Diffusive Shock Acceleration by Multiple Shocks

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Abstract: Diffusive shock acceleration produces a power law momentum distribution $f(p) \alpha\ p^{-b}$, with $b \geq 4$ for a single shock, and $b = 4$ for a single strong shock. It has been shown that the distribution for acceleration at a sequence of identical shocks is flatter, approaching $f(p) \alpha\ p^{-3}$ below a high energy knee, for an arbitrarily large number of shocks. We show how this flatter distribution arises and discuss the range of momenta over which it extends after a finite number of shocks.

1. Introduction

Diffusive shock acceleration is the favoured mechanism for the acceleration of galactic cosmic rays, and of relativistic electrons in some, and maybe most synchrotron sources. This mechanism was proposed independently by several sets of authors at about the same time (Krimsky 1977, Axford, Leer and Skadron 1977, Bell 1978a,b, Blandford and Ostriker 1978), and has been reviewed widely (e.g., Axford 1980, Toptyghin 1980, Drury 1983, Scholer 1985, Melrose 1986, p. 247, Blandford and Eichler 1987). Particular attention has been given to such acceleration at shocks due to supernovae (e.g., Blandford and Ostriker 1980; Moraal and Axford 1983; Bogdan and Völk 1983). A favorable feature of the mechanism is that it implies a power law spectrum similar to the form observed for cosmic rays and to that inferred for synchrotron sources (after correction for interstellar propagation effects). In terms of the momentum distribution function, this is $f(p) \alpha\ p^{-b}$, where p is the momentum and b is the power-law index; one has $b \approx 4.6$ for galactic cosmic rays, and $b = 2\alpha + 3$ for electrons in a synchrotron source with a spectrum $I_\nu\ \alpha\ \nu^{-\alpha}$. The theory implies $b = 3r/(r-1)$, where r is the compression ratio of the shock, which has a maximum value $r = 4$ for strong shocks. Hence the flattest distribution that can be produced by a single shock has $b = 4$. There is no difficulty, in principle, in accounting for spectra steeper than this (e.g., due to shocks with $r < 4$) through steepening due to synchrotron losses, and so on.

It is known that acceleration at a sequence of shocks produces a distribution that is flatter than can be produced by a single shock; specifically, the distribution approaches $f(p)\ \alpha\ p^{-3}$ after an arbitrarily large number of shocks (e.g., White 1985, Achterberg 1990). Such a distribution of relativistic electrons would produce a synchrotron spectrum with $\alpha = 0$, that is, a flat synchrotron spectrum. The model leading to this distribution involves quite specific assumptions. These include: (i) all shocks are identical, that is, all have the same value of r, (ii) decompression occurs between each shock, and (iii) at each shock there is an injection of new particles. Our purpose in this paper is to show in detail how this model leads to a distribution that is flatter than the distribution that results from a single shock. The inclusion of decompression is essential, and the method used by White (1985) is incorrect; the method used here is based on Liouville's theorem.

