THREE-WAVE TURBULENCE IN PULSAR WINDS *

QINGHUAN LUO and D. B. MELROSE
Research Centre for Theoretical Astrophysics, School of Physics, University of Sydney, NSW 2006, Australia

(Received 14 December, 1993; in revised form 6 May, 1994)

Abstract. Three-wave interactions can modify the very bright radio waves propagating through dense, magnetized pulsar winds and prevent radiation from escaping, or severely alter the resulting spectrum. Absence of the signatures of such effects in the very bright radio emission from pulsars implies constraints on the structure of their winds, especially for a spherically-symmetric wind which should block the radiation unless that Lorentz factor of the pairs is $\gg 8 \times 10^2$.

1. Introduction

Nonlinear scattering of radio waves becomes of increasing importance with increasing brightness temperature of the radiation. Pulsars have very high brightness temperatures ($T_B \gtrsim 10^{25}$ K), well over ten orders of magnitude brighter than for most other bright radioastronomical sources. As a consequence, seemingly exotic nonlinear scattering effects can be significant for pulsars. One such effect is induced Compton scattering. Wilson and Rees (1978) considered such scattering by relativistic electrons in the wind of the Crab pulsar, and concluded that it should have observable effects on the pulsar radio spectrum. These authors argued that the absence of the predicted effects implies a constraint on the wind parameters; they concluded that the Lorentz factor in the wind must be much higher than simple theory suggested. Induced Compton scattering in this and related contexts has been discussed more recently by Coppi, Blandford, and Rees (1993) and by Sincell and Krolik (1992), who included the effect of an ambient magnetic field. Another nonlinear scattering effect was discussed by Gedalin and Eichler (1993), motivated by evidence that a seemingly (radio) transparent stellar wind can occult pulsar radio emission propagating through it (e.g., Fruchter and Goss, 1992; Thompson *et al.*, 1994). The nonlinear scattering process involves the radio beam associated with the pulsar emission generating Langmuir turbulence in the wind. This turbulence can scatter the radio waves, destroying the beam, and distorting the radio spectrum. In this sense its effects are analogous to those of induced Compton scattering (Melrose, 1994). Thus nonlinear scattering effects can be important for pulsars in two ways: on the one hand, the absence of predicted signatures can imply constraints on the wind parameters and, on the other hand, the scattering may prevent pulsar radiation from reaching the observer under conditions where the intervening medium would otherwise be regarded as transparent.

* Presented at the 'Cosmic Winds and the Heliosphere' conference in Tucson, Arizona, U.S.A., October 18–22, 1993.

Solar Physics **154:** 187–198, 1994.

In this paper we discuss nonlinear scattering processes involving Langmuir waves in a pulsar wind, concentrating on the effect of the magnetic field. The particular magnetic effect of interest to us is the cyclotron resonance. This resonance occurs in the wind zone for typical radio pulsars. The cyclotron resonance has been invoked to explain the circular polarization of some radio pulsars, due to preferential cyclotron damping of waves of one circular polarization compared with the other when the distribution of electrons is assumed to be different from that for positrons (Kazbegi, Machebeli, and Melikidze, 1991). There is also a resonance in the scattering cross-section at the cyclotron frequency. Here we explore some of the implications of this resonance on the nonlinear scattering processes in the pulsar wind. In particular, the growth rate for Langmuir waves due to a radio beam can be enhanced above the value expected in the absence of a magnetic field. The presence of the cyclotron resonance implies that constraints deduced on the parameters of pulsar winds are more severe when this resonance is taken into account.

For simplicity, we consider an idealized model of the pulsar wind in which there are relativistic electrons and positrons moving with the same bulk velocity, and such that the spread in Lorentz factors in the rest frame of the wind is small. We are interested in the effect of nonlinear scattering in this wind on the pulsar radio emission propagating through it. With the assumption that the wind is cold, the nonlinear processes may be treated in the rest frame of the wind using a standard nonrelativistic theory for the three-wave processes. This assumption may overemphasize the importance of the cyclotron resonance: a spread in Lorentz factors in the rest frame of the wind implies a spread in cyclotron frequencies, which tends to wash out the resonance. Nevertheless, our results should indicate the qualitative nature of the modifications that the magnetic field introduces.

In Section 2 the theory of photon-induced plasma instability (Gedalin and Eichler, 1993; Melrose, 1994) is generalized to take account of the cyclotron resonance. This instability involves growth of Langmuir waves due to a beam of photons, and is closely analogous to the familiar bump-in-tail instability in which a beam of fast electrons causes Langmuir waves to grow. In Section 3 we comment briefly on the effect of the cyclotron resonance on induced photon decay, which involves generation of Langmuir waves by the photons due to a spontaneous-emission-like process. The effect of the scattering near the cyclotron resonance on the photon beam is discussed in Section 4. The results are applied to pulsars in Section 5.

