over which growth occurs, and the ray path that maximizes the growth may be at an oblique angle. For example, suppose that the electrons that drive the ECME are localized in space in a pancake distribution perpendicular to the field lines. Growth at an oblique angle could then be favored due to a competition between two effects: $p_\parallel$-driven ECME favoring growth at $|\cos\theta| \lesssim 0.5$, and the pancake (in coordinate space) distribution of electrons for which the maximum growth length is across the field lines. However, such a model seems contrived when one is attempting to account for observed emission that seems to be confined to a relatively narrow range of oblique angles. Another possibility is that emission occurs at all $\theta \lesssim 60^\circ$ for individual sources, but the only ray paths that intersect the Earth correspond to a specific value of θ. This type of geometric structure has been invoked to account for the arc structure in the *Voyager* data (e.g. Gurnett and Goertz, 1981). However, this is implausible for the $p_\parallel$-driven version of ECME because the emission at all angles is driven by the same electrons, and the faster growing waves at $\theta \approx 0$ should exhaust the supply of free energy before the slower growing waves at larger θ have time to grow significantly.

There are other difficulties in a model for Io-related emission based on this strictly nonrelativistic form of ECME. Perhaps the most important is that the maximum growth rate is much less than that for the semirelativistic version of ECME. As a consequence, $p_\parallel$-driven ECME must be regarded as intrinsically less plausible than $p_\perp$-driven ECME. Another difficulty is that there is no obvious mechanism for forming a distribution function of the form illustrated in Fig. 2. As a consequence, it seems implausible to invoke $p_\parallel$-driven ECME to account for emission at a characteristic oblique angle $\theta \approx 60^\circ$.

Is there an intermediate form of ECME?

In view of $p_\parallel$-driven ECME favoring emission at small angles and $p_\perp$-driven ECME favoring emission at large angles, the question arises as to whether there is an intermediate form of ECME that favors emission at intermediate angles. To discuss this possibility consider the distribution function with positive gradients in both $p_\perp$ and $p_\parallel$. An idealized case of such a distribution is illustrated in Fig. 3: there is a peak that defines a characteristic speed, v_0, and a characteristic pitch angle, α_0, with the contours around the peak elongated along a direction that is at an oblique angle. The most favorable resonance ellipse corresponds to that which samples the maximum positive values of f_Δ. Now, because the coefficient of the $p_\perp$-gradient term in equation (6) is much larger than the coefficient of the $p_\parallel$-gradient term, it is essential that the resonance ellipse samples the region where the $p_\perp$-gradient is large and positive, and it is relatively unimportant if the ellipse passes through a region where the $p_\parallel$-gradient is negative.

The fact that the contribution from the $p_\perp$-gradient is much more heavily weighted than the contribution from the $p_\parallel$-gradient in equation (6) implies that the $p_\parallel$-gradient is unimportant in practice. Granted that the $p_\perp$-gradient dominates, the conditions under which growth maximizes are effectively the same as in the $p_\perp$-driven version of ECME. In effect this means that the intermediate case does not exist, or rather that it exists only for distributions that are improbable or contrived. This argument suggests that the maximum growth for the distribution function illustrated in Fig. 3 is not for the resonance ellipse drawn, which is chosen to have contributions from positive gradients in both $p_\perp$ and $p_\parallel$, but is for an ellipse with center at $v_\mathrm{c} \approx v_0 \cos\alpha_0$ that optimizes the contribution of the $p_\perp$-gradient. Numerical calculations are required to confirm this qualitative conclusion, and initial results (A. Willes, private communication, 1992) support this.

Spiraling-beam-driven ECME at oblique angles

The above arguments lead to the conclusion that the most plausible explanation for ECME at an oblique angle, θ, is the $p_\perp$-driven maser with the free energy concentrated at $v_\parallel/c \approx \cos\theta$. This still leaves several possible distribution functions that are qualitatively different.