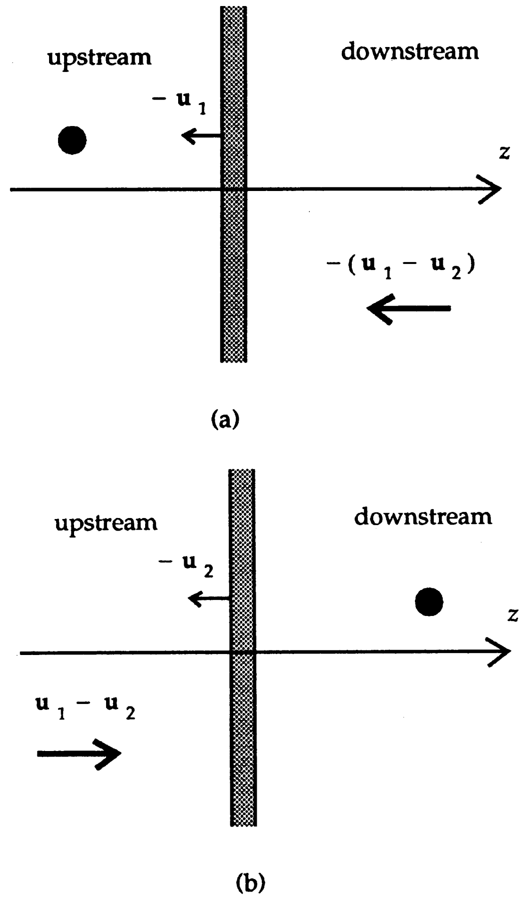

Figure 1: (a) A particle, denoted by the black circle, at rest in the upstream plasma sees the downstream plasma behind the shock approaching at $u_1 - u_2$. (b) As for (a) but for a particle at rest in the downstream plasma.

2. The Asymptotic Distribution

Diffusive shock acceleration results from the continual crossing of a shock by a fast particle that scatters off scattering centers (Alfvén waves) at rest in the plasma on either side of the shock. Let u_1 and u_2 be the flow speeds of the upstream (unshocked) and downstream (shocked) plasmas, respectively. (For a parallel shock with compression ratio r one has $u_2 = u_1/r$.) As illustrated in Figure 1, on diffusing across the shock, a particle always encounters the scattering centers approaching head-on, and so it gains energy on reflecting off them. This is sometimes called first order Fermi acceleration.

The theory of diffusive shock acceleration implies the following form for the downstream distribution function, $f_+(p)$, in terms of the the upstream distribution function, $f_+(p)$:

$$f_+(p) = bp^{-b} \int_0^p p' p'^{(b-1)} f_-(p'), \quad b = \frac{3u_1}{u_2 - u_1} = \frac{3r}{r-1} \qquad (1)$$

Thus, if the shock propagates through a region where the electron distribution is $f_-(p)$, then, after the shock passes, this distribution is transformed into $f_+(p)$. The distribution $f_-(p)$ acts as an injection distribution in this case. Note that there are two qualitatively different sources of particles for the shock acceleration. One is the upstream distribution $f_-(p)$. The other is a distribution of suprathermal particles, denoted $\phi_0(p)$ here, that is created at the shock from the thermal particles.

Consider a sequence of identical shocks. At each shock a new distribution of particles is injected and accelerated, and the particles injected at earlier shocks are accelerated further. To be more specific, we make the following assumptions:

(a) At each shock newly created suprathermal electrons are injected with a monoenergetic distribution $\phi_0(p) \propto \delta(p - p_0)$.

(b) The distribution downstream of the first shock, denoted $f_1(p)$, then follows from (1):

$$f_1(p) = bp^{-b} \int_0^p dq\, q^{b-1}\, \phi_0(q) \qquad (2)$$

(c) All particles accelerated at the first shock are accelerated again when the second shock arrives. Hence the distribution downstream of the second shock includes the particles injected and accelerated at the first shock and reaccelerated at the second shock, plus the particles injected and accelerated at the second shock.

(d) Between each shock a decompression occurs. White (1985) treated this effect incorrectly. The following treatment is based on Liouville's theorem (P. Schneider 1992, private communication). According to Liouville's theorem, f is a constant along the trajectory of a particle in phase space. Let the compression ratio at the shock be written $r = R^{-3}$, so that, during decompression, p changes to $p' = Rp$. Then Liouville's theorem implies that the distribution function, $f'_1(p')$, after decompression is equal to $f_1(p)$ before decompression, and then $p' = Rp$ implies $f'_1(p') = f_1(p/R)$. Hence the injection distribution into the second shock is not $\phi_0(p) + f_1(p)$, as given by (2), but $\phi_0(p) + f'_1(p')$, with

$$f'_1(p) = b(p/R)^{-b} \int_0^{p/R} dq\, q^{b-1}\, \phi_0(q) \qquad (3)$$

(e) The distribution accelerated at the second shock is decompressed, giving a distribution $f'_2(p)$ just before the third shock arrives, and so on for subsequent shocks.

(f) The injection distribution at each shock is assumed to be $\phi_0(p) + K\delta(p - p_0)$, where K and p_0 are constants.