2. Photon-Induced Plasma Instability

To treat three-wave processes we use a semi-classical description in which waves are described by wave quanta with energy $\hbar\omega(\mathbf{k})$, momentum $\hbar\mathbf{k}$, and occupation number N^σ, where σ labels the wave mode. The processes of wave-wave interaction considered here are the three-wave interaction, i.e., $\sigma \leftrightarrow \sigma' + \sigma''$ where σ, σ', σ'' label three modes with frequencies and wave vectors $(\omega,\, \mathbf{k})$, $(\omega',\, \mathbf{k}')$, and

(ω'', $\mathbf{k}''$), respectively. The occupation numbers are N^σ, $N^{\sigma'}$, and $N^{\sigma''}$, respectively. One can write down kinetic equations for these occupation numbers by appealing to detailed balance (e.g., Melrose, 1986).

2.1. Absorption and Diffusion Coefficients

In the following we consider three-wave interactions involving high-frequency waves scattered by a low-frequency wave, $\omega \approx \omega' \gg \omega''$ and $k \approx k' \gg k''$. An important simplifying assumption made here is the differential approximation (cf. Appendix) in which the low-frequency wave (σ'') is emitted and absorbed by the high-frequency wave ($\sigma = \sigma'$). The kinetic equations for the three-wave processes then reduce to kinetic equation closely analogous to those of electrons interactions with Langmuir waves. A beam of photons can cause the Langmuir waves to grow, analogous to the familiar beam instabilities for electrons generating Langmuir waves. The Langmuir waves also scatter the photons, tending to destroy the beam and the flatten the photon spectrum, closely analogous to the effect of Langmuir turbulence on a beam of electrons.

In the differential approximation, quasi-linear interaction between a high-frequency wave and a low-frequency wave is described by absorption and diffusion coefficients which are defined respectively by (Melrose, 1994; cf. Appendix)

$$\Gamma^{\sigma''} = -\int \frac{d\mathbf{k}}{2\pi)^3} w(\mathbf{k}, \mathbf{k}'')\mathbf{k}'' \cdot \frac{\partial N^\sigma}{\partial \mathbf{k}} \tag{1a}$$

and

$$D_{ij} = \int \frac{d\mathbf{k}''}{(2\pi)^3} w(\mathbf{k}, \mathbf{k}'') k_i'' k_j'' N^{\sigma''} , \tag{1b}$$

with $w(\mathbf{k}, \mathbf{k}'')$ the probability for emission of a low-frequency wave (σ'') by the high-frequency photon (σ). This probability is given by

$$w(\mathbf{k}, \mathbf{k}'') = \frac{4\hbar R^{\sigma''}_{\mathbf{k}''} R^\sigma_{\mathbf{k}} R^\sigma_{\mathbf{k}\pm\mathbf{k}''} \, |\alpha^{\sigma\sigma\sigma''}|^2}{\varepsilon_0^3 \, |\omega^2 \omega''|} 2\pi\delta(\omega'' - \mathbf{k}'' \cdot \mathbf{v}_g) , \tag{2}$$

with $\alpha^{\sigma\sigma\sigma''} = e_i^* e_j' e_s'' \alpha_{ijs}$ where α_{ijs} is the quadratic response tensor (Melrose, 1986), and where subscripts i, j, s run over three components of the Cartesian coordinates with the third axis along the magnetic field in the rest frame (the summation over the three components is implied whenever the subscript is repeated). The (unimodular) polarization vectors of the waves, σ, σ', and σ'', are $\mathbf{e}$, $\mathbf{e}'$, and $\mathbf{e}''$, respectively. The quantities $R^{\sigma''}_{\mathbf{k}''}$ and $R^\sigma_{\mathbf{k}}$ are the ratios of the electric to total energy in waves σ'', σ, respectively. In (2), $\mathbf{v}_g = \partial\omega/\partial\mathbf{k}$ is the group velocity of high-frequency wave σ. The condition $\omega'' - \mathbf{k}'' \cdot \mathbf{v}_g = 0$ implied by the δ-function in (2) is derived from the frequency-matching condition for the three-wave interaction:

$$\omega(\mathbf{k}) = \omega(\mathbf{k} \mp \mathbf{k}'') \pm \omega''(\mathbf{k}'') \approx \omega(\mathbf{k}) \mp \mathbf{k}'' \cdot \frac{\partial \omega(\mathbf{k})}{\partial \mathbf{k}} \pm \omega''(\mathbf{k}'') \,, \tag{3}$$

where terms of the order higher in k''/k are neglected.

When the low-frequency waves are Langmuir waves ($\sigma'' = l$), three-wave processes imply that an anisotropic beam of photons can generate Langmuir waves provided that Γ^l is negative in some region of $\mathbf{k}''$. The Langmuir waves can then scatter the photon beam, which is referred to as induced Raman scattering.