One possible distribution function is a loss-cone distribution of the form suggested by Wu and Lee (1979), but scaled up to higher electron velocities. A difficulty with this suggestion is that one would expect such a distribution to contain a range of electron speeds, and hence one would expect ECME at all angles θ satisfying $(v/c)\cos\alpha_0 = \cos\theta$. Moreover, one would expect the distribution to be a decreasing function of increasing v, and hence for the maximum growth to occur at the smallest value of $\cos\theta = (v/c)\cos\alpha_0$ for which electrons are present. It could be that such emission does occur for individual sources, but is not seen because its ray path does not intersect the Earth. Alternatively, it might be that the electron distribution peaks at a speed, $v \sim c/2$, of the order of that inferred. Thus, in order for a loss-cone distribution to be favorable, one needs to combine it with a geometric model which allows only emission at a particular angle to intersect the Earth. (Note that an argument against a similar suggestion was made above in connection with $p_\parallel$-driven ECME which relied on the fact that, for $p_\parallel$-driven ECME, growth at all angles is due to the same electrons. That argument is not relevant here because, for $p_\perp$-driven ECME due to a loss-cone distribution, the emission at a given θ is driven by electrons with a given v, and the free energy in electrons with a given v is not affected by ECME at other angles.)

Another way in which ECME at an oblique angle can be favored is illustrated schematically in Fig. 4, and referred to here as a "spiraling beam". The essential feature that defines a spiraling beam is a peak in the distribution at some point in velocity space, here at $v_\perp = v_0 \sin\alpha_0$, $v_\parallel = v_0 \cos\alpha_0$. The distribution illustrated in Fig. 4 is concentrated at small pitch angles, $\sin\alpha_0 \ll \cos\alpha_0$, but in the following discussion we allow both small pitch angles and moderate ones that have $\sin\alpha_0 \sim \cos\alpha_0$.

The relevant resonance ellipse for the spiraling-beam distribution illustrated in Fig. 4 is a semicircle with center at $v_c \approx v_0 \cos\alpha_0$ and radius $v_R \approx v_0 \sin\alpha_0$. According to equation (2), these properties require:

$$\theta \approx (v_0/c)\cos\alpha_0 \quad \text{and} \quad 2(\omega-\Omega_e)/\Omega_e \approx \cos^2\theta.$$

If the pitch angle is small, this case is closely related to the original case of Wu and Lee (1979), in the sense that the two may be related by a Lorentz transformation along the field lines by a velocity of order $v_0 \cos\alpha_0$. Hence, all the qualitative properties of this case may be inferred by applying the Lorentz transformation to known results. For larger values of α_0, relativistic effects are larger, and the ratio of the growth rate at the second harmonic to that at the fundamental decreases. However, to a first approximation existing results derived for nonrelativistic electrons are likely to be adequate.

In summary, of the three possibilities considered for production of ECME at oblique angles, the last one seems to hold the greatest promise. In the following we develop further the possibility of spiraling-beam distributions.

4. Acceleration of the electrons

In this section, possible ways are considered in which a spiraling-beam distribution function with small pitch angles, as illustrated schematically in Fig. 4, might be produced in the Jovian magnetosphere. In the following discussion, positive α and θ correspond to upwardly (away from Jupiter) propagating electrons and waves, respectively. Observations show that Jovian S-bursts drift from high to low frequency, indicating that the electrons that drive the ECME are probably propagating upwards. One possibility is that the electrons are accelerated near the magnetic equator and mirror near the surface of Jupiter, and a second possibility is that the electrons are accelerated upwards near the surface of Jupiter.

Acceleration near Io

First, let us assume that the electrons are accelerated near the orbit of Io (at $r = 6R_J$). For simplicity, we assume that the Jovian magnetic field is dipolar, and more simply that it varies with radius as $B \propto r^{-3}$. The observed frequency then implies the height of the emission, $r = r_0$, which is typically in the range $1R_J < r_0 < 2R_J$. Letting α_0 be the pitch angle of the fast electrons at height r_0, then anywhere else along their orbit, their pitch angle is determined by $\sin^2\alpha/B =$ constant. At the orbit of Io, one has:

$$\sin\alpha_{\mathrm{Io}} = (r_0/6R_J)^{3/2}\sin\alpha_0. \tag{8}$$

Then a source height in the range $1R_J < r_0 < 2R_J$ implies that, at Io, the pitch angle would be in the range $\alpha_0/15 \lesssim \alpha_{\mathrm{Io}} \lesssim \alpha_0/5$. A beam with a moderate pitch angle, $\alpha_0 \sim 60°$, at the height of emission then has a small pitch angle at Io, $4° \lesssim \alpha_{\mathrm{Io}} \lesssim 12°$. For a beam with smaller pitch angles at the height of emission, α_{Io} is correspondingly smaller. In the following we adopt $\alpha_{\mathrm{Io}} = 5°$ for the sake of discussion.