The distribution injected into the *nth* shock is the injection distribution, $\phi_0(p)$, plus the distributions injected at the n'th shock, and processed through the $(n' + 1)$th to the $(n - 1)$th shocks, for all $n' < n$. The contribution from the injection at the first shock to the distribution downstream of the ith shock (after decompression downstream of the preceding shock) is

$$f'_i(p, p_0) = \frac{Kb^i}{p_0} \left(\frac{p}{R^i P_0}\right)^{-b} \frac{(\ln p/R^i p_0)^{i-1}}{(i-1)!} \qquad (4)$$

The total distribution downstream of the nth shock (after decompression) is

$$f'_n(p) = \sum_{i=1}^{n} f'_i(p, p_0). \qquad (5)$$

Summing over an infinite number of shocks gives

$$f_\infty(p) \propto p^{-3}, \qquad (6)$$

which is the distribution found by White (1985), who used $N(p) \propto p^2 f(p)$ and by Achterberg (1990).

To derive (6), one first writes $f_\infty(p) = p^{-b} S(p)$ and shows that $S(p)$ satisfies the differential equation $p dS(p)/dp = (b - 3)R^{b-3} S(p/R)$. Then one notes that a power law is a solution of this equation and that the only power law index that is acceptable gives $S(p) \propto p^{b-3}$.

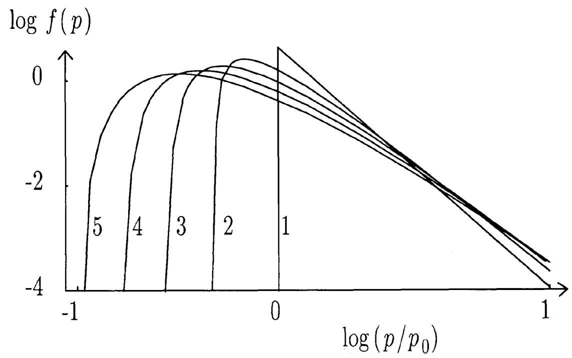

Figure 2: The distribution function of a fixed group of particles injected into the first shock with $p = p_0$ is shown immediately downstream of the first and subsequent shocks up to the fifth; the numbers 1 to 5 label the shock that has just passed.

3. Evolution of the Distribution Function

We have performed numerical calculations for acceleration at five identical, strong shocks. In Figure 2, the solution for the distribution of particles injected at the first shock is shown immediately downstream of the first through the fifth shocks. The curve labeled 1 is the distribution immediately downstream of the first shock. It is a power law $f(p) \propto p^{-4}$ above the injection momentum $p = p_0$, and is zero at $p < p_0$ because no particle loses energy as the shock passes. In between the first and second shocks adiabatic decompression occurs, so that the momentum of every particle decreases according to the scaling $p \to Rp$. When these particles encounter the second shock, their distribution is of the same shape as the distribution labeled 1, but is shifted down so that the cutoff is at $p = Rp_0$. When this distribution is taken as the injection distribution into the second shock, the distribution labeled 2 results immediately downstream of that shock. (Note that we have not included the additional particles injected at the second shock.) The adiabatic decompression and further reacceleration is repeated at subsequent shocks to obtain the distributions labeled 3—5. At each subsequent shock, the peak in the distribution becomes broader, with the maximum moving to smaller p, and with the value of p where the nth curve crosses the $(n - 1)$th curve increasing with increasing n.

In figure 3 we show the sum of the distributions that result from injection at the first to the fifth shocks: the distribution labeled b is the sum of the distributions labeled 1–5 in figure 2. In this case the distribution at $p < p_0$ is not shown. The distribution labeled a, which is included for comparison, is distribution 1 in figure 2 multiplied by five. This is the distribution that would result in the absence of reacceleration at shocks subsequent to that at which the particles are injected. The difference between the two distributions in figure 3 demonstrates the notable qualitative effect of reacceleration at subsequent shocks: it causes the distribution to flatten. This corresponds to a hardening of the