2.2. Cold Plasma Approximation

To examine the absorption of the low-frequency waves by the high-frequency waves requires an explicit evaluation of the absorption coefficient (1a). An important ingredient in (1a) and (1b) is the probability $w(\mathbf{k}, \mathbf{k}'')$, which involves the quadratic response tensor (Melrose, 1986). Simplification to the quadratic response tensor needs to be made to derive useful results. We assume that the plasma in the pulsar wind may be treated as cold in its rest frame. This requires that there be only a small spread in Lorentz factors in this frame. The cold plasma approximation (to the nonlinear response tensor) should be a reasonable approximation provided that the random motions of the particles are not highly relativistic in the rest frame, and provided that the phase speed of the all waves (including the Langmuir waves) is much greater than the typical speeds of the particles.

A cold magnetized plasma can be described conveniently by two parameters, the plasma frequency $\omega_p = (e^2 n_e/\varepsilon_0 m_e)^{1/2}$ (where n_e is the proper electron density, m_e is the rest mass of the electron), and the cyclotron frequency $\omega_B = |e|B'/m_e$ (where B' is the magnetic field in the rest frame). We consider waves propagating approximately along the field line in the rest frame, referred to as the quasi-radial limit by Sincell and Krolik (1992). In this case, transverse waves are circularly polarized with opposite handedness and there also exists a Langmuir wave with frequency $\omega'' \approx \omega_p$.

In the cold plasma approximation and in the rest frame, the quadratic response tensor is given by (Melrose, 1986)

$$\begin{aligned}\alpha_{ijs} = -\sum \frac{e^3 n_e}{2m_e^2} \bigg[& \frac{k_r}{\omega'}\tau_{rj}(\omega')\tau_{is}(\omega'') + \frac{k_r}{\omega''}\tau_{rs}(\omega'')\tau_{ij}(\omega') + \\ & + \frac{k_r'}{\omega}\tau_{ir}(\omega)\tau_{js}(\omega'') + \frac{k_r''}{\omega}\tau_{ir}(\omega)\tau_{sj}(\omega') - \\ & - \frac{k_r''}{\omega'}\tau_{rj}(\omega')\tau_{is}(\omega) - \frac{k_r'}{\omega''}\tau_{rs}(\omega'')\tau_{ij}(\omega) \bigg] \,, \end{aligned} \tag{4}$$

where the summation is made over the particle species: electrons and positrons. In (4) and throughout the following discussion, except where stated, all quantities are referred to the rest frame. The tensor τ_{ij} is defined by

$$\tau_{ij} = \begin{pmatrix} \dfrac{\omega^2}{\omega^2-\omega_B^2} & \dfrac{i\eta\omega\omega_B}{\omega^2-\omega_B^2} & 0 \\ \dfrac{-i\eta\omega\omega_B}{\omega^2-\omega_B^2} & \dfrac{\omega^2}{\omega^2-\omega_B^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{5}$$

with η the sign of the charge. The denominator $\omega^2-\omega_B^2$ in (5) implies a resonance in $\tau_{ij}(\omega)$ at $\omega=\omega_B$. Hence, α_{ijs} as given by (4) diverges at $\omega=\omega_B, \omega'=\omega_B$ or $\omega''=\omega_B$. Near any of these resonances, the dominant terms in the quadratic response tensor are the divergent terms $\sim 1/(\omega^2-\omega_B^2)$ or $1/(\omega'^2-\omega_B^2)$, or $1/(\omega''^2-\omega_B^2)$. Consider the resonances at $\omega\approx\omega'\approx\omega_B\gg\omega''$ with $\omega_p^2/\omega<|\omega-\omega_p|<\omega_p$. Similar to the induced Compton scattering in which the resonance scattering affects preferentially one sense of circular polarization (e.g., Blandford and Scharlemann, 1976), the probability (2) for one sense of circular polarization is different from that for the other. When assuming equal numbers of electrons and the positrons, both senses of circular polarization can be scattered equally since the refractive index in this case reduces to $n_\sigma^2 = 1-2\omega_p^2/(\omega^2-\omega_B^2)$. The refractive index n_σ also has a resonance at $\omega\approx\omega_B$, and this affects the quantities R^σ since $R^\sigma\approx 1/[2n_\sigma\,\partial(\omega n_\sigma)/\partial\omega]$. Near this resonance one has $n_\sigma\,\partial(\omega n_\sigma)\,\partial\omega\approx\omega_p^2/(\omega-\omega_B)^2$ for $\omega\approx\omega_B$ (e.g., Melrose and Sy, 1972). The factor $(\omega-\omega_B)^2$ in R^σ cancels $1/(\omega-\omega_B)^2$ in $|\alpha^{\sigma\sigma\sigma''}|^2$ (with $\sigma''=l$). Hence, near the cyclotron resonance $\omega\approx\omega_B$, using (4) and (5) one has

$$R^\sigma\,|\alpha^{\sigma\sigma l}|^2 \approx \frac{e^6 n_e^2}{4m_e^4c^2}\left(\frac{\omega_B}{\omega''}\right)^2\left(\frac{\omega}{\omega_p}\right)^2\left(\frac{2\omega'}{\omega+\omega_B}\right)^2 . \tag{6}$$

Using (6), one evaluates the probability $w(\mathbf{k},\mathbf{k}'')$ for $\omega=\omega_B$, that is, near the cyclotron resonance.