These arguments imply that a distribution of energetic electrons with small pitch angles is required at the orbit of Io. Such a distribution could be produced through acceleration by a parallel electric field. Suppose, for example, that there is a quasithermal distribution of electrons trapped near the orbit of Io with characteristic momentum p'. Acceleration by a potential drop ϕ gives each electron a parallel energy $p_\parallel^2/2m = e\phi$. In this simple model one has:

$$\sin\alpha_{\mathrm{Io}} = p'/(p'^2+2me\phi)^{1/2}. \tag{9}$$

Given that the initial pitch angle α_{Io} must be small, it follows that the initial energy of the electrons must be dominated by that gained from the potential drop. For example, if the final energy is 60 keV, then with $\alpha_{\mathrm{Io}} = 5°$, the initial energy before acceleration should be ~ 100 eV, corresponding to a temperature ~ 10^6K.

The maximum energy available to an electron in this model is determined by the potential drop. The potential available due to the motion of Io through the Jovian magnetosphere (e.g. Goldreich and Lynden-Bell, 1969) is estimated to be $\phi \approx 400$ kV (Hill *et al.*, 1983). This is somewhat higher than the mildly relativistic energy required to account for emission at oblique angles, and hence for a relatively small eccentricity of the polarization ellipse. However, the details of how this acceleration occurs are not well understood. (The energy provided to the electrons comes ultimately from the rotational energy of the Io–Jupiter system, but this energy loss has a negligible effect on the system itself.) In the original model of Goldreich and Lynden-Bell (1969) a quasistatic potential drop was invoked for the acceleration of electrons, cf. also Smith and Goertz (1978), but the identification of the Io plasma torus implied that an important assumption in this model was invalid. Specifically, the time for return propagation from Io to Jupiter is much longer than the time for Io to move through a distance equal to its diameter (e.g. Gurnett and Goertz, 1981). An alternative theory involves acceleration by Alfvén waves generated by the passage of Io (e.g. Neubauer, 1980), but this appears capable of acceleration only to several keV (Goldstein and

Goertz, 1983). The energy required ($\approx$60 keV) to account for the relatively highly circularly polarized component is intermediate between the values for direct electric field acceleration and acceleration by Alfvén waves.

Acceleration near Jupiter

The distribution illustrated in Fig. 4 might also be produced below the height of emission by a parallel potential drop accelerating electrons upwards. There is evidence that electrons are accelerated upwards away from the terrestrial ionosphere (e.g. Hultqvist *et al.*, 1988). This acceleration is attributed to a downward directed parallel electric field near the ionosphere, e.g. Hultqvist (1988). It is possible that a similar, upward acceleration occurs at the Jovian ionosphere.

A difficulty with this suggestion is that in order to lead to a distribution of the form illustrated in Fig. 4, such an upward-directed electron flux also needs to have a significant perpendicular energy. This is because it is the perpendicular energy that provides the free energy in $p_\perp$-driven ECME. However, a parallel potential drop has no effect on the perpendicular energy, so that the resulting electrons would have small pitch angles, with speed determined by $\frac{1}{2}mv_0^2 \approx e\phi$. Moreover, with $v_\perp$ decreasing as the accelerated electrons propagate upwards into the decreasing magnetic field, the pitch angle α_0 would decrease. Thus, one requires an acceleration process that leads to a significant increase in the perpendicular energy; acceleration by a parallel potential drop alone is inadequate. A model based on electrons accelerated above the height of emission is more favorable because the increase in $p_\perp$ due to downward propagation with $p_\perp^2/B$ = constant provides a natural source of free energy to drive the maser.

In summary, we have identified two forms of distribution function that could plausibly account for the ECME at oblique angles. Both assume $p_\perp$-driven ECME at $\cos\theta \approx (v/c)\cos\alpha_0$. One is a loss-cone distribution, with α_0 being the loss-cone angle, like the distribution invoked by Wu and Lee (1979) for AKR, but extending to higher electron energies. In this case one expects a wide range of speeds, v, and hence a wide range of angles of emission. One must invoke a geometric constraint to exclude all but the inferred angle of emission for each subsource; that is, emission at other angles must not be observable from the Earth. The other distribution is a spiraling-beam distribution. In this case it is assumed that the electrons are accelerated by a parallel potential drop, ϕ, near Io such that one has $v = v_0 = (2me\phi)^{1/2}$. The angle of emission is then related to the value of the potential drop ϕ.