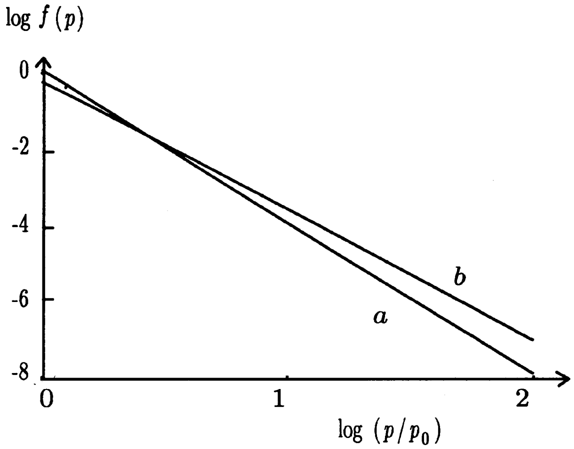

Figure 3: The distribution *(b)* immediately after the fifth shock, summed over the particles injected at shocks 1 to 5, is compared with the distribution *(a)*, which is five times the distribution immediately after the first shock.

energy spectrum, and to a flattening of any resulting synchrotron spectrum.

The summed distribution b in figure 3 is close to the asymptotic form $f(p)\ \alpha\ p^{-3}$ over the limited range shown in figure 3. The distribution after n shocks may be approximated by the power-law form, $\alpha\, p^{-3}$ at low momenta $p > p_0$, steepening to $\alpha\, p^{-b}$ (with $b = 4$ here) at high momenta. The p^{-3} distribution is the sum of the contributions from the injections at different shocks, and at the highest momenta, the largest contribution comes from the broadest of these contributing distributions, which is that injected at the first shock. Before decompression, the relevant contribution is $f_n(p, p_0)$, cf. (4), whose logarithmic derivative may be used to estimate the slope at the highest momenta:

$$\frac{\partial[\ln f_n\,(p,p_0)]}{\partial \ln p} = -b + \frac{n-1}{\ln(p/R^{n-1}p_0)}\ . \qquad (7)$$

It follows that the distribution $\alpha\, p^{-3}$ steepens only slowly toward $\alpha\, p^{-b}$. The slope is $-b + \delta$ at

$$\log(p/p_0) = (n-1)(0.434\delta^{-1} - 0.201). \qquad (8)$$

For example, this implies that the distribution is steeper than $\alpha\, p^{-3.5}$, corresponding to $\delta = 0.5$, for $\log(p/p_0) > 2.67$ after five shocks. This may be seen in figure 3 where the approach of the slope of distribution b to the slope of distribution a at high energies is almost imperceptible.

4. Discussion and Conclusions

Acceleration by multiple shocks is interesting in that it contrasts with a suggestion that is sometimes made: that the hardest distribution that can be produced by diffusive shock acceleration is a power law with $b = 4$, corresponding to the strongest shock, that is, $r = 4$ in (1). An explanation for this is that although the distribution resulting from a δ-function injection at $p = p_0$ at one shock is a power law with $b \geq 4$ for $p > p_0$, when such a power law distribution is subject to reacceleration at the next shock, it produces a distribution that is zero at $p = p_0$, rises to a peak, and then falls off only very slowly toward the power law $f(p)\ \alpha\ p^{-b}$.

The possibility of shock acceleration producing distribution functions of electrons that are flatter than $f(p)\ \alpha\ p^{-4}$ is of interest in connection with synchrotron sources with flat spectra. A synchrotron spectrum $I_\nu\ \alpha\ \nu^{-\alpha}$ with $\alpha = 0.5$ corresponds to an electron distribution function with $b = 4$. Some sources have synchrotron spectra with $\alpha < 0.5$ and it is of interest to know whether acceleration at multiple shocks might provide a realistic explanation for such spectra. The model investigated here is very specific, with all shocks being identical, and with identical injection distributions of new particles at each shock. We are investigating models in which these assumptions are relaxed to investigate the conditions that favor the formation of the distribution $\alpha\ p^{-3}$, which corresponds to a flat synchrotron spectrum, $\alpha = 0$.

Acknowledgments

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