2.3. Photon Induced Plasma Emission

A photon beam traversing the plasma can generate Langmuir waves in a manner closely analogous to the generation of Langmuir waves by a beam of electrons (Melrose, 1994). To see this here, note that in (1a), one can separate the derivative on N^σ with respective to k from the angular derivative, and that the angular derivative gives a negative contribution to Γ^l, leading to growth of the Langmuir waves.

Following Melrose (1994), we consider an axisymmetric photon beam whose photon occupation number is of the form

$$N(\mathbf{k}) = N(k)b(\theta)\,, \quad b(\theta) = \exp[-\theta^2/2\theta_0^2]\,. \tag{7}$$

Thus the photon beam is approximately confined within an angle θ_0 in the rest frame. In the small angle approximation, using (7) and (1a), one finds the absorption coefficient for the Langmuir waves

$$\Gamma^l = -\int \frac{k^2\,\mathrm{d}k\,2\theta\,\mathrm{d}\theta}{(2\pi)^2}\frac{F(\omega,\,\omega_p)}{k''c\sin\chi_0[\theta^2-(\theta''-\chi_0)^2]^{1/2}}\times$$
$$\times\left\{k''\frac{\partial}{\partial k}+\frac{k''(\theta''-\chi_0)}{k\theta}\frac{\partial}{\partial\theta}\right\}N(k)b(\theta)\,, \tag{8}$$

with $\chi_0 = \arccos(\omega_p/k''v_g)$ and where $F(\omega,\,\omega_p)$ is defined by

$$F(\omega,\,\omega_p) = 2\pi r_e c^2\left(\frac{\hbar\omega}{m_e c^2}\right)\left(\frac{\omega}{\omega_p}\right)\,, \tag{9}$$

with $\omega \approx \omega_B$ and where $r_e = e^2/4\pi\varepsilon_0 m_e c^2$ is the classic radius of electron. From (8), negative absorption is possible only for $\theta'' < \chi_0$. This condition requires that the phase speed of the Langmuir waves be close to but smaller than the group speed $v_g \approx c$ of the relevant high-frequency waves, i.e., within the Cerenkov resonance cone. Part of the photon energy is transferred to the low-frequency Langmuir waves, resulting in a Langmuir wave growth. Langmuir waves with k$''$ outside the Cerenkov resonance cone are damped. This is closely analogous to the beam-plasma instabilities in which Langmuir waves can be amplified only when their phase speed is less than the particle speed.

Substituting (7) into (8) and using (9), the growth rate for the Langmuir waves ($\omega'' \approx \omega_p$) in the rest frame is estimated to be

$$|\Gamma^l| \approx \frac{1}{2\sqrt{\pi}}\left(\frac{r_e\omega^2}{c}\right)\left(\frac{\omega}{\omega_p}\right)\left(\frac{\kappa T_B}{m_e c^2}\right)\,, \tag{10}$$

where ω is close to the cyclotron frequency, and where T_B is the brightness temperature of the high-frequency photon beam and κ is the Boltzmann constant. In deriving (10), the approximation $k'' \approx \omega_p/c$ is used. Since the right-hand side of (9) is larger than that for an unmagnetized plasma by a factor $(\omega/\omega_p)^4$, the growth rate (10) is much larger than in the unmagnetized case (Melrose, 1994) for $\omega_B \gg \omega_p$.

3. Induced Photon Decay

Langmuir wave turbulence can also be produced through induced photon decay, which is similar to the spontaneous emission of Langmuir waves by electrons. In the induced photon decay, the production rate of Langmuir waves is proportional to the square of the photon occupation number N (cf. Equation (A11)). In analogy to the unmagnetized case (Melrose, 1994), near the cyclotron resonance $\omega \approx \omega_B$, one has (cf. Appendix)

$$\frac{\mathrm{d}N^{l}}{\mathrm{d}t} = \frac{\Delta\Omega}{8\pi}\left(\frac{\kappa T_B}{m_e c^2}\right)\left(\frac{r_e\omega^2}{c}\right)\left(\frac{\omega}{\omega_p}\right)^2 N\,, \tag{11}$$

where we rewrite $dN^l/dt \sim N^2$ into the form of the one power of N using $N = \kappa T_B/\hbar\omega$. In general the process of generating Langmuir waves competes with various damping processes such as Landau damping, collisional damping, and nonlinear damping. Let Γ_d^l represent the damping rate due to a certain damping process. Then the level of Langmuir waves, balanced by this damping process, is given by

$$N^l = \frac{1}{\Gamma_d^l}\left(\frac{\Delta\Omega}{8\pi}\right)\left(\frac{\kappa T_B}{m_e c^2}\right)\left(\frac{r_e\omega^2}{c}\right)\left(\frac{\omega}{\omega_p}\right)^2 N\,, \tag{12}$$