5. Discussion and conclusions

The observation that Jupiter's decametric radio emission is intrinsically elliptically polarized, with different characteristic axial ratios in different subsources, imposes a tight constraint on the interpretation in terms of ECME. For ECME the relation between the axial ratio, T, of the polarization ellipse and the angle of emission, θ, is $T = \cos\theta$. For the more circularly polarized subsources one requires $|\cos\theta| \approx 0.5$, and this value implies angles intermediate within the range $|\cos\theta| \gtrsim 0.5$ where $p_\parallel$-driven ECME can occur (e.g. Melrose, 1973, 1976) and the values $|\cos\theta| \ll 0.5$ relevant to $p_\perp$-driven ECME (e.g. Wu and Lee, 1979; Hewitt *et al.*, 1982). Three possible interpretations of emission at oblique angles are discussed here. (a) The $p_\parallel$-driven version of ECME can account for emission at an angle $\theta \lesssim 60°$, but it cannot readily account for emission in a specific narrow range of angles. It is possible that some geometric property favors growth at a particular angle, but suggested models seem contrived. The main argument against $p_\parallel$-driven ECME, in comparison with the more familiar $p_\perp$-driven ECME, is that the intrinsic values of the growth rate are relatively small. (b) A possible form of ECME, intermediate between the $p_\parallel$-driven and $p_\perp$-driven forms, that favors emission at intermediate angles is discussed, and it is argued that no such form exists. (c) It then follows that one needs to identify a distribution function which leads to $p_\perp$-driven ECME at oblique angles.

For $p_\perp$-driven ECME to occur at intermediate angles, θ, requires free energy in electrons centered at v_0, α_0, with $\cos\theta = \beta_{\parallel 0} = (v_0/c)\cos\alpha_0$. Two possibilities are discussed; both have the free energy in reflected electrons with a one-sided loss-cone anisotropy. One possibility is that there is a range of speeds v in these electrons, in which case one expects ECME over the range of angles determined by the range of v through $\cos\theta = (v/c)\cos\alpha_0$. In this case, to account for the emission seen from an individual Jovian subsource having a characteristic θ, one requires that this be a favored viewing angle. That is, one requires a geometric model in which only emission at the favored angle intersects the Earth. Another possibility is that the distribution is a spiraling beam, with a peak in velocity space at a particular speed, v_0, as well as at a particular pitch angle α_0. If there is a characteristic value of $v_0\cos\alpha_0$ for each subsource, then there is a characteristic angle of emission determined by $\cos\theta = (v_0/c)\cos\alpha_0$ associated with each subsource. A simple model for the production of a spiraling beam involves acceleration of electrons by a parallel potential drop near Io; this model seems capable of accounting for the required properties. The values of v_0, α_0 are then determined by the potential drop, ϕ, the energy of the electrons prior to acceleration, and the height of the emission region.

A model for a source of *S*-bursts with $T \approx 0.5$, and hence $\theta \approx 60°$, that is based on ECME due to a spiraling beam would have the following features.

(1) Electrons are trapped near the orbit of Io with an equivalent temperature of order 10^6K.

(2) These electrons are accelerated by a parallel potential drop of order 60 kV near the orbit of Io. A somewhat lower potential drop is required to account for the emission of subsources with more elliptical polarization, and hence $\theta > 60°$.

(3) The ECME is produced after these electrons mirror near the surface of Jupiter, so that they have an upward-directed loss cone. The model for ECME may be regarded as a generalization of the conventional loss-cone driven ECME to mildly relativistic energies.

Further development of this suggestion requires a detailed modeling of the distribution function, and an extension of existing calculations of ECME to higher energy electrons. One further point that needs to be addressed is whether this mechanism is effective only for impulsive bursts of acceleration, producing bunches of electrons that might account for S-bursts, or whether it can operate in a quasi-continuous manner to account for the other Jovian subsources that do not involve S-bursts. Finally, in a more detailed version of the spiraling-beam model, in which the differences in $T \approx \cos\theta$ from one subsource to another are explained in terms of a difference in the accelerating potential ϕ, one needs to explain how the inferred differences in ϕ arise.

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