with $\omega \approx \omega_B$ and where $\Delta\Omega$ is the solid angle subtended by the photon beam in the rest frame. The ratio of the energy density of Langmuir waves to the photon energy density is estimated to be $R_{l\sigma} \sim (\kappa T_B/m_e c^2)\,(r_e\omega^2/c)\,(\omega_p/\omega)^2/\Gamma_d^l$. For Landau damping, one has $\Gamma_d^l \sim \exp(-1/2\beta_{\text{th}}^2)/\beta_{\text{th}}^3$ in the rest frame where $\beta_{\text{th}} = V_e/c$ with V_e the thermal speed of the electrons. Assuming a small β_{th} in the pulsar wind, a high level of Langmuir waves is possible (Melrose, 1994).

Unlike the photon-induced plasma instability, in which the anisotropic photon distribution in the rest frame is essential in resulting in Langmuir wave turbulence, the Langmuir waves are produced through induced photon decay even for an isotropic photon distribution. Similar to the unmagnetized case (Melrose, 1994), the production rate (11) can be enhanced when the photon distribution is highly anisotropic since for given number of photons, the highly anisotropic photon beam has a much higher brightness temperature than the isotropic case.

4. The Resonance Scattering

When a sufficient level of Langmuir waves is built up either by the mechanisms discussed in Sections 2 and 3 or by any other mechanism such as the usual two-stream instability, the scattering effect on the photon beam by the Langmuir waves can be important. This can result in both isotropization of the beam and transfer of photons from higher to lower frequencies, distorting the frequency spectrum.

Let W^σ and W^l be the wave energy densities of transverse waves and Langmuir waves, respectively. Substituting (6) into (2) and using (1b), one can estimate the diffusion time scale in the rest frame, viz.,

$$\tau_{\text{diff}} = \left(\frac{D_{ii}}{k^2}\right)^{-1} \approx \frac{c}{r_e\omega^2}\left(\frac{\omega_p}{\omega}\right)\left(\frac{W^\sigma}{W^l}\right)\left(\frac{m_e c^2}{\kappa T_B}\right)\left(\frac{2\pi}{\Delta\Omega}\right), \tag{13}$$

where the photon frequency is assumed to be near the cyclotron resonance $\omega \approx \omega_B$ and where an approximation $k''c \approx \omega'' \approx \omega_p$ is used.

Owing to the saturation condition $W^l/W^\sigma \lesssim 1$, the time scale for Langmuir wave growth, given by $1/\Gamma^l$, is faster than the time scale for the diffusion by a factor $W^\sigma/W^l \gtrsim 1$. Thus the process of generating the Langmuir waves which scatter the photon beam can be sustained.

5. Constraints on Pulsar Wind Models

Let us apply the results derived in the previous sections to pulsar winds. The conventional pulsar wind model may be summarized as follows. (i) Qualitatively there are two regions associated with the pulsar environment, the magnetospheric region well within the light cylinder and the wind region beyond the light cylinder. (ii) Pair production is thought to occur within the light cylinder, e.g., in Ruderman and Sutherland's (1975) model the electron-positron pairs are produced near the stellar surface, and in Arons (1979) model, the pair production occurs continuously up to the light cylinder. (iii) The resulting dense pair plasma flows out through the light cylinder to form a steady, relativistic wind.

For a spherically-symmetric wind model (e.g., Kennel and Coroniti, 1984), the radio emission generated within the light cylinder must propagate through a plasma layer located outside the light cylinder, where the cyclotron resonance is encountered. In the pulsar frame, the location of this resonance region (at the radial distance, r_c, to the star) is estimated to be (Sincell and Krolik, 1992)

$$r_c/r_L \approx \xi \dot{P}_{-14}^{1/2} P^{-5/2}/f_9 \, , \tag{14}$$

with r_L the light cylinder radius, f_9 the frequency in units of 1 GHz, P the period of the pulsar, $\dot{P}_{-14}$ the period derivative in units of 10^{-14}. The parameter ξ was evaluated by Sincell and Krolik (1992) in two limits: the quasi-radial limit, $\xi = \sqrt{\sigma_0}$, and the quasi-toroidal limit, $\xi = \sqrt{\sigma_0/(1+\sigma_0)}$. These limits are determined by whether of the radial or toroidal component of the magnetic field in the rest frame is stronger. The ratio σ_0 depends explicitly on the radial distance to the star. The explicit form of its radial function is model-dependent.

Either the quasi-radial or quasi-toroidal limits can apply depending on the photon frequency (Sincell and Krolik, 1992). This arises from the changeover from a quasi-radial magnetic field nearer the pulsar to a quasi-toroidal field far then from the pulsar, coupled with the fact that the location of the cyclotron resonance, $\omega = \omega_B$, moves outward with decreasing photon frequency. For illustrative purposes, here we assume that the cyclotron resonance occurs in the quasi-radial region.

Consider the possibility of Langmuir wave generation in the pulsar wind. We use subscript 0 to label the quantities in the rest frame. Let γ be the Lorentz factor of the pairs in the pulsar wind seen in the pulsar frame. The time scale for Langmuir wave growth in the pulsar frame is $\tau = \gamma\tau_0$, with $\tau_0 = 1/|\Gamma^l|$. In the small angle approximation the frequency ω, and the electron density n_e in the pulsar frame, are related to the frequency ω_0 and the density n_{0e} in the rest frame by $\omega_0 = \omega\gamma(1 - \mathbf{k}\cdot\mathbf{v}/kc) \approx \omega/2\gamma$ (where $\mathbf{v}$ is the bulk velocity of the plasma) and $n_{0e} = n_e/\gamma$, respectively. The brightness temperature T_{0B} in the rest frame can be related to the photon occupation number N by $\kappa T_{0B} = \hbar\omega_0 N$. Since the photon occupation number is a Lorentz invariant, the brightness temperature is transformed to the pulsar frame in the same way as the frequency. Transforming all relevant quantities in (10) to that in the pulsar frame (note that all quantities in (10) are in the rest frame) and using $\tau = \gamma/|\Gamma^l|$, one has

$$\tau \approx 32\sqrt{\pi}\left(\frac{c}{r_e\omega^2}\right)\left(\frac{\omega_p}{\omega}\right)\left(\frac{m_e c^2}{\kappa T_B}\right)\gamma^{9/2}, \tag{15}$$

where T_B is now the brightness temperature in the pulsar frame and where all relevant frequencies are referred to the pulsar frame. If one takes the thickness Δr_c of a layer of the cyclotron region to be 1% of the light cylinder distance r_L, the variation of cyclotron frequency due to the change of the magnetic field with increasing radial distance is less than $|\omega - \omega'| \approx \omega_p$. The time required for the plasma to travel through Δr_c is $\Delta r_c/c = 10^{-2} r_L/c$. For $T_B \approx 10^{28}$ K, $\omega/2\pi = 1$ GHz, $\omega/\omega_p = 5 \times 10^2$, the Langmuir wave turbulence can be generated by the anisotropic photon beam, provided that the Lorentz factor of the wind satisfies $\gamma < 8 \times 10^2$ (derived by requiring that $\tau < \Delta r_c/c$ with $P = 1$ s).

The diffusion time scale in the pulsar frame is $\tau_{\rm diff} = \gamma\tau_{0\rm diff}$ where we use $\tau_{0\rm diff}$ to redesignate the time scale (13) in the rest frame. For illustrative purposes we take $\Delta\Omega_0 = 0.1$ (the solid angle subtended by the photon beam in the rest frame). Then assuming that Langmuir wave energy density is comparable with the photon energy density, the diffusion time scale is found to be $\tau_{\rm diff} \approx \tau$ with τ given by (15). For the same choice of parameters used to discuss the Langmuir wave generation, the scattering effect on the photon beam by Langmuir waves should be important in the wind plasma with $\gamma \lesssim 8 \times 10^2$.

6. Conclusions and Discussion

We consider the effect of the three-wave interactions on the intense radio emission in the pulsar winds and implied constraint on the parameters of the wind. In particular, we consider the three-wave interactions that involve a photon beam generating Langmuir turbulence and the Langmuir waves scattering the photon beam. The main results can be summarized as follows.

(1) A photon beam can generate Langmuir turbulence analogous to the way that an electron beam generates Langmuir turbulence. This photon-beam-induced Langmuir turbulence can be important in the pulsar wind provided that the Lorentz factors satisfy $\gamma \lesssim 8 \times 10^2$ and provided that the random motion of the electrons (positrons) in the rest frame is not highly relativistic. With these assumptions the cyclotron resonance condition can be satisfied. The growth rate of the Langmuir waves ($\omega'' \approx \omega_p$) is greatly enhanced near the cyclotron resonance ($\omega \approx \omega' \approx \omega_B \gg \omega_p$). We estimate the time scale for the Langmuir wave growth and find it smaller than that for diffusion due to the scattering on the photon beam by the Langmuir waves. Thus the process of Langmuir wave generation and scattering on the photon beam by the Langmuir waves can be sustained.

(2) The Langmuir waves can also be produced through induced photon decay. This process is similar to spontaneous emission of Langmuir waves by electrons. The actual level of the Langmuir waves is balanced by a damping process. A high level of Langmuir turbulence may be obtained provided that the thermal motion of the electrons (positrons) in the wind is not important.

(3) The important consequence of Langmuir wave generation or production is that part of the photon energy is transferred to the Langmuir waves. Thus the photons are transferred from higher to lower frequencies thereby distorting the frequency spectrum. Also, as a result of scattering by Langmuir waves, the photon beam becomes more isotropic.

(4) The fact that we can observe the radiation implies some constraints on the parameters of the pulsar winds. There are two possible ways to avoid strong scattering effects: (a) the Lorentz factors are much larger than 8×10^2 in the scattering region, or (b) the wind structure has such a geometry that does not occult the radiation. The first of these implies a constraint that is qualitatively similar to that deduced by Wilson and Rees (1978) for induced Compton scattering. The second puts a constraint on the spherically-symmetric wind models since for spherical-symmetric wind there is always a layer of cyclotron resonance region (at the distance r_c from the pulsar), and within this resonance region the strong scattering occurs.

Appendix. Quasi-Linear Equations for Three-Wave Interactions

When a three-wave process involves a high-frequency wave scattered by a low-frequency wave, the wave-interaction resembles quasi-linear interaction in a wave-particle. The kinetic equations for a three-wave process can be written in terms of the probability $u^{\sigma\sigma'\sigma''}(\mathrm{k}, \mathrm{k}', \mathrm{k}'')$ of the coalescence process $\sigma + \sigma'' \to \sigma'$ (Melrose, 1986):

$$\frac{\mathrm{d}N^{\sigma}(\mathrm{k})}{\mathrm{d}t} = \int \frac{\mathrm{dk}'}{(2\pi)^3}\frac{\mathrm{dk}''}{(2\pi)^3} u^{\sigma\sigma'\sigma''}(\mathrm{k}, \mathrm{k}', \mathrm{k}'') \times \\ \times \{N^{\sigma'}(\mathrm{k}')N^{\sigma''}(\mathrm{k}'') - N^{\sigma}(\mathrm{k})\,[N^{\sigma'}(\mathrm{k}') + N^{\sigma''}(\mathrm{k}'')]\}\,, \qquad \text{(A1)}$$

$$\frac{\mathrm{d}N^{\sigma'}(\mathrm{k}')}{\mathrm{d}t} = -\int \frac{\mathrm{dk}}{(2\pi)^3}\frac{\mathrm{dk}''}{(2\pi)^3} u^{\sigma\sigma'\sigma''}(\mathrm{k}, \mathrm{k}', \mathrm{k}'') \times \\ \times \{N^{\sigma'}(\mathrm{k}')N^{\sigma''}(\mathrm{k}'') - N^{\sigma}(\mathrm{k})\,[N^{\sigma'}(\mathrm{k}') + N^{\sigma''}(\mathrm{k}'')]\}\,, \qquad \text{(A2)}$$

$$\frac{\mathrm{d}N^{\sigma''}(\mathrm{k}'')}{\mathrm{d}t} = -\int \frac{\mathrm{dk}}{(2\pi)^3}\frac{\mathrm{dk}'}{(2\pi)^3} u^{\sigma\sigma'\sigma''}(\mathrm{k}, \mathrm{k}', \mathrm{k}'') \times \\ \times \{N^{\sigma'}(\mathrm{k}')N^{\sigma''}(\mathrm{k}'') - N^{\sigma}(\mathrm{k})\,[N^{\sigma'}(\mathrm{k}') + N^{\sigma''}(\mathrm{k}'')]\}\,, \qquad \text{(A3)}$$

with

$$u^{\sigma\sigma'\sigma''}(\mathbf{k},\,\mathbf{k}',\,\mathbf{k}'') = \frac{4\hbar R^{\sigma''}_{\mathbf{k}''} R^{\sigma}_{\mathbf{k}} R^{\sigma'}_{\mathbf{k}\pm\mathbf{k}''}\,|\alpha^{\sigma\sigma\sigma''}|^2}{\varepsilon_0^3\,|\omega^2\omega''|}\times$$
$$\times(2\pi)^4\delta(\mathbf{k}-\mathbf{k}'-\mathbf{k}'')\delta[\omega(\mathbf{k})-\omega'(\mathbf{k}')-\omega''(\mathbf{k}'')]\,. \quad \text{(A4)}$$

Assuming $\sigma = \sigma'$, (A1) and (A2) can be combined into a single equation:

$$\frac{\mathrm{d}N^{\sigma}(\mathbf{k})}{\mathrm{d}t} = \int \frac{\mathrm{d}\mathbf{k}'}{(2\pi)^3}\frac{\mathrm{d}\mathbf{k}''}{(2\pi)^3} u^{\sigma\sigma\sigma''}(\mathbf{k},\,\mathbf{k}',\,\mathbf{k}'')\times$$
$$\times\{N^{\sigma}(\mathbf{k}')N^{\sigma''}(\mathbf{k}'') - N^{\sigma}(\mathbf{k})\,[N^{\sigma'}(\mathbf{k}') + N^{\sigma''}(\mathbf{k}'')]\}-$$
$$-u^{\sigma\sigma\sigma''}(\mathbf{k},\,\mathbf{k}',\,-\mathbf{k}'')\{N^{\sigma}(\mathbf{k})N^{\sigma''}(\mathbf{k}'')-$$
$$-N^{\sigma}(\mathbf{k}')\,[N^{\sigma}(\mathbf{k}) + N^{\sigma''}(\mathbf{k}'')]\}]\,. \quad \text{(A5)}$$

In analogy with the discussion by Melrose (1994), one may separate terms on the right-hand sides of Equations (A5) and (A3) into those that are proportional to $N^{\sigma''}$ and those that are independent of it The former terms describe induced emission of σ'' waves (IE), and the terms independent of $N^{\sigma''}$ are called induced photon decay (IPD). Therefore from (A5) and (A3) one has

$$\frac{\mathrm{d}N^{\sigma}(\mathbf{k})}{\mathrm{d}t} = \left.\frac{\mathrm{d}N^{\sigma}(\mathbf{k})}{\mathrm{d}t}\right|_{\mathrm{IE}} + \left.\frac{\mathrm{d}N^{\sigma}(\mathbf{k})}{\mathrm{d}t}\right|_{\mathrm{IPD}}\,, \quad \text{(A6)}$$

$$\frac{\mathrm{d}N^{\sigma''}(\mathbf{k}'')}{\mathrm{d}t} = \left.\frac{\mathrm{d}N^{\sigma''}(\mathbf{k}'')}{\mathrm{d}t}\right|_{\mathrm{IE}} + \left.\frac{\mathrm{d}N^{\sigma''}(\mathbf{k}'')}{\mathrm{d}t}\right|_{\mathrm{IPD}}\,. \quad \text{(A7)}$$

For $\omega \approx \omega' \gg \omega''$, $k \approx k' \gg k''$, one has the following differential approximation:

$$N^{\sigma}(\mathbf{k}') = N^{\sigma}(\mathbf{k}\mp\mathbf{k}'') \approx \left[1 \pm \mathbf{k}''\cdot\frac{\partial}{\partial\mathbf{k}} + \frac{1}{2}\left(\mathbf{k}''\cdot\frac{\partial}{\partial\mathbf{k}}\right)^2\right] N^{\sigma}(\mathbf{k})\,. \quad \text{(A8)}$$

Then from (A5)–(A8), for induced plasma emission, one has the quasi-linear equations

$$\left.\frac{\mathrm{d}N^{\sigma''}(\mathbf{k}'')}{\mathrm{d}t}\right|_{\mathrm{IE}} = -\Gamma^{\sigma''}N^{\sigma''}(\mathbf{k}'')\,, \quad \text{(A9)}$$

$$\left.\frac{\mathrm{d}N^{\sigma}(\mathbf{k})}{\mathrm{d}t}\right|_{\mathrm{IE}} = \frac{\partial}{\partial k_i}\left[D_{ij}(\mathbf{k})\frac{\partial N^{\sigma}(\mathbf{k})}{\partial k_j}\right]\,, \quad \text{(A10)}$$

where the absorption and diffusion coefficients are defined, respectively, by (1a) and (1b). For induced photon decay, one has

$$\left.\frac{\mathrm{d}N^{\sigma''}(\mathbf{k}'')}{\mathrm{d}t}\right|_{\mathrm{IPD}} = \int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^3} w(\mathbf{k},\, \mathbf{k}'')\, [N^{\sigma}(\mathbf{k})]^2\,, \tag{A11}$$

where $w(\mathbf{k},\, \mathbf{k}'')$ is defined by (2). For $\sigma'' = l$, (A11) describes the production of Langmuir waves through induced photon decay, cf. Equation (11).

References

Arons, J.: 1979, *Space Sci. Rev.* **24**, 437.
Blandford, R. D. and Scharlemann, E. T.: 1976, *Monthly Notices Roy. Astron. Soc.* **174**, 59.
Coppi, P., Blandford, R., and Rees, M. J.: 1993, *Monthly Notices Roy. Astron. Soc.* **262**, 603.
Fruchter, A. S. and Goss, W. M.: 1992, *Astrophys. J.* **384**, L47.
Gedalin, M. and Eichler, D.: 1993, *Astrophys. J.* **406**, 629.
Kazbegi, A. Z., Machebeli, G. Z., and Melikedze, G. I.: 1991, *Monthly Notices Roy. Astron. Soc.* **253**, 377.
Kennel, C. F. and Coroniti, F. V.: 1984, *Astrophys. J.* **283**, 694.
Melrose, D. B.: 1986, *Instabilities in Space and Laboratory Plasmas*, Cambridge University Press, Cambridge.
Melrose, D. B.: 1994, *J. Plasma Physics* **51**, 13.
Melrose, D. B. and Sy, W. N.: 1972, *Astrophys. Space Sci.* **17**, 343.
Ruderman, M. A. and Sutherland, P. G.: 1975, *Astrophys. J.* **196**, 51.
Sincell, M. W. and Krolik, J. H.: 1992, *Astrophys. J.* **395**, 553.
Thompson, C., Blandford, R. D., Evans, C. R., and Phinney, E. S.: 1994, *Astrophys. J.* **422**, 304.
Wilson, D. B. and Rees, M. J.: 1978, *Monthly Notices Roy. Astron. Soc.* **185**, 297.